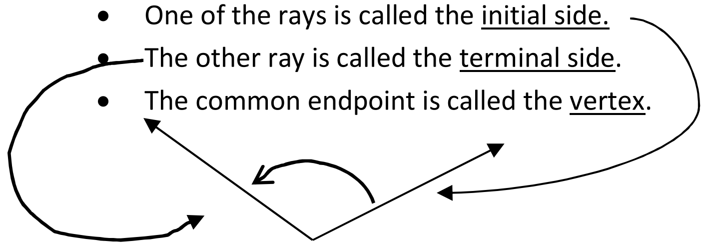
Chapter 4 – Trigonometric Functions

4.1 Angles and Radian Measure

Def: An angle is formed by 2 rays that have a common endpoint.

- One of the rays is called the initial side.
- The other ray is called the terminal side.
- The common endpoint is called the vertex.



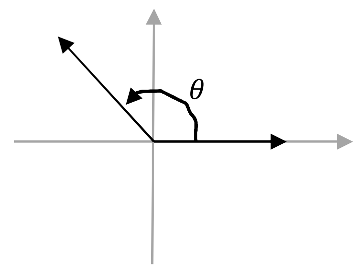
Angles are usually named with 3 points
- One on the initial side
- The vertex
- One on the terminal side

Angles are also labeled using Greek letters.
The most common are: α, β, θ

The arrow near the vertex shows the direction from the initial side to the terminal side.

Def: An angle is in **standard position** if the vertex is at the origin (0,0) and the initial side is the *positive x-axis.*

- Think of the initial side being rotated around the origin to the terminal side.
- CLOCKWISE = NEGATIVE direction
- COUNTER CLOCKWISE = POSITIVE direction

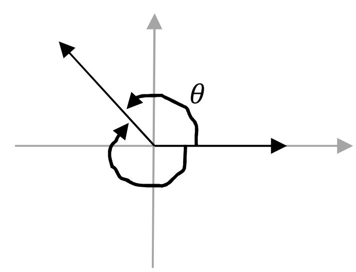


Def: A quadrantal angle is an angle whose terminal side is on an axis.

If the terminal side is:	Counter Clockwise	Clockwise
Positive x-axis:	0 or 360	0 or –360
Positive y-axis:	90	–270
Negative x-axis:	180	–180
Negative y-axis:	270	–90

Def: Two angles, in standard position, are coterminal if they share the same terminal side but have different measures.

Ex. 215° and -145° (sum of abs. value=360) | 30° and 390° (difference = 360)

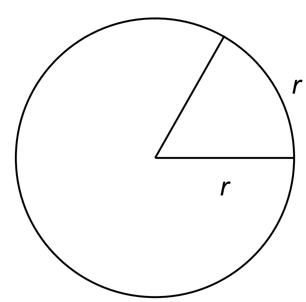


Ex. Find the measure of an angle coterminal to

1. 85°
2. -100°
3. 482°

Radian Measure:

Def: An angle of 1 radian is the measure of a central angle of a circle such that the length of the arc intercepted by the angle is equal to length of the radius of the circle.



$$C = 2\pi r$$

How does 1 radian compare to degrees?

If you solve the circumference for r you get $\Longrightarrow$ $\dfrac{C}{2\pi} = r$

The circumference is the distance around the circle which is $\Longrightarrow$ $r = \dfrac{360^o}{2\pi}$

Simplifying……we get $\Longrightarrow$ $r = \dfrac{180}{\pi} \approx 57.3^o$

We can use this fraction to help convert from degrees to radians:

To convert: radians to degrees: multiply radians by $\frac{180}{\pi}$

Degrees to radians: multiply degrees by $\frac{\pi}{180}$

Ex. Convert to radians:

1. 240°
2. 150°
3. -45°
4. 180°

Ex. Convert to degrees:

1. $\frac{2\pi}{3}$
2. $\frac{5\pi}{4}$
3. $\frac{\pi}{5}$
4. $\frac{-5\pi}{6}$

Ex. On a set of axes, draw an angle in standard position whose measure is

1. $\frac{2\pi}{3}$
2. $-\frac{9\pi}{4}$

<u>Formula</u>: <u>Length of an Arc of a circle</u>

Let *r* be the length of the radius of a circle and θ is the non-negative measure of a central angle, in radians. The length of the arc intercepted by the central angle is:

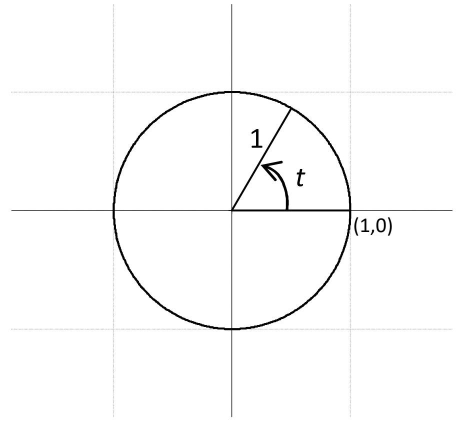
$$S = r\theta$$

Ex. A circle has a radius of 10 in. Find the length of the arc intercepted by a central angle of 120°.

Ex. A circle has a central angle of 45° intercepts and arc whose length is 8 in, find the radius of the circle.

Homework: Pg. 472-474 #7 – 11 odd, 15 – 39 (3's), 41 – 65 odd, 72, 74, 93, 95

4.2 Trigonometric Functions: The Unit Circle



Def: A unit circle is a circle whose radius is 1 unit and whose center is at the origin.

- The unit circle's equation is: $x^2 + y^2 = 1$

- Suppose you have a unit circle with a central angle measuring t radians. By the formula for arc length, the of the intercepted arc would be:

$$S = r\theta = 1 \times t = t$$

- The length of the arc is the same as the measure of the angle.

Th: In a unit circle, the radian measure of a central angle is equal to the length of the intercepted arc.

Def: Given a unit with a central angle in standard position whose terminal side intersects the unit circle at the point P(x,y).

(a) The sine of angle θ, written $\sin\theta$, is the *y*-coordinate of P.
(b) The cosine of angle θ, written $\cos\theta$, is the *x*-coordinate of P.

