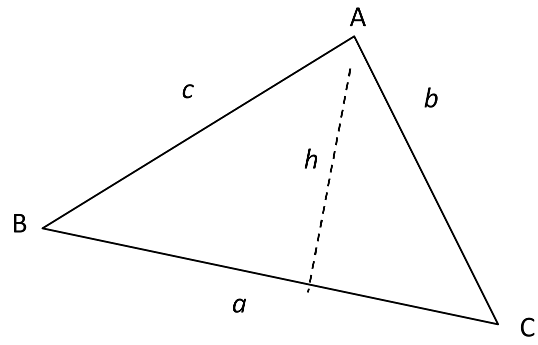
Chapter 6 – Additional Topics in Trigonometry
6.1 The Law of Sines

An *oblique triangle* is a triangle that does not contain a right angle.

Given ΔABC as shown to the right:



Let h be the length of the height drawn from A to BC

This forms two right triangles, using right triangle trig, we get

$$\sin B = \tfrac{h}{c}$$
$$\sin C = \tfrac{h}{b}$$

these lead to the following equations after cross multiplying:

$$h = c\sin B$$
$$h = b\sin C$$

Since h is equal to both of these then

$$b\sin C = c\sin B$$
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Similarly, we can prove the same with side a and angle A, this leads to **<u>The Law of Sines</u>**:

<u>Theorem</u>: The Law of Sines

If a, b, and c are the lengths of the sides of a triangle and A, B, and C are the angles opposite these sides, then

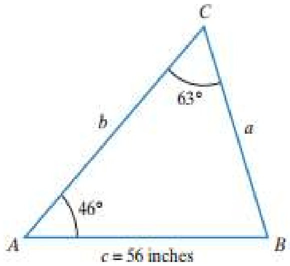
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The ratio of the length of the side of any triangle to the sine of the angle opposite that side is the same for all sides of the triangle.

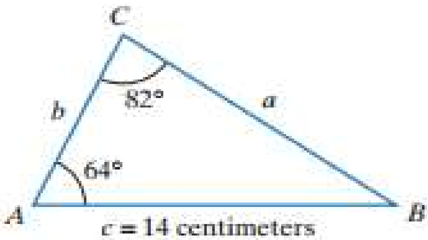
This law can be used to find the remaining parts of an oblique triangle when we know either of the following:

1. two sides and an angle opposite one of the sides (SSA)
2. two angles and any side (AAS or ASA)

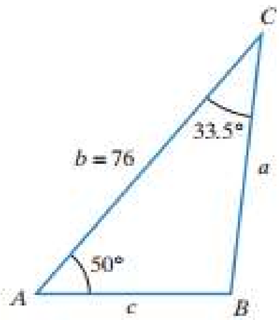
Ex. Solve the triangle shown below with $A = 46°$, $C = 63°$, and $c = 56$ inches. Round lengths to nearest tenth.



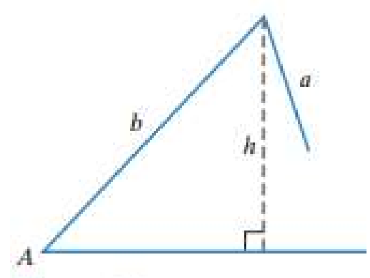
Ex. Solve the triangle shown below with $A = 64°$, $C = 82°$, and $c = 14$ cm. Round lengths to the nearest tenth.



Ex. In ΔABC, shown below, with $A = 50°, C = 33.5°$, and $b = 76$. Solve the rest of the triangle. Round lengths to the nearest tenth.



<u>Application</u>: The Ambiguous Case



Consider a triangle in which a, b, and A are given. You can determine the number of possible triangles doing the following:

1. Calculate the height h, using $h = b \sin A$
2. Test the height vs. side a to determine the # of triangles:
   - (i) If $a < h$: 0 triangles
   - (ii) If $a = h$: 1 triangle (right triangle)
   - (iii) If $h < a < b$: 2 distinct triangles
   - (iv) If $a > h$ and $a > b$, then 1 triangle

Ex. Determine the number of possible triangles given the following: Solve the rest of the triangle.

1. $A = 43°, a = 81,$ and $b = 62$

2. $A = 57°, a = 33,$ and $b = 26$

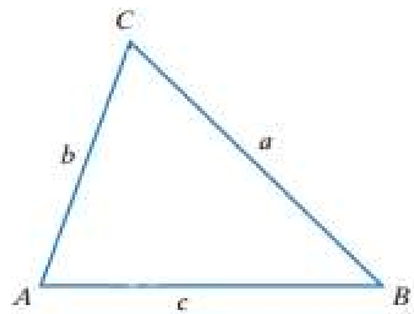
3. $A = 40°, a = 54,$ and $b = 62$

<u>Th</u>: The Area of an Oblique Triangle

Given ΔABC with known sides a and b, and included angle C, the area of the triangle is

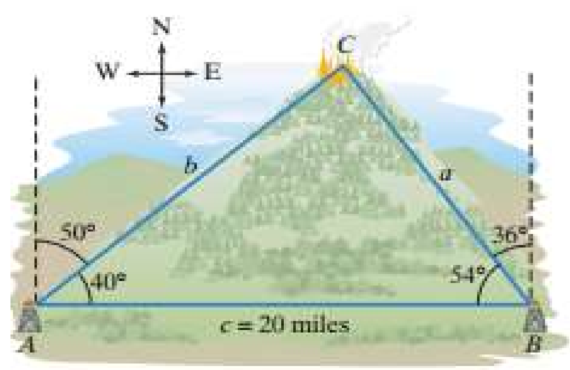
$$Area = \frac{1}{2}\,ab\sin C$$

Likewise, $Area = \frac{1}{2}\,ac\sin B$ and $Area = \frac{1}{2}\,bc\sin A$



Ex. Find the area of the triangle have two sides of length 10 and 24 meters and an included angle of 62°

Ex. Two fire-lookout stations are 20 miles apart, with Station B directly East of Station A. Both stations spot a fire on a mountain to the north. The bearing from Station A to the fire is $N50°E$ (which means 50° east of north). The bearing from Station B to the fire is $N36°W$ (36° west of North). How far, to the nearest tenth of a mile, is the fire from Station A?
(A map is shown to the right)



