Note: No fractions or decimal coefficients

Chapter 7 – Systems of Equations and Inequalities
7.1 Systems of Linear Equations in Two Variables

Recall: The General Equation of a Line is $Ax + By = C$ or $Ax+By+C=0$, where $A, B,$ and C are real integers.

Ex. $3x + 2y = 9$ $\quad\quad$ $5x - 7y = 12$

Def: Two or more linear equations graphed on the same coordinate axes are called a **system of linear equations**.

- The **solution of a system of linear equations** is the ordered pair point (x, y) such that it satisfies all equations of the system. (Graphically, it is the point of intersection)

Point must satisfy all the equations

Ex. Consider the system:

$$x + 2y = 2$$
$$x - 2y = 6$$

Which point (4, -1) or (-4,3) is the solution?

Put the point into both equations and see if it is true:

$4 + 2(-1) = 4 - 2 = 2$ *check*

$4 - 2(-1) = 4 + 2 = 6$ *check*

Satisfies both equations so it is the solution to the system.

To solve a system of linear equations:

Method #1: **The Substitution Method**

1. Solve one of the two equation for one of the variables (i.e. y = …. or x = …)
2. Using the other equation, substitute the expression in for the variable you solved for.
3. Solve the equation containing the one variable
4. Substitute the solution into the equation found in step 1 to get the other variable.
5. Check the solution into both the equations to see if it is correct.

Ex. Solve the following system of equations:

1. $5x - 4y = 9$
   $x - 2y = -3$

$x = 2y - 3$
$5(2y - 3) - 4y = 9$
$10y - 15 - 4y = 9$
$6y - 15 = 9$
$6y = 24$
$y = 4$

$x = 2y - 3$
$x = 2(4) - 3$
$x = 5$

2. $3x + 2y = 4$
   $2x + y = 1$

$y = 1 - 2x$
$3x + 2(1 - 2x) = 4$
$3x + 2 - 4x = 4$
$2 - x = 4$
$-x = 2$
$x = -2$

$y = 1 - 2x$
$y = 1 - 2(-2)$
$y = 1 + 4$
$y = 5$

To solve a system of linear equations (con't):

Method #2 **The Elimination Method (also called the Addition Method)**

1. Make sure both equations are of the form: $Ax + By = C$
2. Choose a variable to eliminate (I usually choose the ones with different signs or with a 1 as a coefficient)
3. If necessary: multiply each equation by the coefficient of the opposite equation. If the terms have the same sign, make sure of the multipliers is negative.
4. Add up the "column" of the two new equations (this will eliminate a variable)
5. Solve the resulting equation
6. Substitute the solution found in step 5 into one of the two equations and solve for the other variable.
7. Check the solution (from steps 5 and 6) into the equation not used in step 6. Then check it into the other equation.

Ex. Solve by eliminating

1. $x + 2y = 48$

   $x - y = -3$ => multiply by 2 (coeff. of top)

   $2x - \cancel{2y} = -6$

   $x + \cancel{2y} = 48$ Add columns

   $3x = 42$

   $x = 14$

   $14 - y = -3$

   $-y = -17$

   $y = 17$

2. $4x + 5y = 3$ ➔ x3

   $2x - 3y = 7$ ➔ x5

   $12x + \cancel{15y} = 9$

   $10x - \cancel{15y} = 35$ Add columns

   $22x = 44$

   $x = 2$

   $12(2) + 15y = 9$

   $24 + 15y = 9$

   $15y = -15$

   $y = -1$

   Try eliminating the x

Note: If a method is not specified, choose the best one for the job………

Ex. Solve the system:

1. $5x - 2y = 4$ ➔ x2

   $10x + \cancel{4y} = 7$ *Elimination Method*

   $10x - \cancel{4y} = 8$

   $20x = 15$

   $x = \frac{15}{20}$ => $x = \frac{3}{4}$

   $10\left(\frac{3}{4}\right) - 2y = 4$

   $\frac{15}{2} - 2y = 4$

   $15 - 4y = 8$

   $-4y = -7$

   $y = \frac{7}{4}$ *They all aren't pretty!!*

2. $y = 3x - 2$

   $15x - 5y = 10$ *Substitution*

   $15x - 5(3x - 2) = 10$

   $15x - 15x + 10 = 10$

   $10 = 10$

   There is no variable left in the equation.

   If this happen, you have two choices, the statement will either be:

   1. *TRUE – every point on the line is a solution (the two equations are the same line)*
   2. *FALSE – no solution (lines are parallel)*

Homework: Pg. 742 #1-4, 6-30(3's), 31, 35, 38, 41, 43, 45, 47

7.1 Systems of Linear Equations (Day 2)

Ex. Solve the system:

1. $x + 4y = 14$
   $2x - y = 1$

$-2x - 8y = -28$

$-9y = -27$
$y = 3$

$x + 4(3) = 14$
$x + 12 = 14$

$x = 2$

2. $y = 4x + 1$
   $3x + 2y = 13$

$3x + 2(4x + 1) = 13$
$3x + 8x + 2 = 13$
$11x = 11$
$x = 1$

$y = 4(1) + 1$
$y = 5$

Applications of Systems of Equations

Ex. A chemist is working on a flu vaccine needs to mix 10% sodium-iodine solution with a 60% sodium-iodine solution to obtain 50ml of a 30% sodium-iodine solution. How many mL of each solution are needed to create the 30% solution?

Strategies for Solving Using Systems:
1. Read the problem carefully. Use variables to represent unknown quantities.
2. Write a system of equations for each condition
3. Solve the system and answer the question
4. Check to see if it is reasonable.

Mixture Problems

Let x = amt of 10% solution *y = amt of 60% solution*

$x + y = 50$ *First equation is the amount of solutions*

$10x + 60y = 30(50)$ *2nd equation is the mixture (% * amt+% * amt)=% * amt*

$-60x - 60y = -3000$

$-50x = -1500$ *Solve the system*

$x = 30$

$y = 20$

Mixture problems are always done this way:
$\%_1\ (amt_1) + \%_2(amt_2) + \ldots = Final\%(final\ amt)$

$amt_1 + amt_2 + \ldots = final\ amt$

Ex. Mr. Jankowski wishes to make 12 L of a 36% acid solution for a lab. He has in his supplies an 18% acid solution and a 45% acid solution of the same types. How many liters of each solution must he use to get his 12 liters?