- Because of the definition:
  - this leads to the following quadrantal angle values:

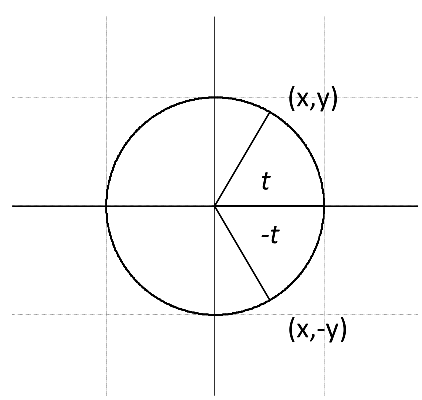
Function	0	90	180	270
sin	0	1	0	-1
cos	1	0	-1	0

  - What are the possible values for $y = \sin\theta$?________________
  - What are the possible value for $y = \cos\theta$?_________________

- Sine and cosine of "special angles": (These values have to be memorized)

Function	0	30 $(\frac{\pi}{6})$	45 $(\frac{\pi}{4})$	60 $(\frac{\pi}{3})$	90 $(\frac{\pi}{2})$	180 (π)	270 $(\frac{3\pi}{2})$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0

Given this definition, the unit circle whose equation is $x^2 + y^2 = 1$ can also be written with the equation: $\sin^2\theta + \cos^2\theta = 1$



How do the values of $\sin(-t)$ and $\cos(-t)$ compare to $\sin t$ and $\cos t$?

- Using the diagram to the right, you can see that both angles intercept points (x,y) and (x,-y). Therefore:

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$

- By definition this means:
  1. $y = \sin x$ is an odd function
  2. $y = \cos x$ is an even function.

Ex. What is the value of

1. $\sin(-30)$
2. cos (—60)

Ex. If $\sin t = \frac{3}{5}$ and $0 \le t < \frac{\pi}{2}$, find cos t.

Other trig functions:

A. Quotient Functions:

1. tan θ – tangent of angle θ: $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. cot θ – cotangent of angle θ: $\cot\theta = \frac{\cos}{\sin\theta}$

B. Reciprocal Function:

1. csc θ – cosecant of angle θ: $\csc\theta = \frac{1}{\sin\theta}$
2. sec θ – secant of angle θ: $\sec\theta = \frac{1}{\cos\theta}$

- This can now lead to 3 Pythagorean Identities:
  1. $\sin^2\theta + \cos^2\theta = 1$
  2. $1 + \tan^2\theta = \sec^2\theta$ ← found by dividing $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$
  3. $1 + \cot^2\theta = \csc^2\theta$ ← found by dividing $\sin^2\theta + \cos^2\theta = 1$ by $\sin^2\theta$

Ex. Given $\sin t = \frac{2}{5}$ and $\cos t = \frac{\sqrt{21}}{5}$, find the values of the other 4 trig functions.

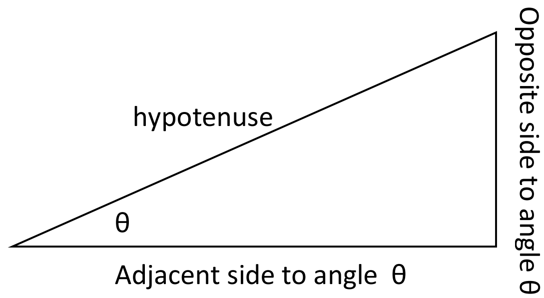
Ex. If $\sin t = \frac{5}{13}$ and $0 \le t < \frac{\pi}{2}$, find the value of the other 5 trig functions.

Homework: Pg. 486-488 #1-4, 5-13o, 11-42 (3's), 53-69odd

4.3 Right Triangle Trigonometry

- When studying the right triangle, you can use trigonometry to help find missing values.

Def: Parts of a Right Triangle – based on angle θ



Def: Right Triangular Trig Functions

1. $\sin\theta = \frac{opposite}{hypotenus}$
2. $\cos\theta = \frac{adjacent}{hypotenuse}$
3. $\tan\theta = \frac{opposite}{adjacent}$
4. $\csc\theta = \frac{hypotenus}{opposite}$
5. $\sec\theta = \frac{hypotenuse}{adjacent}$
6. $\cot\theta = \frac{adjacent}{opposite}$

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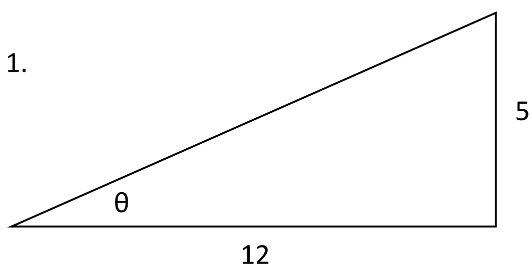
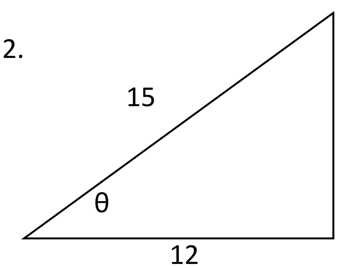
Ex. Find the value of all six trig functions for the triangle below:

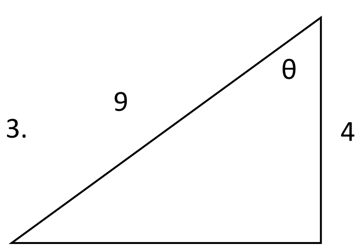
1.



2.



3.



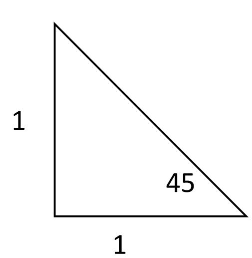
Finding an Angle given the value of a trig function:

- Divide the fraction into a decimal
- Using your calculator, use the $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ functions to find the measure of the acute angle
- If you want $\csc^{-1}$, $\sec^{-1}$, or $\cot^{-1}$, use the reciprocal property of each to help you.

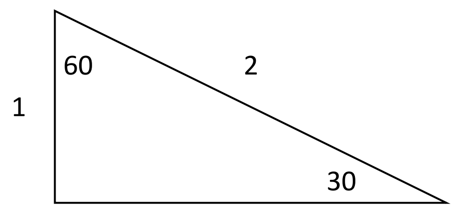
Ex. Find the measure of the acute angle which satisfies:

1. $\sin\theta = \frac{3}{5}$ 2. $\csc\theta = 3$

Ex. Use the right triangle below to find sin 45, cos 45, and tan 45, in simplest radical form



Ex. Use the right triangle below to find sin 30, cos 30, sin 60, cos 60, tan 30, and tan 60, in simplest radical form



Property: The value of a trigonometric function of θ is equal to the cofunction of the complement of θ.

fcn θ = cofcn(90 − θ)

ex. sin 30 = cos 60 sec 56 = csc 34

ex. Write an expression that is equivalent to

1. $\csc\frac{\pi}{3}$
2. $\cot\frac{\pi}{12}$

Def: Angle of Elevation – this is the angle formed by a horizontal line and a line of sight to an object above

Angle of Depression – this the angle formed by a horizontal line and a line of sight to an object below

Ex. Sighting the top of a building, a surveyor measures the angle of elevation to be 22°. If the transit (the device find the angle) is 5 ft above the ground and 300 feet from the building, find the height of the building.