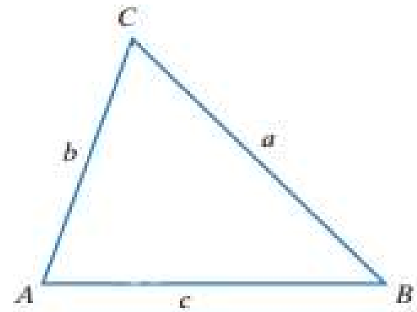
Homework: Day 1: pg. 651-655 #1-4, 10, 15, 18-30(3's), 34, 38, 39
Day 2: pg. 651-655 #5, 14, 20, 33, 35, 37, 42, 50, 52, 57

6.2 The Law of Cosines

Th: The Law of Cosines

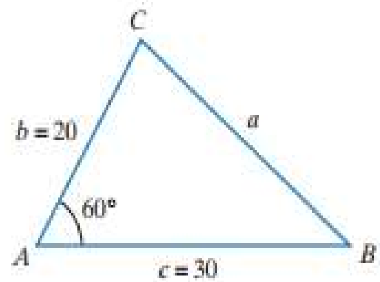
If *A, B,* and *C* are the measures of the angles of a triangle, and *a, b,* and *c* are the lengths of the sides opposite these angles, then

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

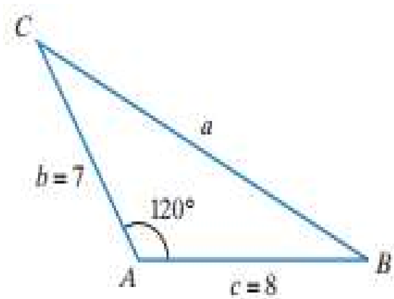


The square of a side of a triangle equals the sum of the squares of the other two sides minus twice their product times the cosine of their included angles.

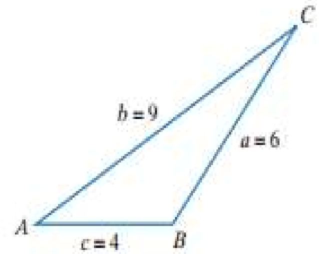
Ex. Given ΔABC shown to the right, with $A = 60°, b = 20,$ and $c = 30$. Find the missing parts of the triangle. Round lengths and angle measures to the nearest degree.



Ex. Given ΔABC shown to the right, with $A = 120°, b = 7,$ and $c = 8$. Find the missing parts of the triangle. Round lengths and angle measures to the nearest degree.



Ex. Solve triangle ABC if $a = 6, b = 9,$ and $c = 4$. Round angle measures to the nearest degree.



<u>Th</u>: Heron's Formula for the Area of a Triangle

The area of a triangle with sides *a, b,* and *c* is

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

Where *s* is one-half the perimeter (called the *semi-perimeter*): $s = \frac{1}{2}(a+b+c)$

Ex. Find the area of the triangle whose sides are 12 yds, 16 yds, and 24 yds. Round the area to the nearest tenth.

Ex. Find the area of the triangle whose sides are 6 m, 16 m, and 18 m. Round to the nearest tenth of a square meter. Also, find the measure of the smallest angle of the triangle.

Homework: pg. 661-663 #1-7odd, 9-30 (3's), 31, 33, 41, 43-44, 49

6.3 Polar Coordinates

<u>recall</u>: The *<u>rectangular coordinate system</u>* is a method of graphing that assigns a set of ordered pairs, called *<u>coordinates</u>* for every point in the plane.

- the ordered pair are designated (x, y)

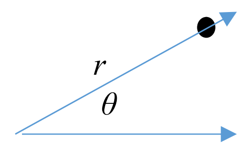
There is another way of assigning points in the plane that uses trigonometry. It is called ***<u>Polar Coordinates</u>***

<u>Def</u>: The Polar Coordinate System

- Instead of a set of axes, the Polar Coordinate System is based on a one ray axis that starts at the origin (called the ***<u>pole</u>***) at extends to the right.
    - This is designated as 0° or 0 radians
- Points are plotted using a set of coordinates (r, θ), where r is the directed distance from the pole and Θ is the measure of the angle from the axis (+ is counter clockwise, – is clockwise)

.

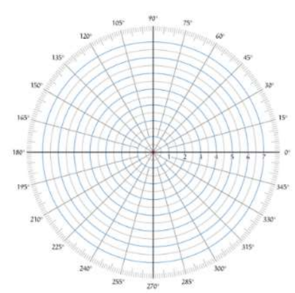
Example: (4,30°) would place a point 4 units from the pole and 30° in a counter-clockwise direction from the 0 axis.

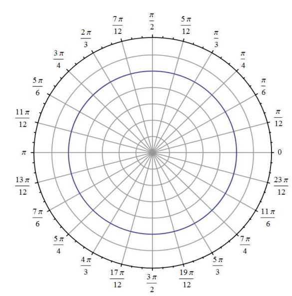


Note: r can be negative

(r, θ) are called *<u>Polar Coordinates</u>*

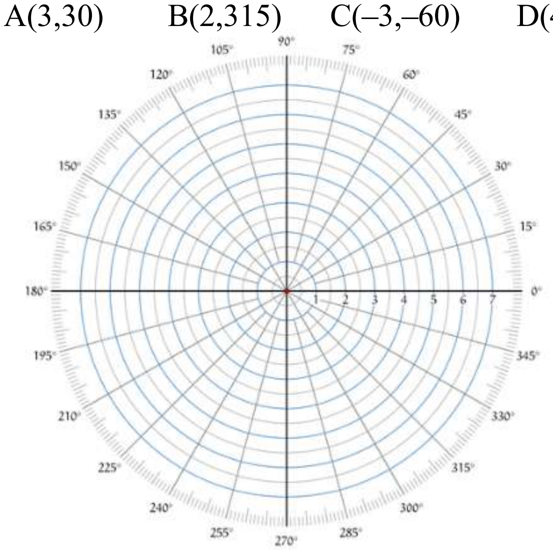
As with Rectangular Coordinates, Polar Coordinates has a specific graphing set up:





Ex. On the graph paper below, plot the following points: (r, θ)

A(3,30) B(2,315) C(–3,–60) D(4, 180) E(2.5, 210) F(0,–120)



The Sign of r and where to plot

The point $P(r, \theta)$ is located $|r|$ units from the pole. If $r > 0$, then the point lies on the terminal side of Θ. If $r < 0$, then the points lies along the ray opposite the terminal side.