Let x = amt of 18% acid y = amt of 45% acid

$x + y = 12$ 432
$18x + 45y = 36(12)$
$-18x - 18y = 216$
$27y = 216$
$y = 8$
$x = 12 - y = 12 - 8 = 4$

8 L of 18% acid and 4 L of 45% acid

Motion Problem

Ex. When a small airplane flies with the wind, it can travel 450 miles in 3 hours. When the same plane flies in the opposite direction, against the wind, it takes 5 hours to fly the same distance. Find the average velocity of the plane in no wind and the average velocity of the wind. Recall: $D = RT$

Let p = average velocity of the plane in no wind. *w = average velocity of the wind*

Add velocities - faster

With the wind: $(p + w)(3) = 450 \rightarrow 3p + 3w = 450 \rightarrow 15p + 15w = 2250$

Against wind: $(p - w)(5) = 450 \rightarrow 5p - 5w = 450 \rightarrow 15p - 15w = 1350$

Subtract velocities - slower

Plane's average velocity with no wind is 120 mph $30p = 3600 \rightarrow p = 120$

Wind's average velocity is 30 mph $(120 + w)(3) = 450$

$120 + w = 150$
$w = 30$

Functions of Business:

A company produces and sells *x* units of a particular product.

Revenue Function: Amount of money earned from selling x units:

$$\boldsymbol{R(x) = (price)x}$$

Cost Function: Amount of money needed to produce x units:

$$\boldsymbol{C(x) = fixed\ cost + (price\ to\ make)x}$$

Def: The break even point for a company is where $R(x) = C(x)$, where the cost to produce x units equals the revenue earned by selling x units.

Ex. A company is planning to manufacture new wheelchairs. The fixed cost will be \$500,000 and it will cost \$400 to produce each wheelchair. Each wheelchair will sell for \$600.

(a) Write the cost function, C(x), of producing x units.
(b) Write the revenue function, R(x), from the sale of x units.
(c) How many wheelchairs must be produced and sold to break-even?

(a) $C(x) = 500000 + 400x$
(b) $R(x) = 600x$
(c) $R(x) = C(x)$
$600x = 400x + 500000$
$200x = 500000$
$x = 2500$ wheelchairs needed to be produced to break even

Def: The profit, P(x), generated from producing and selling x units of a product is given by the following function: $P(x) = R(x) - C(x)$, where R(x) is the revenue function and C(x) is the cost function.

If P(x) < 0, then the company would suffer a loss (or losing money)
If P(x) > 0, then the company would make money (gain a profit).
If P(x) = 0, then the company breaks even.

Ex. A company that manufactures running shoes has a fixed cost of \$300,000. Additionally, it costs \$30 to produce each pair of shoes. Shoes are sold at \$80 per pair.

(a) Write the cost, revenue, and profit functions for producing and selling x pairs of shoes.
(b) How many pairs of shoes are needed to be produced and sold to break even?

(a) $C(x) = 300000 + 30x$
$R(x) = 80x$
$P(x) = 80x - (30x + 300000) = 50x - 300000$

(b) Break even: $P(x) = 0$
$50x - 300000 = 0$
$50x = 300000$
$x = 6000$ pair of shoes are needed to be produced and sold to break even

Homework: Pg. 742 – 746 #7, 14, 26, 35, 40, 44, 46, 55, 56, 58, 62, 65, 67, 68, 73, 75 76, 85, 91

7.2 Systems of Linear Equations in Three Variables

Def: A linear equation in three variables is an equation of the form:

$$Ax + By + Cz = D$$

Ex. $3x - 2y + 5z = 9$

To Solve a Linear System in Three Variables - Elimination Method

1. Make sure all the equations are of the form $Ax + By + Cz = D$
2. Using a pair of equations, eliminate of the variables.
3. Using another pair of equations, eliminate the same variable.
4. Solve the resulting two equations in two variables using elimination or substitution. (Get both values)
5. Substitute the two values into one of the three equations and solve for the remaining variable.
6. Use a different equation for the first check

Ex. Solve the system:

1. $5x - 2y - 4z = 3$ **A**
   $3x + 3y + 2z = -3$ **B**
   $-2x + 5y + 3z = 3$ **C**

A $5x - 2y - 4z = 3$
3xB $6x + 6y + 4z = -6$
____
$11x + 4y = -3$ **D**

3xA $15x - 6y - 12z = 9$
4xC $-8x + 20y + 12z = 12$
____
$7x + 14y = 21$ **E**

−7xD $-77x - 28y = 21$
11xE $77x + 154y = 231$
____
$126y = 252$
$y = 2$

$11x + 4(2) = -3$
$11x + 8 = -3$
$11x = -11$
$x = -1$

$5(-1) - 2(2) - 4z = 3$
$-5 - 4 - 4z = 3$
$-9 - 4z = 3$
$-4z = 12$

$z = -3$

Check:
Using B:
$3(-1) + 3(2) + 2(-3) = -3$
$-3 + 6 - 6 = -3$
$-3 = -3$

2. $x + 4y - z = 20$
   $3x + 2y + z = 8$
   $2x - 3y + 2z = -16$

A $x + 4y - z = 20$
B $3x + 2y + z = 8$
____
$4x + 6y = 28$ **D**

2xA $2x + 8y - 2z = 40$
C $2x - 3y + 2z = -16$
____
$4x + 5y = 24$ **E**

D $4x + 6y = 28$
−E $-4x - 5y = -24$
____
$y = 4$

$4x + 6(4) = 28$
$4x + 24 = 28$
$4x = 4$
$x = 1$

$3(1) + 2(4) + z = 8$
$3 + 8 + z = 8$
$11 + z = 8$
$z = -3$

Check:
Using A:
$1 + 4(4) - (-3) = 20$
$1 + 16 + 3 = 20$
$20 = 20$

3. $x + z = 8$
$x + y + 2z = 17$
$x + 2y + z = 16$

Try yourself:
Answer: $x = 3 \quad y = 4 \quad z = 5$

4. $2y - z = 7$
$x + 2y + z = 17$
$2x - 3y + 2z = -1$

Try yourself:
Answer: $x = 4 \quad y = 5 \quad z = 3$

Homework: Day 1 – Pg. 753 – 755 #5, 11, 15, 17, 25, 39 Day 2: pg. 753-756 #7, 9, 13, 16, 22, 55

7.3 Partial Fractions

Ex. Express as a single fraction: $\frac{3}{x-4} - \frac{2}{x+2}$

Recall: For any fractions $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$

$$\frac{3(x+2) - 2(x-4)}{(x-4)(x+2)} = \frac{3x + 6 - 2x + 8}{(x-4)(x+2)} = \frac{x+14}{(x-4)(x+2)}$$

Notice the denominator is the product of the two denominators.