Homework: Pg. 498 – 500 #1-8, 9-19o, 22-28e, 30, 33-41o, 53, 54, 57

4.4 Trigonometric Functions of Any Angle

Recall: if the terminal side of an angle θ, in standard position, passes through the point P(x,y) on the unit circle, then

(a) $\cos\theta = x$
(b) $\sin\theta = y$

But what if the point it passes through is NOT on the unit circle?

Def: Let θ be an angle in standard position and let P = (x, y) be a point on the terminal side of θ. If r is the distance P is from the origin, i.e. $r = \sqrt{x^2 + y^2}$, then the six trig function of θ are defined:

$\sin\theta = \frac{y}{r}$ $\qquad$ $\csc\theta = \frac{r}{y}$

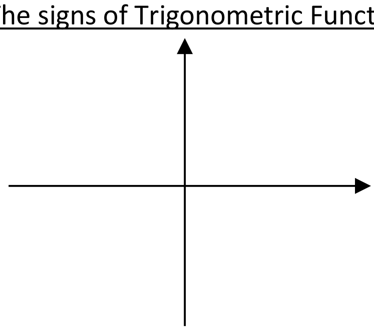
$\cos\theta = \frac{x}{r}$ $\qquad$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x},\ \ x \neq 0$ $\qquad$ $\cot\theta = \frac{x}{y},\ \ y \neq 0$

Ex. Let P(3, −5) be a point on the terminal side of θ. Find each of the six trigonometric functions of θ.

Ex. Let P(−1, −3) be a point on the terminal side of θ. Find each of the six trigonometric functions of θ.

The signs of Trigonometric Functions:



Ex. If $\tan\theta < 0$ and $\cos\theta > 0$, in which quadrant does θ lie?

Ex. If $\sin\theta < 0$ and $\cos\theta < 0$, in which quadrant does θ lie?

Ex. Given $\tan\theta = -\frac{2}{3}$ and $\cos\theta > 0$, find $\cos\theta$ and $\csc\theta$.

Ex. Given $\sin\theta = -\frac{3}{5}$ and $\cos\theta < 0$, find $\tan\theta$ and $\sec\theta$.

Def: Let θ be an angle in standard position that lies in a quadrant. Its **reference angle** is the positive acute angle α, formed by the terminal side and the ***x-axis***.

Ex.

Quadrant	Reference angle of θ
I	$\alpha = \theta$
II	$\alpha = 180 - \theta$
III	$\alpha = \theta - 180$
IV	$\alpha = 360 - \theta$
> 360	Subtract 360 over and over
< -360	Add 360 over and over

Ex. Find the reference for each of the following angles:

1. 345°
2. $\frac{5\pi}{6}$
3. $-135°$
4. $\frac{4\pi}{3}$
5. 487°

Using Reference Angles to Evaluate Trig Functions: The "Q-R-S" Method

1. Convert the angle to a positive acute angle using the "Q-R-S" Method
   a. Q = Quadrant – what quadrant is the angle in?
   b. R = Reference Angle – what is the reference angle?
   c. S = sign – is the trig function positive or negative in that quadrant
2. Rewrite the trig function with the new positive acute angle and the correct sign in front.
3. Evaluate

Ex. Express as a function of a positive acute angle

1. sin 215°
2. cos 310°
3. $\tan\frac{4\pi}{3}$
4. cos −263°

Ex. Find the exact value of each of the following

1. sin 150°
2. tan 135°
3. cos 510°
4. csc 300°
5. $\tan\frac{11\pi}{6}$
6. $\sin\frac{17\pi}{4}$
7. $\cos\left(-\frac{5\pi}{3}\right)$
8. $\sec\left(-\frac{7\pi}{4}\right)$

Ex. Find all possible values of θ, $0 \le \theta < 2\pi$, such that

1. $\cos\theta = -\frac{\sqrt{2}}{2}$
2. $\sin\theta = \frac{\sqrt{3}}{2}$
3. $\csc\theta = -2$

Homework: Pg. 513 #5-100 (5's)

4.5 Graphs of the Sine and Cosine Functions

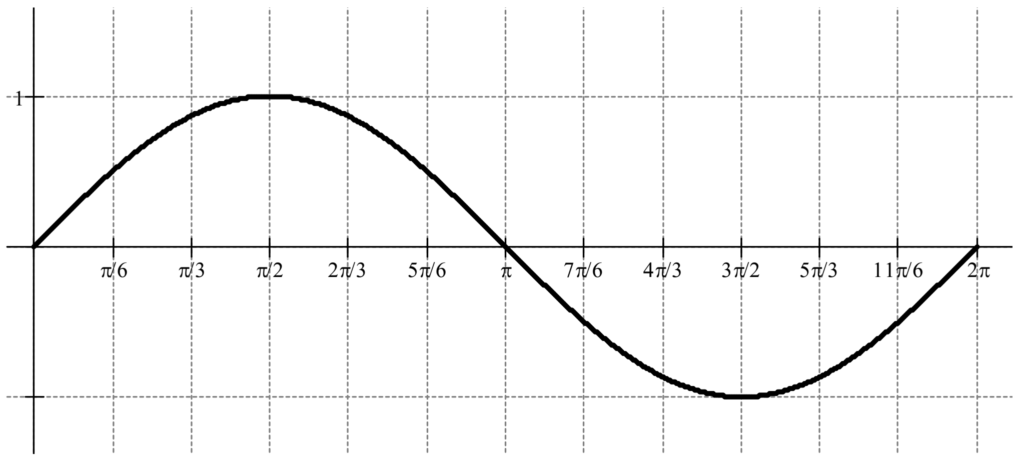
Objective: SWBAT graph: $y = A\sin Bx$ and $y = A\cos Bx$

The graph of $y = \sin x$:

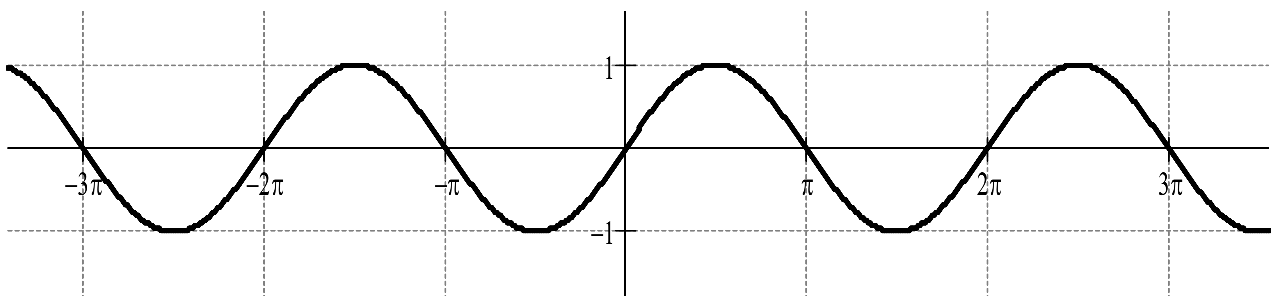
Here is a table for the sine function:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0
	0	.5	.866	1	.866	.5	0	-.5	-.866	-1	-.866	-.5	0

If we graph these points onto a set of axes, we get:



A complete cycle of the curve, also called the **period** is 2π units in length. A complete view of the curve looks like this:



This kind of curve is referred to as *periodic* because it repeats over and over.

From the graph we obtain the following information regarding $y = \sin x$:

1. The Domain is $(-\infty, \infty)$ - the set of all reals
2. The Range is $[-1,1]$ - therefore the $\sin x$ is never higher than 1 or lower than -1.
3. The period is 2π - that is how often it repeats
4. The function is an ODD FUNCTION – symmetric about the origin.

There are 5 "Key Points":

When the sin x =

$0{:}\ x = 0, \pi, 2\pi$

$1{:}\ x = \frac{\pi}{2}$

$-1{:}\ x = \frac{3\pi}{2}$

Variations of $y = \sin x$:

$\boldsymbol{y = A\sin Bx}$: This graph has a vertical and horizontal dilation.

Given the equation above:

A = amplitude of the curve – how high/low the curve goes.

B = Frequency – how many cycles of the curve are seen in 360° or 2π units

Because the frequency is not 1, the period is calculated using the following formula:

Period = $\frac{2\pi}{B}$ or $\frac{360}{B}$

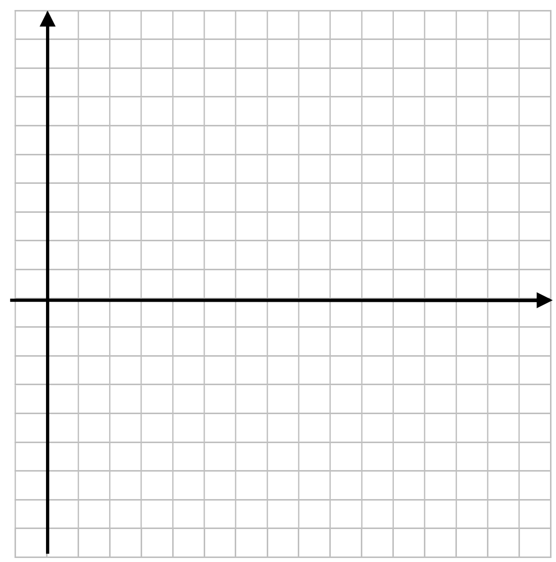
Ex. State the Amplitude, Frequency, and Period for each of the following:

1. $y = 5\sin 2x$ 2. $y = 2\sin 3x$ 3. $y = 4\sin\frac{3}{4}x$

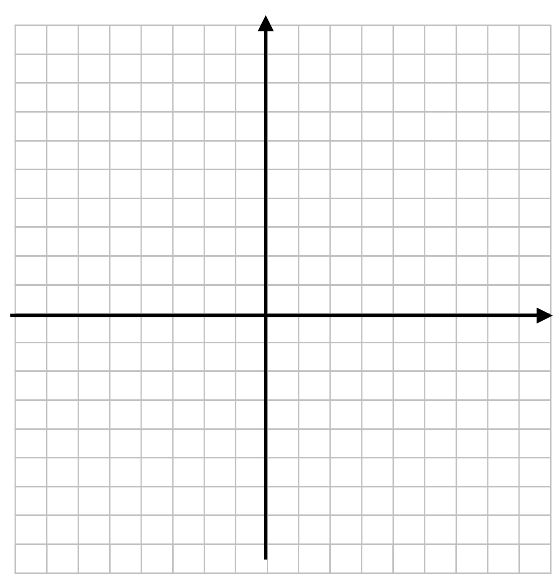
<u>To graph $y = A\sin Bx$</u>:

1. Identify the amplitude and the period
2. Find the x values of the 5 key points:
   a. Divide each of the key x coordinates by the frequency
   b. Add the period to each key x coordinate until you get out of the domain
3. Plot each of the key points
4. Sketch the curve

Ex. Graph $y = 2\sin 3x$ on the interval [0,2π]



Ex. Graph $y = 3\sin 2x$ on the interval $[-\pi, \pi]$

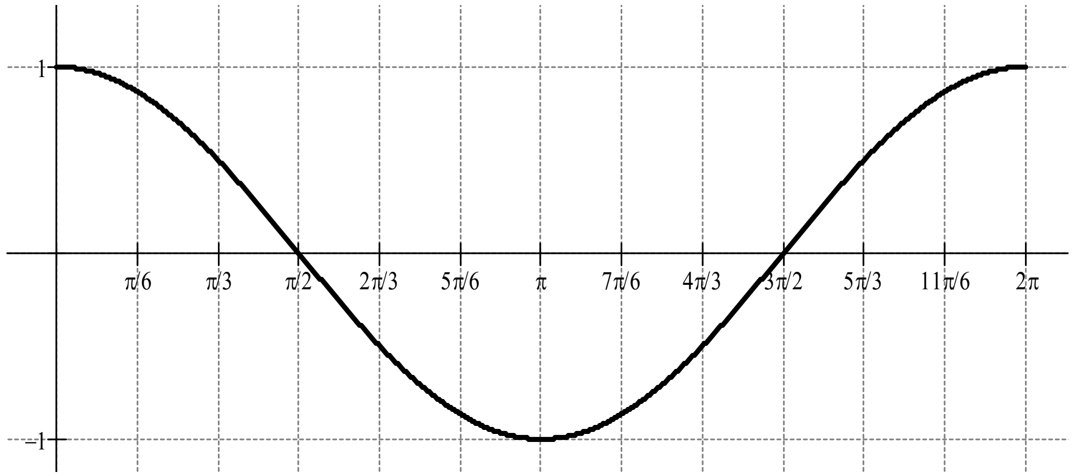


The graph of $y = \cos x$:

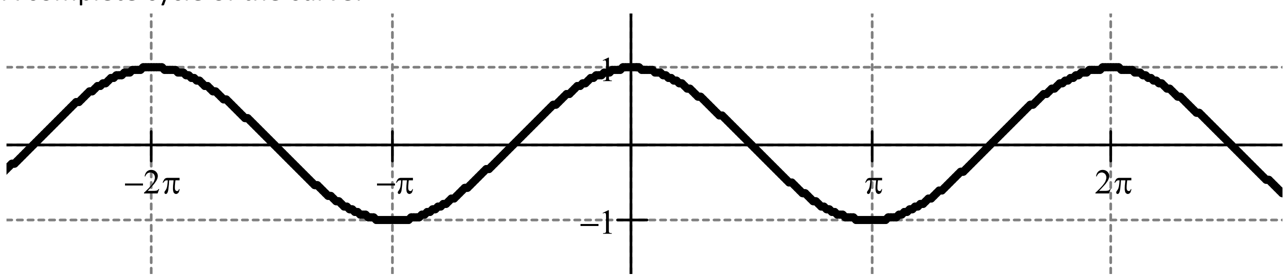
Here is a table for the sine function:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1
	1	.866	.5	0	-.5	-.866	-1	-.866	-.5	0	.5	.866	1

If we graph these points onto a set of axes, we get:



A complete cycle of the curve:



From the graph we obtain the following information regarding $y = \cos x$:

1. The Domain is $(-\infty, \infty)$ - the set of all reals
2. The Range is $[-1,1]$ - therefore the $\cos x$ is never higher than 1 or lower than -1.
3. The period is 2π - that is how often it repeats
4. The function is an EVEN FUNCTION – symmetric about the origin.

The only difference are the 5 key points:

When the cos x =

$$1:\ x = 0, 2\pi$$

$$0:\ x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$-1:\ x = \pi$$

Variations of $y = \cos x$:

$\boldsymbol{y = A\cos Bx}$: This graph has a vertical and horizontal dilation.

Given the equation above:

A = amplitude of the curve – how high/low the curve goes.

B = Frequency – how many cycles of the curve are seen in 360° or 2π units

Because the frequency is not 1, the period is calculated using the following formula:

Period = $\frac{2\pi}{B}$ or $\frac{360}{B}$

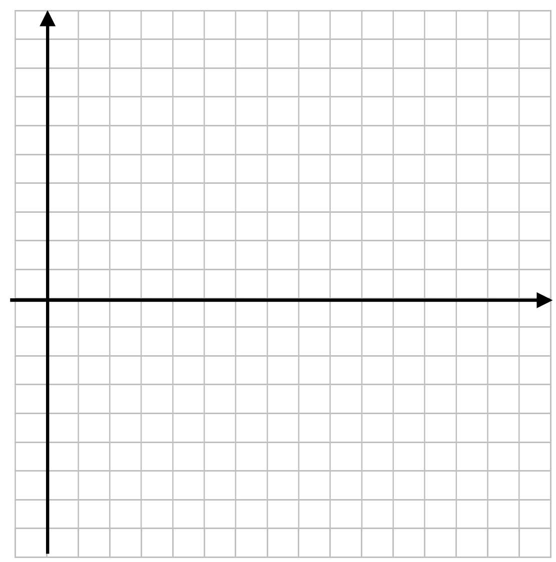
Ex. State the Amplitude, Frequency, and Period for each of the following:

1. $y = 4\cos 2x$ 2. $y = 6\cos 6x$ 3. $y = 2\cos\frac{1}{2}x$

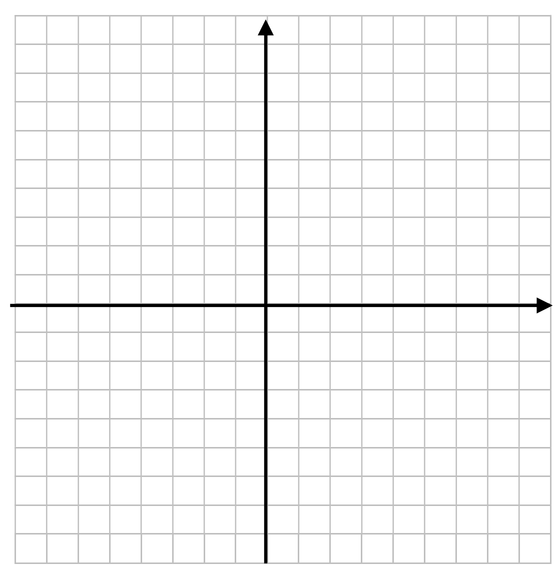
To graph $y = A\cos Bx$:

1. Identify the amplitude and the period
2. Find the x values of the 5 key points:
   a. Divide each of the key x coordinates by the frequency
   b. Add the period to each key x coordinate until you get out of the domain
3. Plot each of the key points
4. Sketch the curve

Ex. Graph $y = 3\cos 2x$ on the interval [0,2π]



Ex. Graph $y = 4\cos\frac{1}{2}x$ on the interval $[-\pi, \pi]$



Homework: **Day 1: Pg. 533 #1-15odd** **Day 3: Pg. 533 #2-40 (4's), 52 ,54, 56**

Day 2: Pg. 533 #31-41 odd, 53, 55

4.5b Graphs of the Sine and Cosine Functions (Day 2)