If $r = 0$, then place the point at the pole.

Property: If n is any integer, the point (r,θ) can be represent as:

$$(r,\ \theta) = (r,\ \theta + 2n\pi) \quad \text{or} \quad (r,\ \theta) = (-r,\ \theta + \pi + 2n\pi)$$

Ex. Find another representation of the point $\left(2, \frac{\pi}{3}\right)$

(a) r is positive and $2\pi < \theta < 4\pi$

(b) r is negative and $0 < \theta < 2\pi$

(c) r is positive and $-2\pi < \theta < 0$

Property: Relationship between Polar and Rectangular Coordinates

$(x,\ y)$ and $(r,\ \theta)$ are the same point if

$$x = r\cos\theta \qquad x^2 + y^2 = r^2$$

$$y = r\sin\theta \qquad \tan\theta = \frac{y}{x}$$

Ex. Find the rectangular coordinates with the following polar coordinates:

a. $\left(2, \frac{3\pi}{2}\right)$ b. $\left(-8, \frac{\pi}{3}\right)$ c. $(3, \pi)$

Ex. Express each of the following rectangular coordinates as polar coordinates

a. $(-1, \sqrt{3})$ b. $(3, -\sqrt{3})$

c. $(4,3)$ d. $(0, -4)$

Def: A polar equation is an equation whose variables are r and θ.

Ex. $r = \frac{5}{\cos\theta + \sin\theta}$ and $r = 3\csc\theta$

To convert equations from Rectangular to Polar:
1. Replace x with $r\cos\theta$
2. Replace y with $r\sin\theta$

Ex. Convert each of the following equations to Polar:

1. $x + y = 5$
2. $(x-1)^2 + y^2 = 1$
3. $y = x^2 - 2x + 1$
4. $xy = 1$

To convert equations from Polar to Rectangular:
Use the formulas
$x = r\cos\theta$ $\quad y = r\sin\theta$
$x^2 + y^2 = r^2$ $\quad \tan\theta = \frac{y}{x}$

Ex. Convert each of the following polar equation to rectangular

1. $r = 5$
2. $\theta = \frac{\pi}{4}$
3. $r = 3\csc\theta$
4. $r = -6\cos\theta$

Homework: Day 1: pg. 672 – 673 #1 – 20, 23, 26
Day 2: pg. 673 #33 – 72 (3's)

If all done in 1 day:

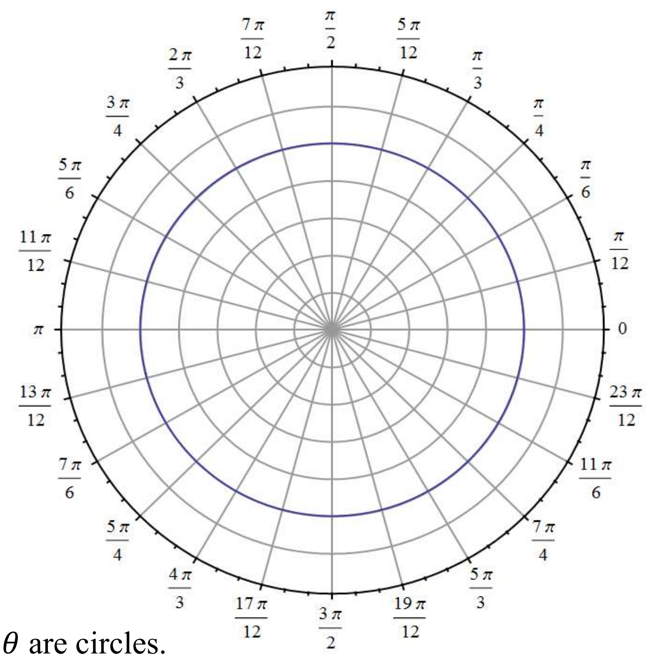
pg. 672–673 #1-25odd, 33-72(3's)

6.4 Graphs of Polar Equations

To graph a polar equation:

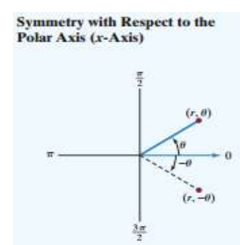
1. Make a table containing the angles and find r based on the value of θ.\
2. Plot each point on the graph paper.

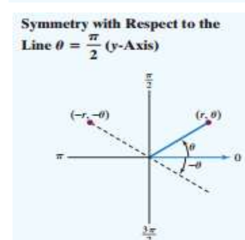
Ex. Graph the polar equation $r = 4\cos\theta$ on the graph provided. Express the equation in rectangular.

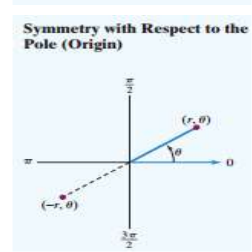


<u>Property</u>: The graphs of $r = a\cos\theta$ and $r = a\sin\theta$ are circles.

(a is the diameter of the circle)







Symmetry in Polar Coordinates: (ALWAYS CHECK!)