The original two fractions in the above example are called ***partial fractions***.

To express a rational expression as a partial fraction – There are four cases

1. The denominator is a product of distinct linear factors
2. The denominator is a product of linear factors, so of which are repeated.
3. The denominator has prime quadratic factors, none repeated
4. The denominator has a quadratic factor that is repeated.

Distinct Linear Factors:

Def: A linear factor is a factor of the form $ax + b$.

If a rational expression $\frac{P(x)}{Q(x)}$ has a denominator consisting of *n* distinct (unique) linear factors:

$$\frac{P(x)}{Q(x)} = \frac{P(x)}{(a_1x+b_1)(a_2x+b_2)\dots(a_nx+b_n)}$$

can be broken down into *n* partial fractions:

$$= \frac{A_1}{(a_1x + b_1)} + \frac{A_2}{(a_2x + b_2)} + \frac{A_3}{(a_3x + b_3)} + \cdots + \frac{A_n}{(a_nx + b_n)}$$

Ex. Write as partial fractions:

1. $\frac{x+14}{x^2-2x-8}$

First factor the denominator, then split up into separate fractions.

$$\frac{x+14}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$$
$$\frac{x+14}{(x-4)(x+2)} = \frac{A(x+2) + B(x-4)}{(x-4)(x+2)}$$
$$\frac{x+14}{(x-4)(x+2)} = \frac{Ax + 2A + Bx - 4B}{(x-4)(x+2)}$$
$$\frac{x+14}{(x-4)(x+2)} = \frac{(A+B)x + (2A-4B)}{(x-4)(x+2)}$$

Using the numerators:

$A + B = 1$
$2A - 4B = 14$
$4A + 4B = 4$

$6A = 18$

$A = 3$
$3 + B = 1$
$B = -2$

$$\frac{3}{x-4} + \frac{-2}{x+2}$$

2. $\frac{5x-1}{x^2+x-12}$

$$\frac{5x-1}{(x-3)(x+4)} = \frac{A}{x-3} + \frac{B}{x+4}$$
$$\frac{5x-1}{(x-3)(x+4)} = \frac{A(x+4) + B(x-3)}{(x-3)(x+4)}$$
$$\frac{5x-1}{(x-3)(x+4)} = \frac{Ax + 4A + Bx - 3B}{(x-3)(x+4)}$$
$$\frac{5x-1}{(x-3)(x+4)} = \frac{(A+B)x + (4A-3B)}{(x-3)(x+4)}$$

Using the numerators:

$A + B = 5$
$4A - 3B = -1$
$3A + 3B = 15$

$7A = 14$

$A = 2$

$2 + B = 5$

$B = 3$

$$\frac{2}{x-3} + \frac{3}{x+4}$$

Linear Factors which Repeat:

If a rational expression $\frac{P(x)}{Q(x)}$ has a denominator consisting of *n* repeating linear factors:

$$\frac{P(x)}{Q(x)} = \frac{P(x)}{(ax+b)^n}$$

can be broken down into *n* partial fractions:

$$= \frac{A_1}{(ax+b)} + \frac{A_2}{(ax+b)^2} + \frac{A_3}{(ax+b)^3} + \cdots + \frac{A_n}{(ax+b)^n}$$

Note: Include one fraction for EACH power of $(ax+b)$

Ex. Express as partial fractions:

Each numerator is multiplied by the other denominators so A is multiplied by $(x-3)^2$, B by $x(x-3)$, and C by x

1. $\frac{x-18}{x(x-3)^2} \quad = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$$\frac{x-18}{x(x-3)^2} = \frac{A(x-3)^2 + Bx(x-3) + Cx}{x(x-3)^2}$$

$$\frac{x-18}{x(x-3)^2} = \frac{A(x^2-6x+9) + Bx^2 - 3B + Cx}{x(x-3)^2} = \frac{Ax^2 - 6A + 9A + Bx^2 - 3B + Cx}{x(x-3)^2}$$

$$\frac{x-18}{x(x-3)^2} = \frac{(A+B)x^2 + (-6A-3B+C)x + 9A}{x(x-3)^2}$$

$A + B = 0$

$-6A - 3B + C = 1$ $\qquad -6(-2) - 3(2) + C = 1$

$9A = -18 \quad \rightarrow \quad A = -2$ $\qquad 12 - 6 + C = 1$

$C = -5$

$$\boxed{\frac{x-18}{x(x-3)^2} = \frac{-2}{x} + \frac{2}{x-3} + \frac{-5}{(x-3)^2}}$$

$-2 + B = 0 \quad \rightarrow \quad B = 2$

2. $\frac{x+2}{x(x-1)^2} \quad = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

$$\frac{x+2}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2 + Bx(x-1) + Cx}{x(x-1)^2}$$

$$\frac{x+2}{x(x-1)^2} = \frac{A(x^2-2x+1) + Bx^2 - Bx + Cx}{x(x-1)^2} = \frac{Ax^2 - 2Ax + A + Bx^2 - Bx + Cx}{x(x-1)^2}$$

$$\frac{x+2}{x(x-1)^2} = \frac{(A+B)x^2 + (-2A-B+C)x + A}{x(x-1)^2}$$

$A + B = 0$
$-2A - B + C = 1$
$A = 2$

$$\boxed{\frac{x+2}{x(x-1)^2} == \frac{2}{x} + \frac{-2}{x-1} + \frac{3}{(x-1)^2}}$$

$A + B = 0$
$2 + B = 0$
$B = -2$

$-2A - B + C = 1$
$-2(2) - (-2) + C = 1$
$-4 + 2 + C = 1$
$C = 3$

Homework: Pg 756 #9, 12, 13, 17, 21, 23 **(Optional Day 2: Pg 756 #11, 14, 18, 20, 22, 28)**

7.3 Partial Fractions (Day 2)

Prime quadratic factors, none repeated:

Def: A prime quadratic expression is a quadratic expression of the form $ax^2 + bx + c$ which cannot be factored.