Objective: SWBAT graph:
$y = A\sin Bx$ and $y = A\cos Bx$

4.6 Graphs of Other Trigonometric Functions and 4.7 Inverse Trig Functions (Day 1)

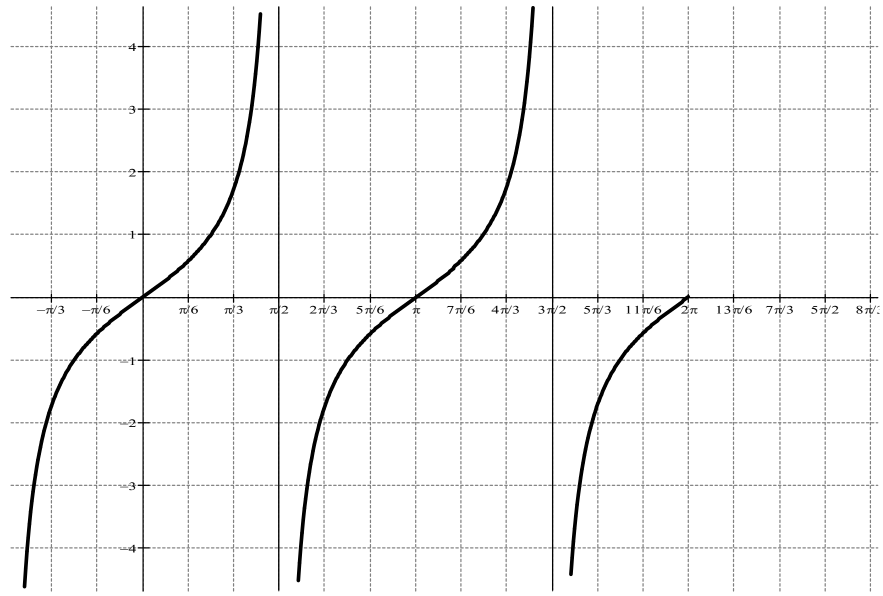
4.6 Graphs of Other Trigonometric Functions

<u>The graph of $y = \tan x$:</u>

Here is a table for the tangent function:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
Tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	**	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	**
	0	0.6	1	1.732	**	-1.732	-1	-0.6	0	0.6	1	1.732	**

Here is the graph:



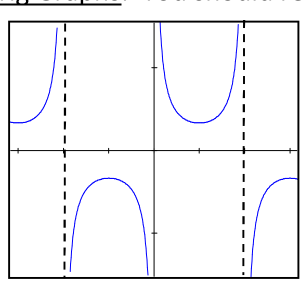
The vertical lines at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ are <u>vertical asymptotes</u>. The curve will get closer and closer to the line without touching it.

<u>Characteristics</u>:

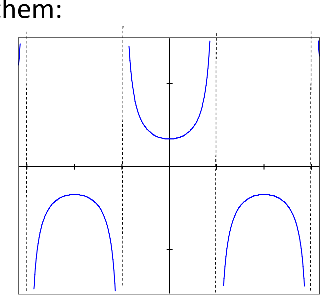
- **Period: π**
- **Domain: ALL REALS except odd multiples of $\frac{\pi}{2}$**
- **Range: ALL REALS**
- **Vertical Asymptotes: odd multiples of $\frac{\pi}{2}$**
- ***x*-intercepts: midway between asymptotes (integral values of π)**
- **Odd function: Symmetric about the origin**

Note: You only need worry about the graph of $y = \tan x$ not $y = A \tan Bx$ for Regents Exam and this course.

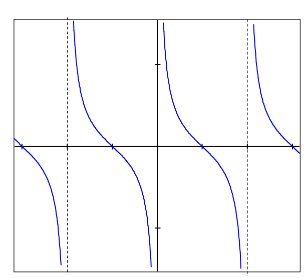
<u>Other Trig Graphs</u>: You should recognize them:



y = csc *x*



y = sec *x*



y = cot *x*

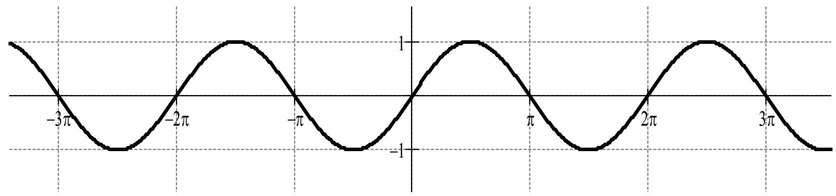
4.7 Inverse Trigonometric Functions (Day 1)

Recall: A function has an inverse if it is one-to-one.

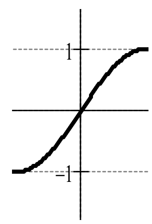
- This means the function must satisfy both the vertical and horizontal line test, which trig functions do not.
- To fix this problem, we restrict the domains of the trig functions to a domain that would be one-to-one.

<u>The Inverse Sine Function</u>:

- Here is the sine curve:



- If we restrict the domain to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, we would have this:



- This section of the curve is one to one. To find the inverse, you would switch the x and y from the equation $y = \sin x$. This would become :

  $x = \sin y$

  Solving for y, we would need to write the function with a different notation:

  $y = \sin^{-1} x$ or $y = \arcsin x$

  - You read it as "inverse sine" or "arc sine" of x
  - You think: "The angle whose sin is "

Ex. Find each of the following:

1. $\sin^{-1}\frac{1}{2}$ 2. $\sin^{-1}\frac{\sqrt{3}}{2}$ 3. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ 4. $\sin^{-1}\frac{3}{5}$

5. $\sin^{-1}\left(-\frac{1}{2}\right)$ 6. $\sin^{-1} 1$ 7. $\sin^{-1} 0.234$ 8. $\sin^{-1} 2$

Homework: **Pg. 563 #1-6, 19-22, 31, 33, 35**

Graph on a set of axes from $-2\pi \le x \le 2\pi$:

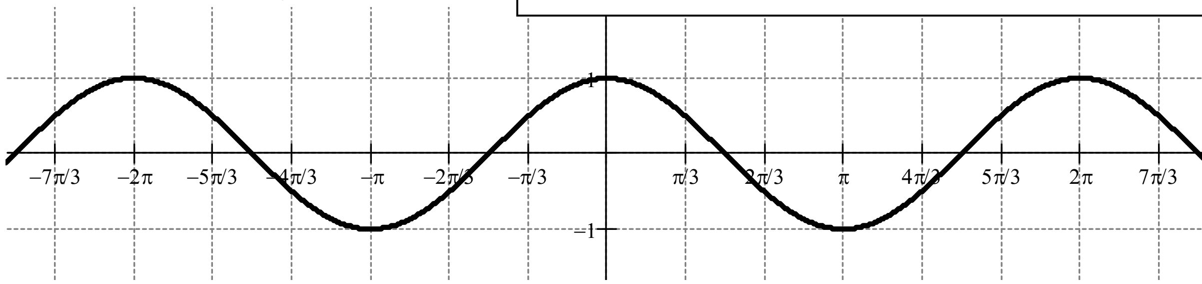
1. $y = 3\sin 2x$ 2. $y = 2\cos\frac{1}{2}x$ 3. $y = -4\sin 3x$

4.7b Inverse Trigonometric Functions (Day 2)