1. **<u>Symmetry with Respect to the Polar Axis (*x*-axis)</u>:**
   Replace θ with $-\theta$. If an equivalent equation results, it is symmetric with respect to polar axis

2. **<u>Symmetry with Respect to the Line</u> $\boldsymbol{\theta = \frac{\pi}{2}}$ (The *y*-axis)**
   Replace (r,,θ) with $(-r, -\theta)$. If an equivalent equation results, then it is symmetric over $\theta = \frac{\pi}{2}$

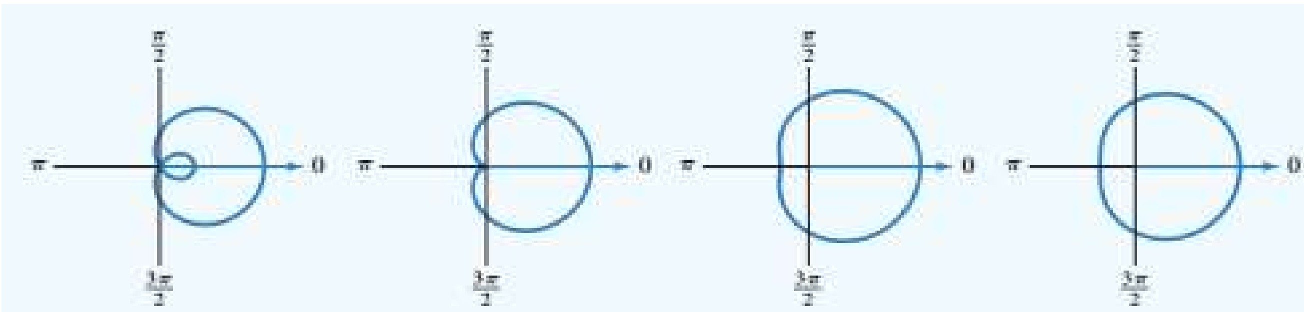
3. **<u>Symmetry with Respect to the Pole (Origin)</u>**
   Replace r with $-r$. *If an equivalent equation results, the graph is symmetric*

<u>Def</u>: A **<u>limiçon</u>** is a graph with the following equation:

$$r = a \pm b\sin\theta \quad \text{or} \quad r = a \pm b\cos\theta \quad \text{for } a > 0 \text{ and } b > 0$$

The ratio $\frac{a}{b}$ determines the limiçon's shape:

$\frac{a}{b} < 1$	$\frac{a}{b} = 1$	$1 < \frac{a}{b} < 2$	$\frac{a}{b} \geq 2$



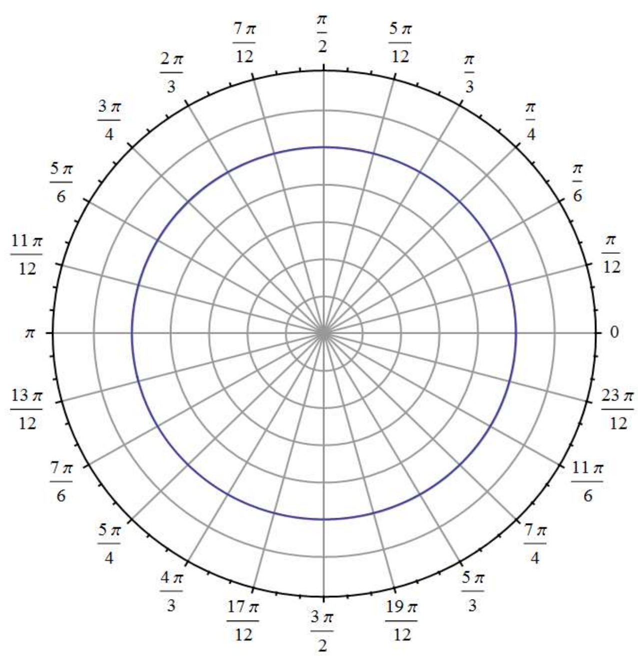
Inner Loop	Heart Shaped (Cardioid)	Dimpled (no inner loop)	No dimple or inner loop

Ex. Graph the polar equation: $r = 4\sin 2\theta$

(i) Check the symmetry

(ii) Make a table for $0 \leq \theta \leq \frac{\pi}{2}$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
r					

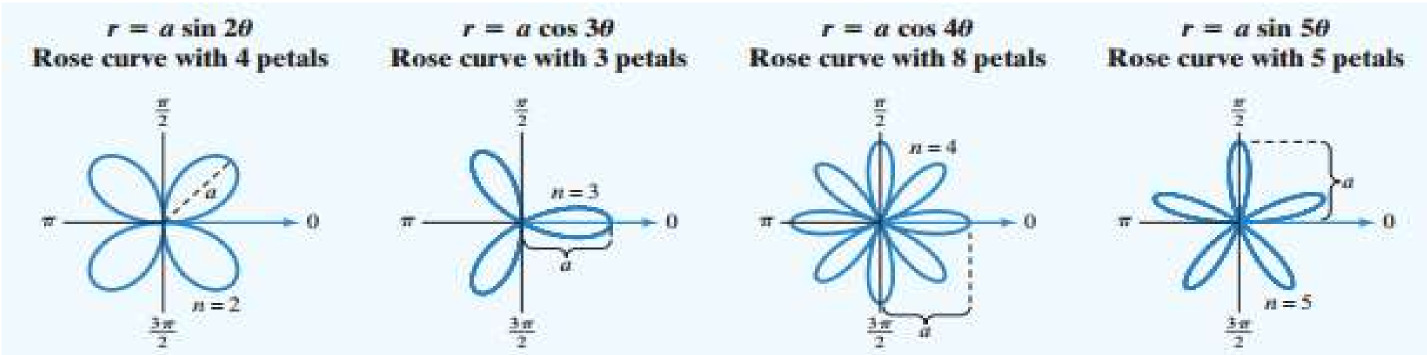


(iii) Plot these points and use the symmetry found to plot the remaining points.

Def: A **rose curve** is a polar graph of the form:

$$r = a\sin n\theta \quad \text{or} \quad r = a\cos n\theta \quad \text{for } a \neq 0$$

If n is *even*, the rose will have 2 petals. If n is *odd*, then it will have n petals.