In other words: $b^2 - 4ac < 0$ or $b^2 - 4ac$ is not a perfect square

If $ax^2 + bx + c$ is a prime quadratic factor of the denominator of $\frac{P(x)}{Q(x)}$, then the list of partial fractions will consist of a term of the form:

$$\frac{Ax+B}{ax^2+bx+c}$$

FOIL!!!

Ex. Write as partial fractions:

1. $\frac{3x^2+17x+14}{(x-2)(x^2+2x+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4}$

$$\frac{3x^2+17x+14}{(x-2)(x^2+2x+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4} = \frac{A(x^2+2x+4)+(Bx+C)(x-2)}{(x-2)(x^2+2x+4)} = \frac{Ax^2+2Ax+4A+Bx^2-2Bx+Cx-2C}{(x-2)(x^2+2x+4)}$$

$$\frac{3x^2+17x+14}{(x-2)(x^2+2x+4)} = \frac{(A+B)x^2+(2A-2B+C)+(4A-2C)}{(x-2)(x^2+2x+4)}$$

$A + B = 3$
$2A - 2B + C = 17$
$4A - 2C = 14$

So double the top equation and add it to the 3rd one and eliminate B

$2A + 2B \quad = 6$
$2A - 2B + C = 17$

$4A + C \quad = 23$

Then multiply the 3rd equation by –1 and add to the result

$-4A + 2C = -14$

$3C = 9$
$C = 3$

$4A - 2C = 14$
$4A - 2(3) = 14$
$4A = 20$
$A = 5$

$A + B = 3$

$5 + B = 3$
$B = -2$

$$\boxed{\frac{3x^2+17x+14}{(x-2)(x^2+2x+4)} = \frac{5}{x-2} + \frac{-2x+3}{x^2+2x+4}}$$

2. $\frac{8x^2+12x-20}{(x+3)(x^2+x+2)}$

Try yourself:
Answer: $\frac{2}{x+3} + \frac{6x-8}{x^2+x+2}$

Repeated Prime quadratic factors:

If the denominator of $\frac{P(x)}{Q(x)}$ has a repeated prime quadratic factor, $(ax^2+bx+c)^n$, then the list of partial fractions will consist of fractions:

$$\frac{P(x)}{(ax^2+bx+c)^n} = \frac{A_1x+B_1}{(ax^2+bx+c)} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \cdots + \frac{A_nx+B_n}{(ax^2+bx+c)^n}$$

Note: Include one fraction for EACH power of ax^2+bx+c

Ex. Write as partial fractions

1. $\frac{5x^3-3x^2+7x-3}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2}$

$\frac{5x^3-3x^2+7x-3}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2}$ multiply both sides by $(x^2+1)^2$

$5x^3-3x^2+7x-3 = (Ax+B)(x^2+1)+Cx+D = Ax^3+Bx^2+Ax+B+Cx+D$

$5x^3-3x^2+7x-3 = Ax^3+Bx^2+(A+C)x+(B+D)$

$A=5$ $B=-3$ *this is from the first two terms*

$A+C=7$

$B+D=-3$

$5+C=7 \rightarrow C=2$

$-3+D=-3 \rightarrow D=0$

$$\frac{5x^3-3x^2+7x-3}{(x^2+1)^2} = \frac{5x-3}{x^2+1} + \frac{2x}{(x^2+1)^2}$$

This is probably the fastest way to do this. When multiplying by the common denominator (from the left side), all the fractions reduce and become "non-fractions". Makes the polynomial a little more workable.

2. $\frac{2x^3+x+3}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$

$\frac{2x^3+x+3}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} \quad \times\ (x^2+1)^2$

$\cancel{(x^2+1)^2}\frac{2x^3+x+3}{\cancel{(x^2+1)^2}} = (x^2+1)^{\cancel{2}}\frac{Ax+B}{\cancel{x^2+1}} + \cancel{(x^2+1)^2}\frac{Cx+D}{\cancel{(x^2+1)^2}}$

$2x^3+x+3 = (Ax+B)(x^2+1)+(Cx+D)$

$2x^3+x+3 = Ax^3+Bx^2+Ax+B+Cx+D$

$2x^3+x+3 = Ax^3+Bx^2+(A+C)x+(B+D)$

$A=2$

$B=0$ (no squared term)

$A+C=1 \rightarrow 2+C=1 \quad C=-1$

$B+D=3 \rightarrow 0+D=3 \quad D=3$

This is the "missing" step:

$$= \frac{2x}{x^2+1} + \frac{-(x-3)}{(x^2+1)^2}$$

then factor '–' out front of the fraction

$$\frac{2x^3+x+3}{(x^2+1)^2} = \frac{2x}{x^2+1} + \frac{-x+3}{(x^2+1)^2} = \frac{2x}{x^2+1} - \frac{x-3}{(x^2+1)^2}$$

Homework: Pg 765-766 #27, 28, 31, 33, 35, 42

(Optional Day 2: Pg. 765-766 #24, 26, 34, 36, 40, 41)

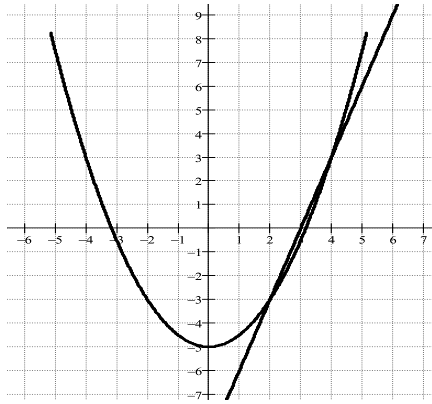
7.4 Systems of Nonlinear Equations in Two Variables (Nonlinear Systems)

Ex. Solve the system:

$$x^2 = 2y + 10$$
$$3x - y = 9$$

$y = 3x - 9$
$x^2 = 2(3x - 9) + 10$
$x^2 = 6x - 18 + 10$
$x^2 - 6x + 8 = 0$
$(x - 2)(x - 4) = 0$

$x = 2$	$x = 4$
$y = 3(2) - 9$	$y = 3(4) - 9$
$y = -3$	$y = 3$
$(2, -3)$	$(4, 3)$



In this example, you need to solve one of the two equations for one of the variables and substitute it into the other equation.