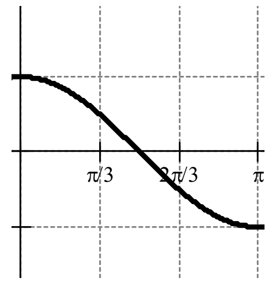
Objective:
SWBAT to use inverse trig functions to find solutions to trig expressions

The Inverse Cosine Function: $y = \cos^{-1} x$

- Here is the cosine curve ($y = \cos x$):



- As with the sine function it is not one-to-one. We will take a section of the curve and restrict the domain so it is one to one. This section is from 0 to π (180°). This results in the following:



Def: The inverse cosine function, denoted $\cos^{-1} x$, is the inverse of the restricted cosine function $y = \cos x$, on the domain $0 \le x \le \pi$.

$$y = \cos^{-1} x \text{ means } \cos y = x$$

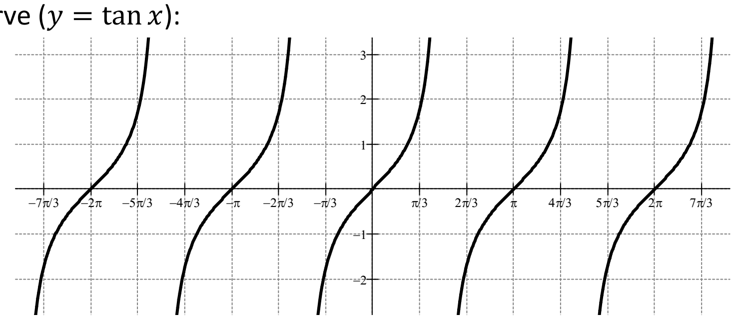
- Remember: $\cos^{-1} x$ means the angle whose cosine is x ➔ The result is an angle (in degrees or radians)
  - If x < 0 then the angle will be a second quadrant angle. Use Q-R-S if need be.

Ex. Find the value of each of the following:

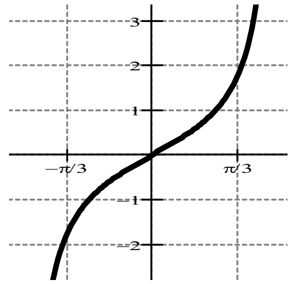
1. $\cos^{-1} \frac{\sqrt{3}}{2}$
2. $\cos^{-1} \left(-\frac{\sqrt{2}}{2}\right)$
3. $\cos^{-1} \left(-\frac{1}{2}\right)$
4. $\sin\left(\cos^{-1} \frac{3}{5}\right)$
5. $\cos^{-1}\left(\cos \frac{\pi}{6}\right)$
6. $\tan\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$

The Inverse tangent Function: ($y = \tan^{-1} x$

- Here is the cosine curve ($y = \tan x$):



- As with the sine function it is not one-to-one. We will take a section of the curve and restrict the domain so it is one to one. This section is from $-\frac{\pi}{2}$ (−90°)to $\frac{\pi}{2}$ (90°). This results in the following:



Def: The <u>inverse tangent function</u>, denoted $\tan^{-1} x$, is the inverse of the restricted tangent function $y = \tan x$, on the domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

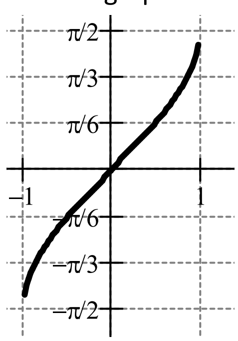
$$y = \tan^{-1} x \text{ means } \tan y = x$$

- Remember: $\tan^{-1} x$ means the angle whose tangent is x ➔ The result is an angle (in degrees or radians)
  - If x < 0 then the angle will be a negative angle.

Ex. Find the value of each of the following:

1. $\tan^{-1}\frac{\sqrt{3}}{3}$
2. $\tan^{-1}(-\sqrt{3})$
3. $\tan(\tan^{-1}(-1))$
4. $\sin\left(\tan^{-1}\frac{3}{5}\right)$
5. $\tan^{-1}(\cos\pi)$
6. $\cot\left(\tan^{-1}\frac{6}{5}\right)$

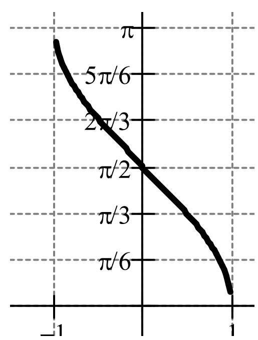
- Here are the graphs of the inverse trig functions:



$y = \sin^{-1} x$

D:

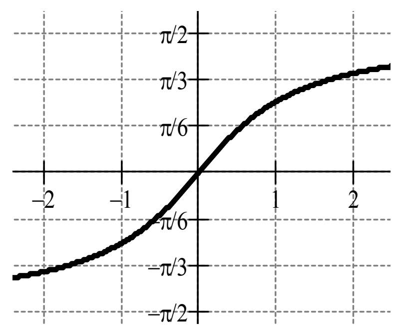
R:



$y = \cos^{-1} x$

D:

R:



$y = \tan^{-1} x$

D:

R:

Ex. Write each of the following expressions as an algebraic expression in *x*

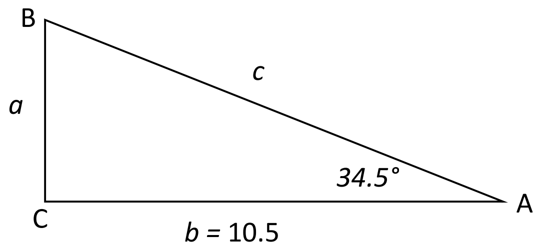
1. $\cos(\sin^{-1} x)$
2. $\sec(\tan^{-1} x)$

Homework: Pg. 563 #7-17odd, 24-30even, 33-73odd

4.8 Applications of Trigonometric Functions

Solving Right Triangles: Given of a right triangle and the length of a side, you can find all the other parts.

Ex. Solve the right triangle shown below. Round all lengths to two decimal places.



Ex. From a point on level ground 125 ft from the base of a tower, the angle of elevation is 57.2°. Approximate the height of the tower to the nearest foot.

Ex. A kite flies at a height of 30 ft when 65 feet of string is out. If the string is in a straight line, find the angle that it make with the ground. Round your answer to the nearest tenth of a degree.