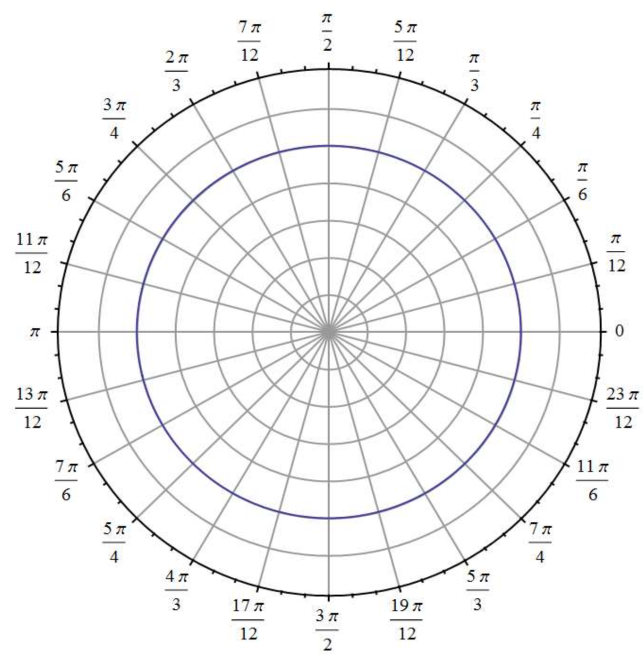


Ex. Graph the polar equation: $r^2 = 4\sin 2\theta$

(i) Check the symmetry

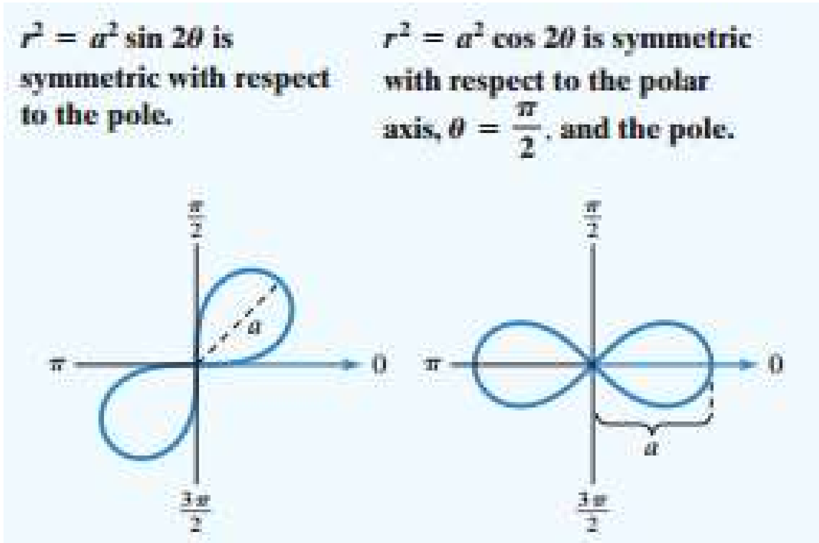
(ii) Make a table for $0 \le \theta \le \frac{\pi}{2}$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
r					



<u>Def</u>: A **<u>lemniscate</u>** is a polar graph of the form:

$r^2 = a^2 \sin 2\theta$ or $r^2 = a^2 \cos 2\theta$ with $a \neq 0$



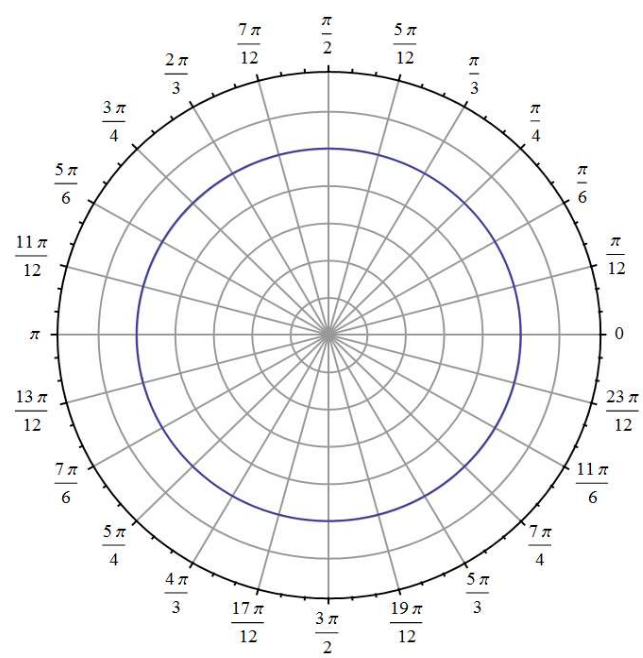
Propellars

Ex. Graph the polar equation $r^2 = 4\cos 2\theta$

(i) Check the symmetry

(ii) Make a table for $0 \leq \theta \leq \frac{\pi}{2}$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
r					



Homework: pg. 683 #1-6, 7, 10, 12, 15-30 (5's)

6.5 Complex Numbers in Polar Form: DeMoivre's Theorem

Recall: The number, ***i***, is an imaginary number defined $i = \sqrt{-1}$

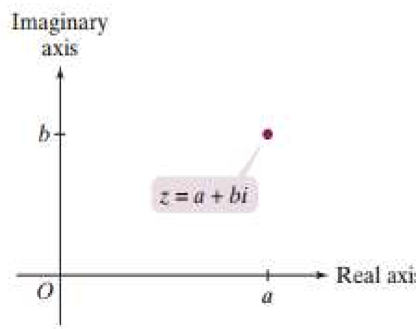
Note: Numbers that contain i are called ***complex numbers***

Def: A **complex number** is a numerical expression of the form: $a + bi$ for any real numbers *a* and *b*, with $b \neq 0$

Ex. $6 + 3i$ $\quad -4 - \frac{3}{4}i$ $\quad i$

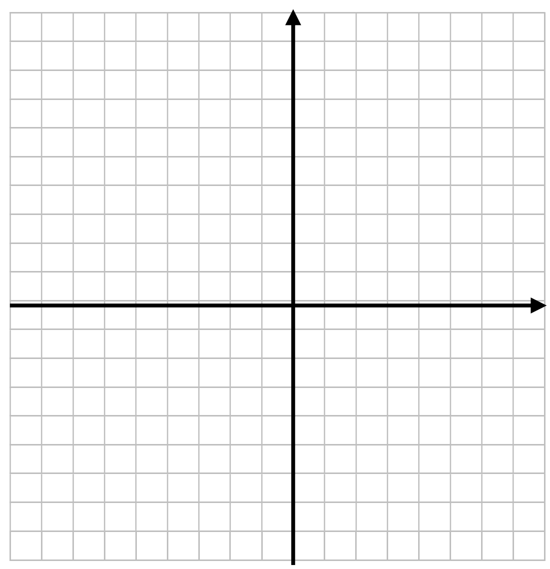
Def: The **complex plane** is a coordinate system consisting of two axes.