It does not matter which equation or which variable, just don't solve for the squared term in the quadratic equation.

Check the solution in BOTH equations

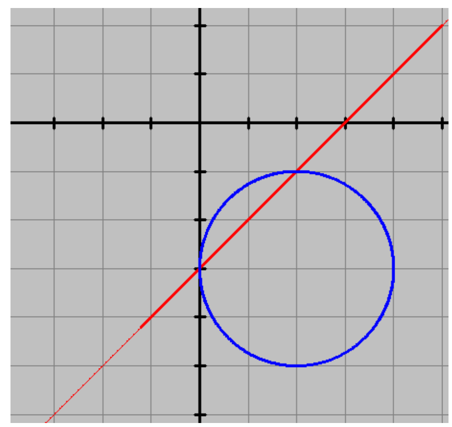
<u>Graphically</u>: The two equations are a parabola and a line intersecting in two points. (see graph to right)

Ex. Solve the system

$x - y = 3 \longrightarrow y = x - 3$
$(x - 2)^2 + (y + 3)^2 = 4$

$(x - 2)^2 + (x - 3 + 3)^2 = 4$
$x^2 - 4x + 4 + x^2 = 4$
$2x^2 - 4x = 0$
$2x(x - 2) = 0$

$x = 0$	$x = 2$
$y = 0 - 3$	$y = 2 - 3$
$y = -3$	$y = -2$
$(0, -3)$	$(2, -2)$



Ex. Solve the system

A $4x^2 + y^2 = 13$ **Ellipse**
B $x^2 + y^2 = 10$ **Circle**

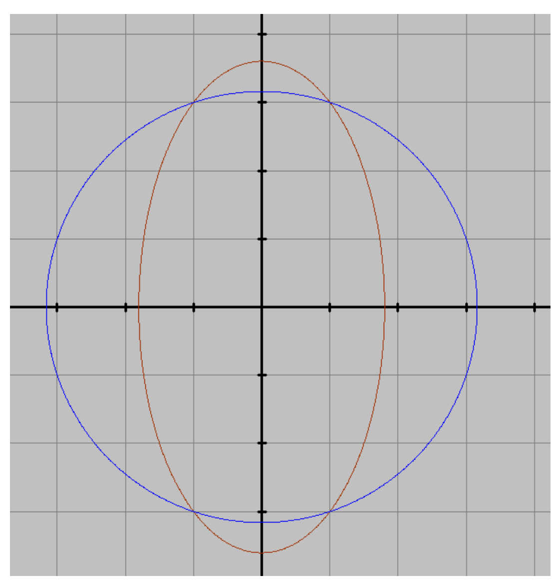
You can use the elimination method if you have common squared terms. Eliminate the same way as with a system of linear equation.

A: $4x^2 + y^2 = 13$
−4B: $-4x^2 - 4y^2 = -40$

$-3y^2 = -27$
$y^2 = 9$

$y = \pm 3$

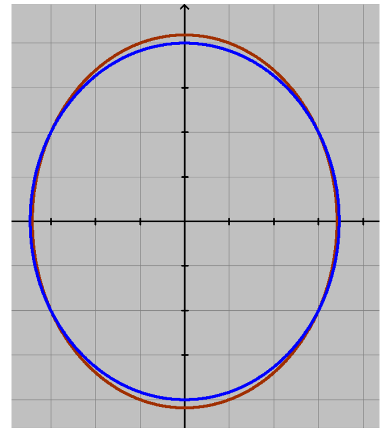
$x^2 + (3)^2 = 10$	$x^2 + (-3)^2 = 10$
$x^2 + 9 = 10$	$x^2 + 9 = 10$
$x^2 = 1$	$x^2 = 1$
$x = \pm 1$	$x = \pm 1$
$(1,3)$ $(-1,3)$	$(1, -3)$ $(-1, -3)$



Ex. Solve the system

$$3x^2 + 2y^2 = 35$$
$$4x^2 + 3y^2 = 48$$

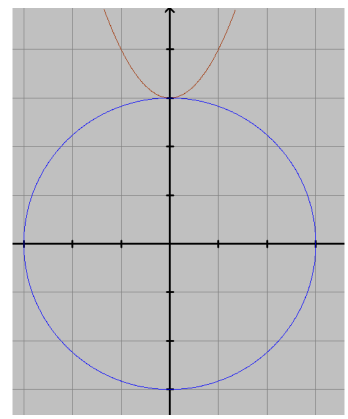
Try yourself:
Answer: (3, 2), (3, –2), (–3, 2), and (–3, –2)



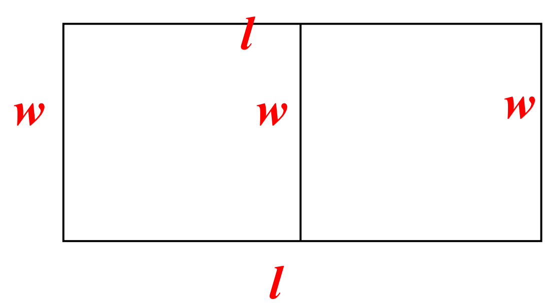
Ex. Solve the system

$$y = x^2 + 3$$
$$x^2 + y^2 = 9$$

Try yourself:
Answer: (0, 3)



Ex. You have 36 yards of fencing to build a rectangular enclosure shown in the diagram to the right. Some the fencing will be used to an internal divider. If you would like to enclose 54 square yards, what are the dimensions of the rectangular enclosure?