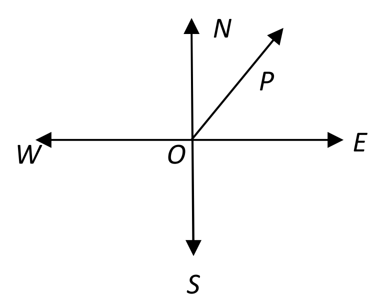
Ex. A guy wire is 13.8 yards long and is attached from the ground to a pole 6.7 yards above the ground. Find the angle, to the nearest tenth of a degree, that the wire makes with the ground.

Two Right Triangle Problem:

Ex. You are taking your first hot air balloon ride. Your friend is standing on level ground 100 feet away from your launch point, making a video. At one one instance, the angle of elevation from the video camera to the balloon is 31.7°. One minute later, the angle of elevation is 76.2°. How far did the balloon travel, to the nearest tenth of a foot in that minute?

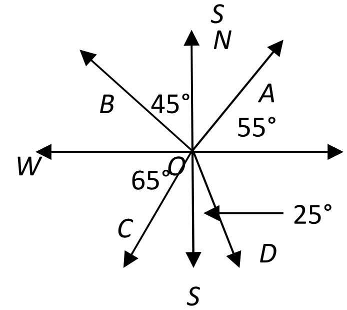
Ex. You are standing on level ground 800 feet from Mount Rushmore looking at the face of George Washington. The angle of elevation to the bottom the sculpture is 32° and the angle of elevation to the top is 35°. What is the height of the sculpture of Washington's face to the nearest tenth of a foot?

Def: The *bearing* from point O to point P is the acute angle formed by $\overrightarrow{OP}$ and a North-South line.



Ex. Use the diagram to the right to find the bearing from *O* to

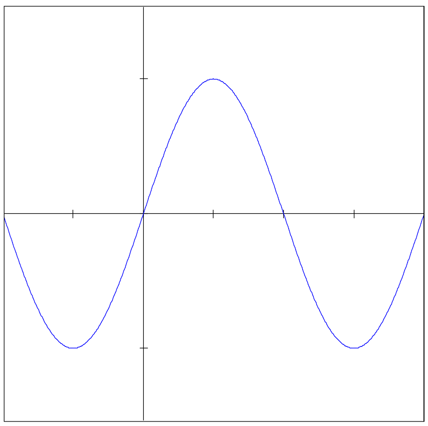
(a) *B*
(b) *A*
(c) *D*



Homework: Pg. 574-575 #1-15odd, 29-35odd, 41, 44, 48

Graphs of Trigonometric Functions

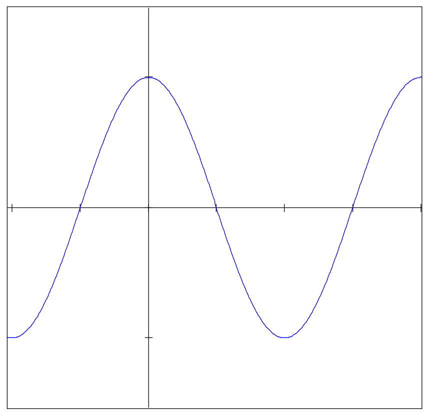
- Each of the following graphs should be recognized on sight:



$y = \sin x$

Domain: $x \;\varepsilon$ Reals

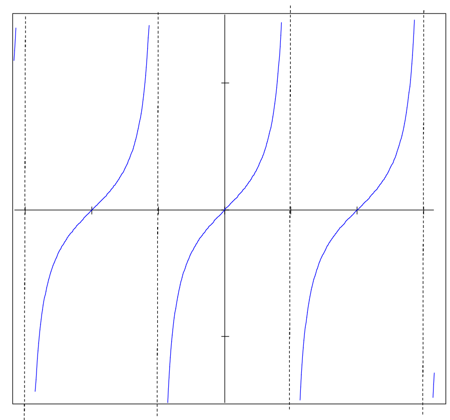
Range: $-1 \le y \le 1$



$y = \cos x$

Domain: $x \;\varepsilon$ Reals

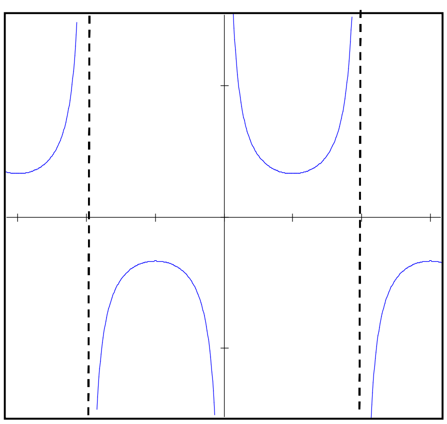
Range: $-1 \le y \le 1$



$y = \tan x$

Domain: $x \neq \frac{(2n-1)\pi}{2}$

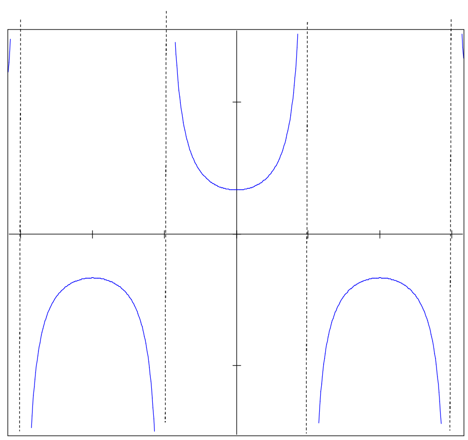
Range: $y \;\varepsilon$ Reals



$y = \csc x$

Domain: $x \neq 0, \pi, 2\pi, \ldots$

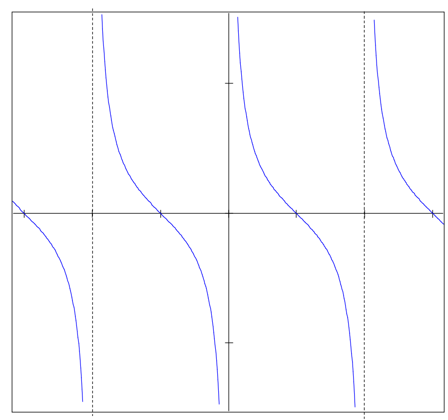
Range: $y \le -1$ or $y \ge 1$



$y = \sec x$

Domain: $x \neq \frac{(2n-1)\pi}{2}$

Range: $y \le -1$ or $y \ge 1$



$y = \cot x$

Domain: $x \neq 0, \pi, 2\pi, \ldots$

Range: $y \;\varepsilon$ Reals

- ALL TRIG GRAPHS are to use RADIAN MEASURE (not degrees)