- The horizontal axis is called the **real axis**.
- The vertical axis is called the **imaginary axis**.
- Every complex number corresponds to a point in the complex plane
- To plot a complex number in the complex plane:
  1. Go along the real axis and find *a*.
  2. Go up the imaginary axis to the coefficient of *i* and place the point.



Ex. On the complex plane to the right, plot and label each of the following complex numbers

A: $z = 3 + 4i$
B : $z = -1 - 2i$
C: $z = -3$
D: $z = -4i$



Th: The Absolute Value of a Complex Number

The **absolute value** of a complex number $a + bi$ is

$$|z| = |a + bi| = \sqrt{a^2 + b^2}$$

Ex. Determine the absolute value of each of the following complex numbers

(a) $z = 3 + 4i$ $\qquad$ (b) $z = -1 - 2i$

<u>Th</u>: Polar Form of a Complex Number

The complex number $z = a + bi$ is written in *polar form* as:

$$z = r(\cos\theta + i\sin\theta)$$

where $a = r\cos\theta$, $b = r\cos\theta$, $r = \sqrt{a^2 + b^2}$, and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

r is called the **<u>modulus</u>** of the complex number z and the angle θ is called the **<u>argument</u>** of the complex number with $0 \le \theta \le 2\pi$

ex. Write the following complex numbers in polar form:

(a) $z = -2 - 2i$ (b) $z = -1 + i\sqrt{3}$ (c) $z = 3i$

Ex. Write $z = 2(\cos 60 + i\sin 60)$ in rectangular form

Ex. Write $z = 4(\cos 30 + i\sin 30)$ in rectangular form

<u>Th</u>: The Product of Two Complex Number in Polar Form

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be two complex numbers in polar form. Their product $z_1 z_2$ is

$$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

To multiply two complex numbers, multiply the moduli(radii) and add the arguments(angles)

Ex. Find the product of $z_1 = 4(\cos 50 + i\sin 50)$ and $z_2 = 7(\cos 100 + i\sin 100)$

Ex. Find the product of $z_1 = 6(\cos 40 + i\sin 40)$ and $z_2 = -5(\cos 20 + i\sin 20)$

<u>Th</u>: The Quotient of Two Complex Numbers in Polar Form

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be two complex numbers in polar form. Their quotient, $\frac{z_1}{z_2}$ is

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

To divide two complex numbers, divide the moduli(radii) and subtract the arguments(angles)

Ex. Find the quotient $\frac{z_1}{z_2}$ if the complex numbers. Leave the answer in polar form

$$z_1 = 12(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}) \qquad z_1 = 4(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$$

Ex. Find the quotient $\frac{z_1}{z_2}$ if the complex numbers. Leave the answer in polar form

$$z_1 = 50(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}) \qquad z_1 = 5(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$$

Powers of Complex Numbers in Polar Form – DeMoivre's Theorem

Let $z = r(\cos\theta + i\sin\theta)$ be a complex number in polar form. If *n* is a positive integer, then *z* to the *n*th power, z^n, is

$$z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

Ex. Find $[2(\cos 20 + i\sin 20)]^6$. Write the answer in rectangular form, *a* + *bi*

Ex. Find $[2(\cos 30 + i\sin 30)]^5$. Write the answer in rectangular form, *a* + *bi*

Ex. Find $(1 + i)^6$ using DeMoivre's Theorem. Write the answer in rectangular form, *a* + *bi.*

Ex. Find $(1+i)^4$ using DeMoivre's Theorem. Write the answer in rectangular form, $a + bi$.

Since $[2(\cos 20 + i \sin 20)]^6 = 64(\cos 120 + i \sin 120)$, then $2(\cos 20 + i \sin 20)$ is called the **complex sixth root** of $64(\cos 120 + i \sin 120)$.

<u>Th</u>: DeMoivre's Theorem for Finding Complex Roots

Let w= $r(\cos\theta + i\sin\theta)$ be a complex number in polar form. If $w \neq 0$, w has n distinct complex nth roots given by:

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta+2\pi k}{n}\right) + i\sin\left(\frac{\theta+2\pi k}{n}\right)\right] \text{ radians}$$

or

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta+360°k}{n}\right) + i\sin\left(\frac{\theta+360°k}{n}\right)\right] \text{ degrees}$$

Where $k = 0, 1, 2, \ldots, n-1$

Ex. Find all the complex fourth roots of $16(\cos 120 + i \sin 120)$. Write the roots in polar form with θ in degrees.

Ex. Find all the cube roots of 8. Write the roots in rectangular form.

Homework: pg. 696-698 #3-36(3's), 37, 43, 44, 46, 47, 53, 57, 65, 68, 72,

6.6 Vectors

<u>Def</u>: Quantities that involve both a magnitude and a direction are called ***vector quantities*** or **<u>vectors</u>**.

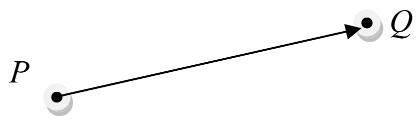
Ex. Velocity, force, acceleration

Vectors are normally denoted with boldface with an arrow over a single letter. Ex. $\vec{\mathbf{v}}$

<u>Def</u>: Quantities that only involve a magnitude are called ***scalar quantities*** or **<u>scalars</u>**.

Ex. Temperature, lengths, area, …

<u>Def</u>: A line segment to which a direction has been assigned is called a **<u>directed line segment</u>**.

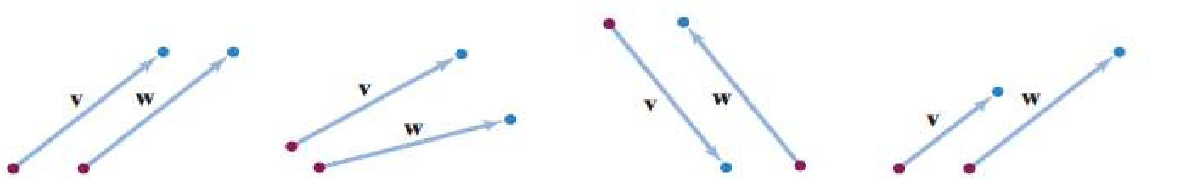


This segment is denoted $\overrightarrow{PQ}$ (from P to Q)

P is called the **<u>initial point</u>**(or ***tail***) and Q is called the **<u>terminal point</u>**(or ***head***).