Let l = length of the enclosure
Let w = width of the enclosure

$A = lw = 54$

$2l + 3w = 36 \quad \rightarrow \quad l = \dfrac{36 - 3w}{2}$

$\left(\dfrac{36 - 3w}{2}\right)(w) = 54$

$(36 - 3w)(w) = 108$

$36w - 3w^2 = 108$

$0 = 3w^2 - 36w + 108$

$0 = 3(w^2 - 12w + 36)$

$0 = 3(w - 6)(w - 6)$

$0 = 3 \quad w = 6 \quad w = 6$

$lw = 54 \quad \rightarrow \quad 6l = 54 \quad \rightarrow \quad l = 9$

So, the width of the enclosure is 6 yards and the length of the enclosure is 9 yards

6 yds x 9 yds

Homework: Pg. 774 – 775 #3, 7, 11, 21, 23, 29, 35, 43, 45
(Optional Day 2: Pg. 774 – 775 #5, 9, 19, 24, 30, 32, 38, 44, 46)

7.5 Systems of Inequalities

Def: A linear inequality in two variables is a mathematical expression of the form: $Ax + By > C$

Note: The > can be any inequality operator: <, >, ≤, ≥, or ≠

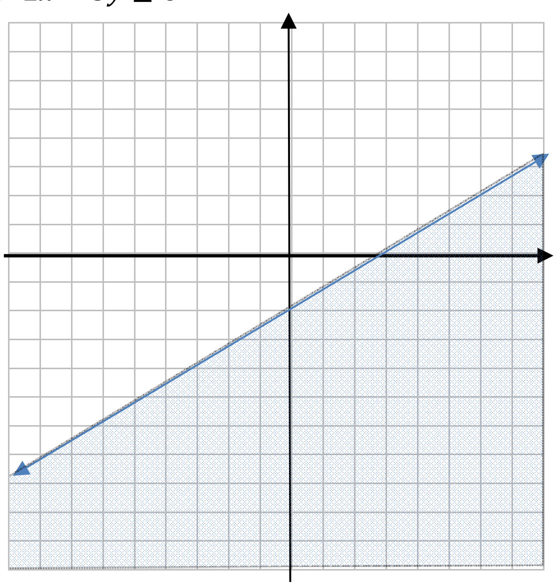
The solution is any ordered pair (x, y) that makes the inequality true. There are usually an infinite number of solutions for an inequality.

To graph a linear inequality:

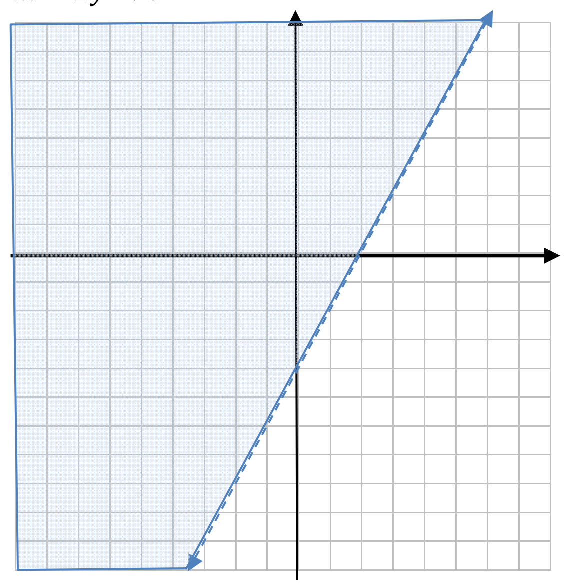
1. Graph the inequality as if it were an equality, $Ax + By = C$.
   - If the inequality is < or >, use a dashed line
   - If the inequality is ≤ or ≥, use a solid line
2. Choose a test point by selecting any point not on the line, substitute this point into the inequality
   - If the statement is true, shade the side of the line that the point lies on
   - If the statement is false, shade the side of the line that the point does not lie on

Ex. Graph the inequality

1. $2x - 3y \geq 6$

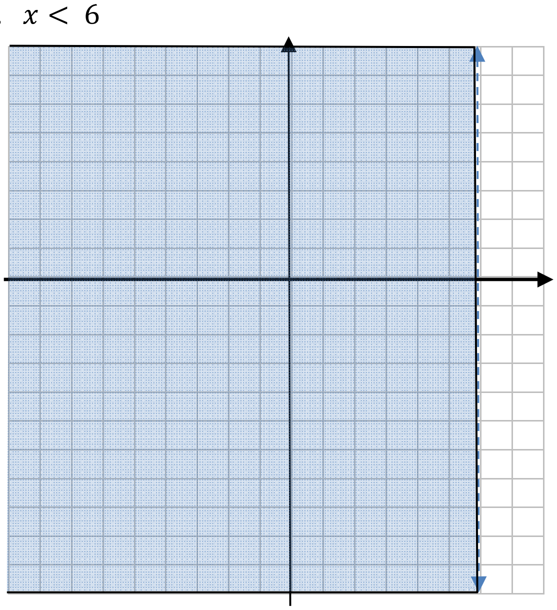


2. $4x - 2y < 8$

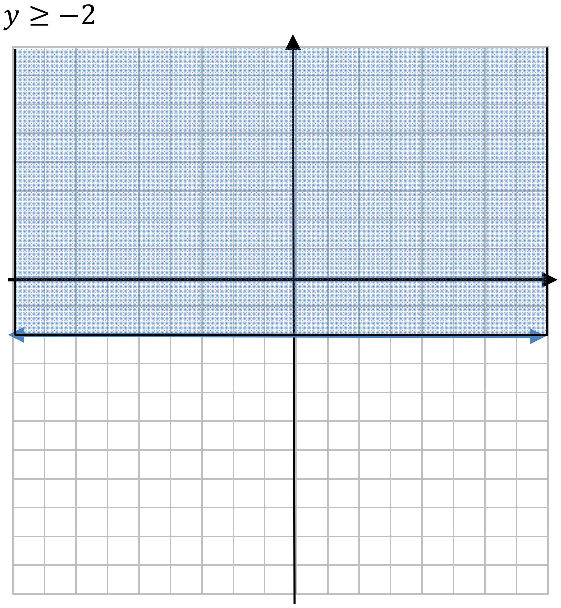


Choosing a direction to shade without a test point:

1. $x < 6$



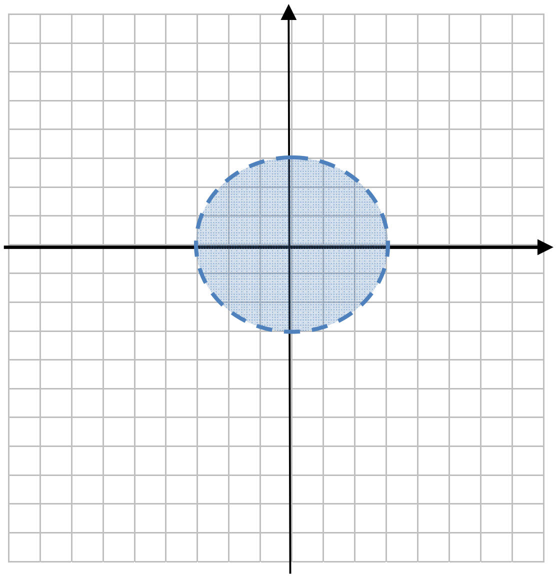
2. $y \geq -2$



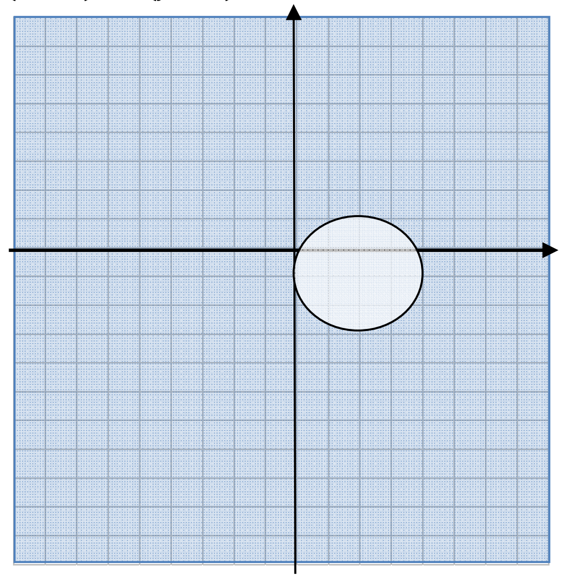
What about a nonlinear inequality? No different

Ex. Graph the inequality:

1. $x^2 + y^2 < 9$



2. $(x-2)^2 + (y+1)^2 \geq 4$

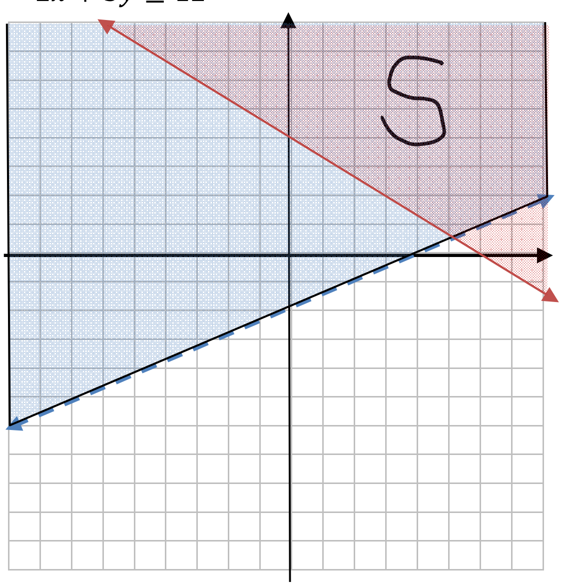


Graphing a System of Inequalities:

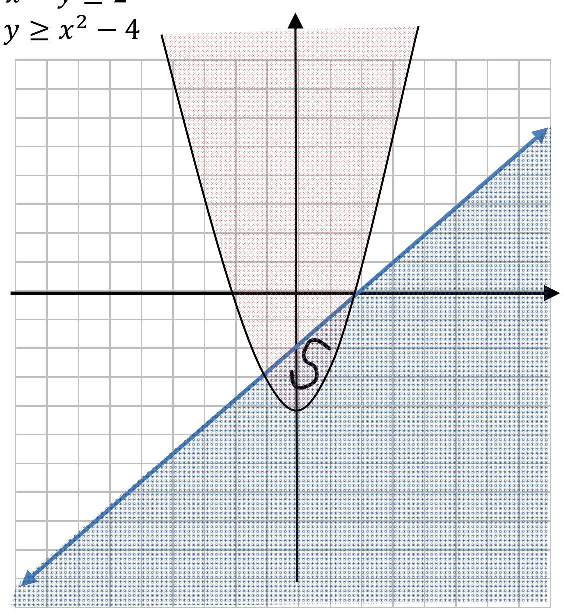
Graph the two inequalities on the same axes. Where the shading overlaps is the solution.

Ex. Graph the solution set of the each of the following systems. Label the solution with an ***S***.

1. $x - 2y < 4$
   $2x + 3y \geq 12$



2. $x - y \geq 2$
   $y \geq x^2 - 4$



Homework: Pg. 787 #3-8, 19, 21, 27, 31, 40, 45, 47, 57

7.6 Linear Programming

An important application of systems of linear inequalities is **linear programming**.

- This is a method for solving a system of inequalities in which a particular quantity must be maximized or minimized.
- Linear programming is one of the most widely used tools in management science.
  - Helps businesses allocate resources to manufacture products to maximize profit.

Def: An objective function is an algebraic expression in two or more variables describing a quantity that must be maximized or minimized.

Ex. Bottled water and medical supplies are to be shipped to survivors of an earthquake by plane. Each container of bottled water will serve 10 people and each med kit will aid 6 people. If *x* represents the number of bottles to be shipped and *y* represents the number of med kit, write the objective function that models the number of people that can be helped.