<u>Def</u>: The **<u>magnitude</u>** of a directed segment, $\overrightarrow{PQ}$, is its length. This is denoted, $\|\overrightarrow{PQ}\|$. It is the distance from P to Q. Magnitudes are **NEVER** negative.

Two vectors, $\vec{v}$ and $\vec{w}$ have 4 possible relationships:



(a) **v** = **w** because the vectors have the same magnitude and same direction.

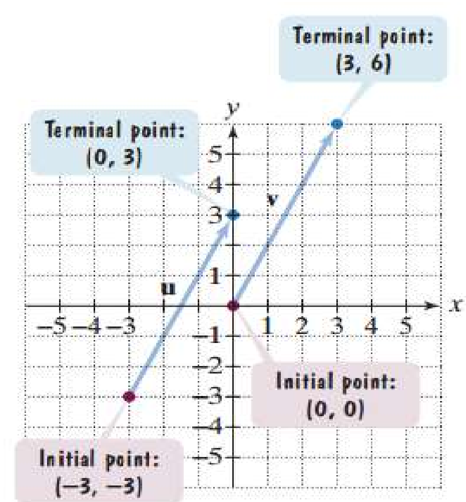
(b) Vectors **v** and **w** have the same magnitude, but different directions.

(c) Vectors **v** and **w** have the same magnitude, but opposite directions.

(d) Vectors **v** and **w** have the same direction, but different magnitudes.

Ex. Use the figure to the right to show **u = v**

(to show same direction – show same slope)

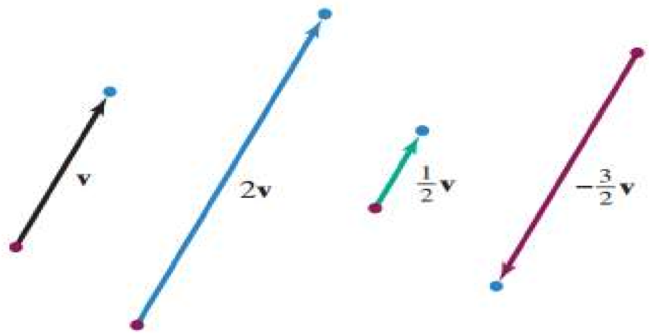


Property: Scalar Multiplication

If k is a real number and v is vector, then kv is called **scalar multiple** **of the vector v.** The magnitude and direction of k**v** is determined as follows:

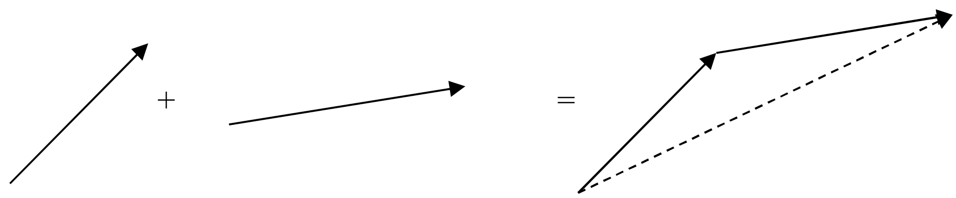
- The magnitude of k**v** is $|k|\ \|\mathbf{v}\|$ (absolute value of k times magnitude of **v**)
- If $k > 0$, then the direction remains the same
- If $k < 0$, then the direction is opposite the original direction

Ex.

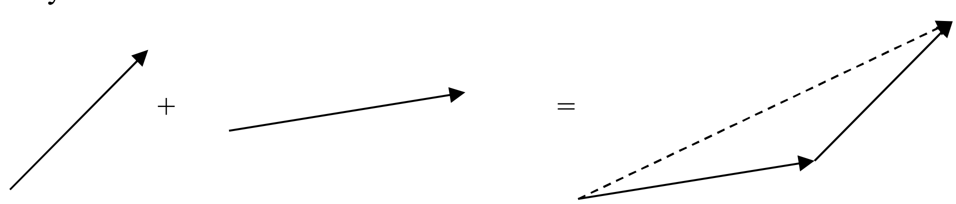


Adding Vectors:

Geometrically, you ***add vectors*** by connecting the head of one vector to the tail of another:



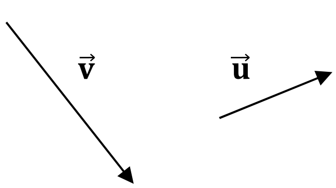
The dashed line is the sum of the two vector, called the ***resultant vector***. It does not matter which order you connect the vectors:



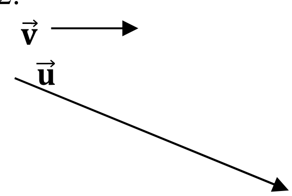
Your resultant vector will still have the same magnitude and direction.

Ex. Given vectors **v** and **u**, draw the resultant vector for **v** + **u**

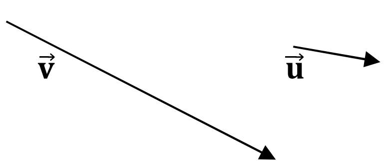
1.



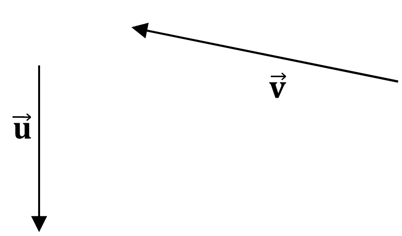
2.



3.



4.



Difference of Two Vectors, $\vec{\mathbf{v}} - \vec{\mathbf{u}}$

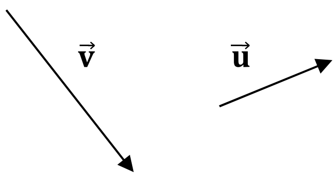
$\vec{\mathbf{v}} - \vec{\mathbf{u}} = \vec{\mathbf{v}} + (-\vec{\mathbf{u}})$ – The difference of two vectors is the same as adding the scalar opposite of **u**.

To do this geometrically (quickly): Put the head of **u** at the head of **v** and connect the tail of **v** t o the tail of **u**:

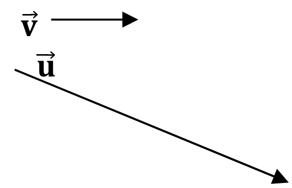
By putting the head of **u** at the head of **v**, you create –**u**

Ex. Given vectors **v** and **u**, draw the resultant vector for $\vec{\mathbf{v}} - \vec{\mathbf{u}}$

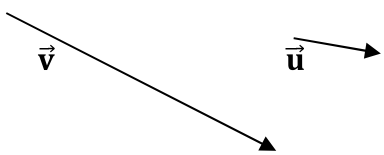
1.



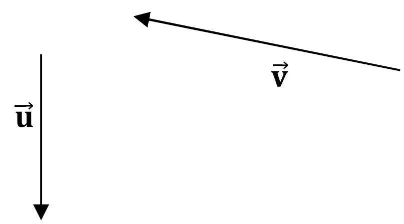
2.



3.



4.

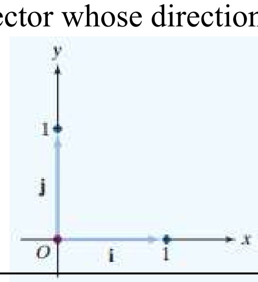


Vectors in the Rectangular Coordinate System

- Vectors can be shown in the rectangular coordinate system

<u>Def</u>: The i and j Unit Vectors

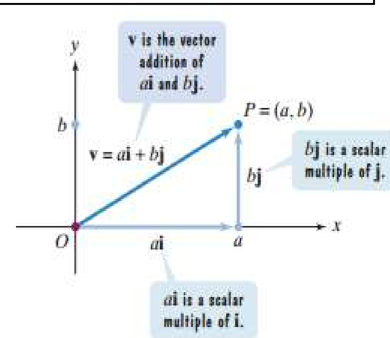
Vector **i** is the unit vector whose direction is along the positive x-axis with a magnitude of 1.
Vector **j** is the unit vector whose direction is along the positive y-axis with a magnitude of 1.



All vectors can be represented in terms of **i** and **j**.

Consider vector **v** with initial point at (0,0) and terminal point at $P(a, b)$.

We can present **v** using **i** and **j** as $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$



Property: Representing Vectors in Rectangular Coordinates

Vector **v**, from (0,0) to (a, b), is represented by

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j}$$

The real numbers a and b are called the **scalar components of v.**

- a is the **horizontal compenent** of **v**
- b is the **vertical component** of **v**

The vector sum $a\mathbf{i} + b\mathbf{j}$ is called the **linear combination** of vectors **i** & **j**

The magnitude of $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ is $\|\mathbf{v}\| = \sqrt{a^2 + b^2}$

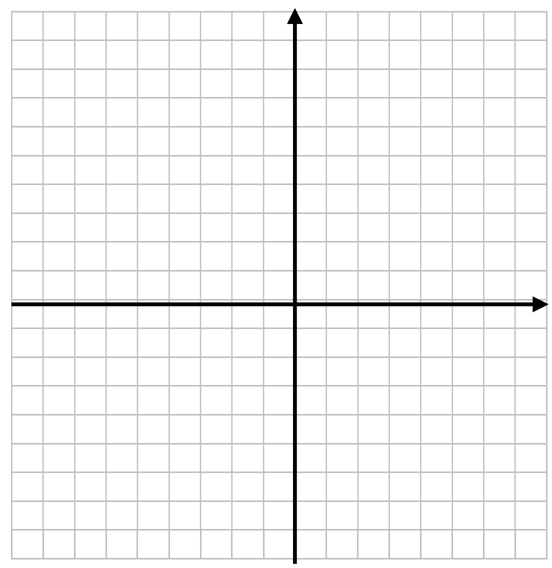
Ex. (a) One the set of axes, sketch each vector and find its magnitude.

$\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$

$\mathbf{u} = 3\mathbf{i} + 3\mathbf{j}$

(b) On the axes, sketch **w** = **v** + **u**

(c) What is linear combination and magnitude of **w**?



Property: Representing Vectors in Rectangular Coordinates

Vector **v** with initial point $P_1(x_1,y_1)$ and terminal point $P_2(x_2,y_2)$ is equal to the vector

$$\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$$

Ex. Write vector **v** in terms of **i** and **j** if initial point P_1=(3, –1) and terminal point P_2= (–2,5). What is the magnitude?

Operations with Vectors in Terms of i and j

1. **Adding/Subtracting Vectors**

   If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$, then

   $$\mathbf{v} \pm \mathbf{w} = (a_1 \pm a_2)\mathbf{i} + (b_1 \pm b_2)\mathbf{j}$$

   ex. If $\mathbf{v} = 7\mathbf{i} + 3\mathbf{j}$ and $\mathbf{w} = 4\mathbf{i} - 5\mathbf{j}$, find $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$.

2. **Scalar Multiplication with Vectors**

   If $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ and k is a real number, then the scalar multiplication of the vector **v** and that scalar k is

   $$k\mathbf{v} = ka\mathbf{i} + kb$$

   ex. If $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$, find

   (a) $6\mathbf{v}$ (b) $-3\mathbf{v}$

   ex. If $\mathbf{v} = 7\mathbf{i} + 3\mathbf{j}$ and $\mathbf{w} = 4\mathbf{i} - 5\mathbf{j}$, find $6\mathbf{v} + 5\mathbf{w}$ and $3\mathbf{v} - 2\mathbf{w}$.

Def: The Zero Vector

A vector whose magnitude is 0 is called the **zero vector, 0**. The zero vector is assigned no direction. In terms of **i** and **j**: $\mathbf{0} = 0\mathbf{i} + 0\mathbf{j}$

Properties of Vector Addition and Scalar Multiplication

If **u, v,** and **w** are vectors and *c* and *d* are scalars, then the following properties are true:

1. **Commutative Property: u + v = v + u**
2. **Associative Property: (u + v) + w = u + (v + w)**
3. **Additive Identity: u + 0 = 0 + u = u**
4. **Additive Inverse: u + (–u) = (–u) + u = 0**
5.