Let x = # of water bottles *Let y = # of med kits* *z = The number of people helped*

 # helped: 10x *6y* $z = 10x + 6y$

Ex. A company manufactures bookshelves and desks for computers. Let *x* represent the number of bookshelves manufactured daily and *y* be the number of desks manufactured daily. The company's profits are $25 per bookshelf and $55 per desk. Write the objective function models the company's total daily profit, *z*, from *x* bookshelves and *y* desks.

Profit from Bookshelves = 25x *Profit from desks = 55y*

Total Daily Profit: $z = 25x + 55y$

Def: Constraints are restrictions that can occur in situations.

Ex. In the first example:

(a) planes can only carry so much weight or volume. Suppose each plane can carry no more than 80,000 pounds. The bottled water weighs 20 lbs per container and a med kit weighs 10 lbs. Let *x* represent the number of bottled water to be shipped and *y* be the number of med kits. Write an inequality to model this constraint.

Weight of the bottled water = $20x$ *Weight of the med kits =* $10y$

Total weight of shipped items $z = 20x + 10y \leq 80{,}000$

(b) Each plane can only carry a total volume of supplies that does not exceed 6,000 cubic feet. Each water bottle is 1 cubic foot and each med kit has a volume of 1 cubic foot. Using x and y, like in part (a), write an inequality that models this constraint

$1x + 1y \leq 6000 \rightarrow x + y \leq 6000$

Ex. From the second example:

(a) To maintain high quality, the company should not manufacture more than a total of 80 bookshelves and desks per day. Write an inequality for this constraint.

$1x + 1y \leq 80 \rightarrow x + y \leq 80$

(b) To meet customer demand, the company must manufacture between 30 and 80 bookshelves per day, inclusive. Also, the company must manufacture between 10 and 30 desks per day, inclusive. Write an inequality that models this constraint.

$30 \leq x \leq 80$ $10 \leq y \leq 30$

Solving a Linear Programming Problem

Let $z = ax + by$ be an objective function that depends on x and y. Also, z is subject to a number of constraints on x and y. If a maximum or minimum value of z exists, it can be determined as follows:

1. Graph the system of inequalities representing the constraints.
2. Find the value of the z at each CORNER or VERTEX of the graphed region. The max and min of the objective function will occur at one or more of these corner points.

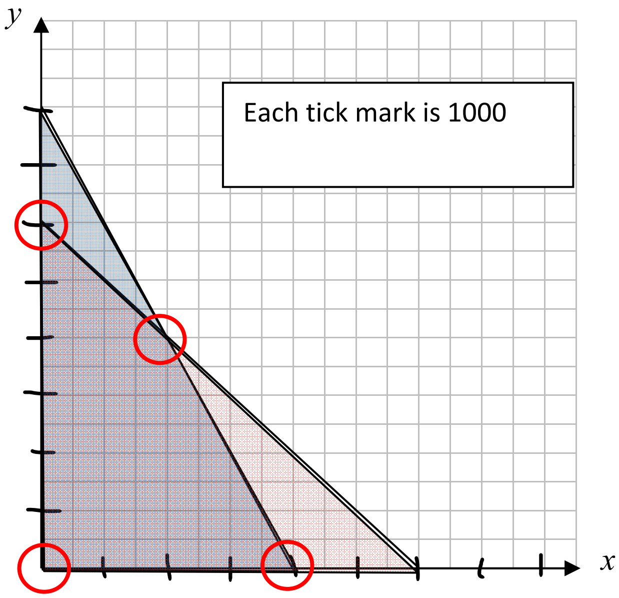
Ex. Ok, determine the number of bottles of water and med kits should be sent to maximize the number of earthquake victims who can be helped.

$20x + 10y \leq 80{,}000$
$x + y \leq 6000$

$$z = 10x + 6y$$

(0,6000): $z = 10(0) + 6(6000) = 36{,}000$
(0,0): $z = 10(0) + 6(0) = 0$
(4000,0): $z = 10(4000) + 6(0) = 40{,}000$
(2000,4000): $z = 10(2000) + 6(4000) = 44{,}000$ *MAX*

To maximize the number of victims helped:
2000 water bottles and 4000 med kits



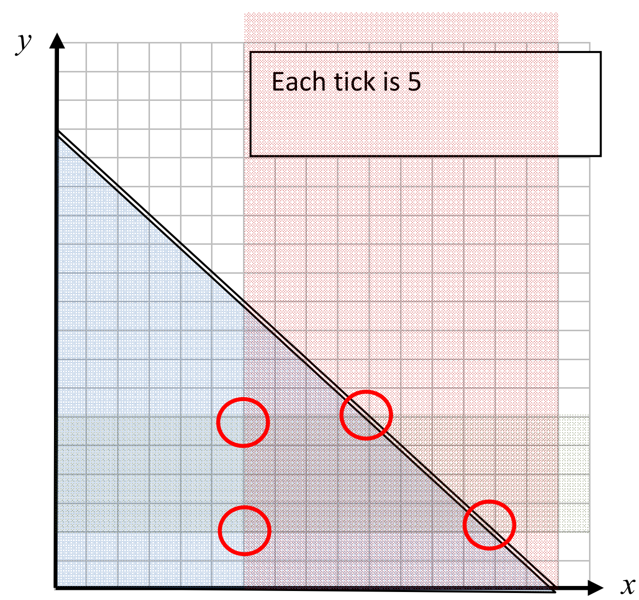
Ex. How many bookshelves and desks should be manufactured per day to obtain the maximum profit? What is the maximum daily profit?

$30 \leq x \leq 80$ $10 \leq y \leq 30$
$x + y \leq 80$

$$z = 25x + 55y$$

(30,10): $z = 25(30) + 55(10) = 1{,}300$
(30,30): $z = 25(30) + 55(30) = 2{,}400$
(50,30): $z = 25(50) + 55(30) = 2{,}900$ * MAX*
(70,10): $z = 25(70) + 55(10) = 2{,}300$

To maximize profit: 50 bookshelves and 30 desks.



Homework: Pg. 795-796 #1-4, 7, 11, 15, 17, 21