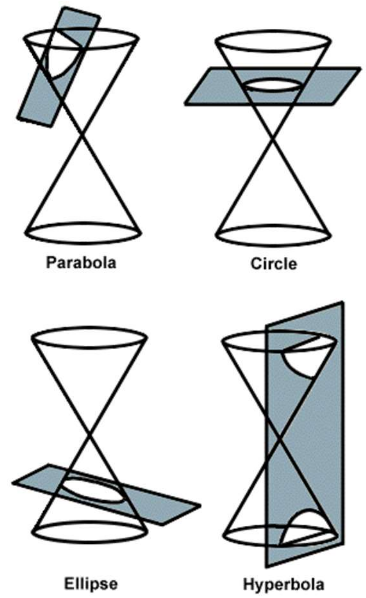
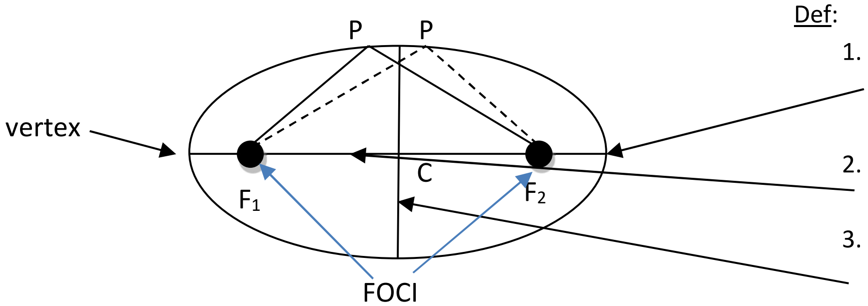
Chapter 9 – Conic Sections and Analytical Geometry

9.1 The Ellipse

- A property of right circular cones is that if a plane intersects a cone, the intersection will be a particular curve (see figure to the right):
  - If the plane is parallel to the circular base of the cone, it will result in a circle on the plane.
  - If the plane is perpendicular to the base of the cone, it will result in a hyperbola.
  - If the plane slices through the side of the code not parallel to the base, it will result in an ellipse.
  - If the plane slices through the base and the side of the code, the result is a parabola.



Def: An ellipse is the set of all points *P* (a locus), in a plane the sum of whose distances from two fixed points F_1 and F_2, is constant. These two fixed points are called the foci (plural for focus). The midpoint of the segment connecting the foci is the center of the ellipse.



Def:

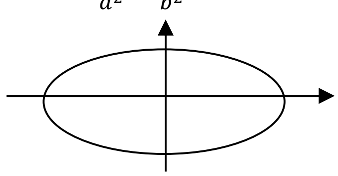
1. The line through the foci intersects the ellipse at two points, called vertices (singular: vertex)
2. The segment that connects the vertices is called the major axis.
3. The segment that passes through the center and is perpendicular to the major axis is called the minor axis.

Major axis is longer than the minor axis.

Standard Form of the Equation of an Ellipse:

Centered at the origin: (2a is length of major axis, 2b is length of minor axis)

Position 1: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



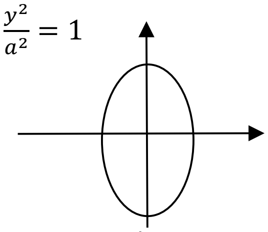
Major axis is horizontal

Vertices: $(\pm a, 0)$

Endpoints of Minor axis: $(0, \pm b)$

Foci: $(\pm c, 0)$

Position 2: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$



$c^2 = a^2 - b^2$

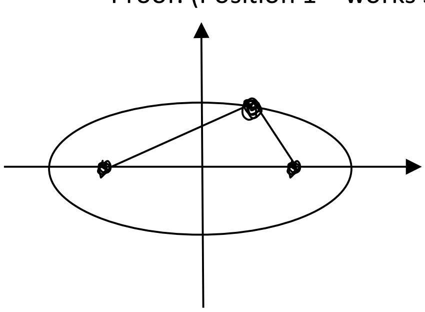
Major axis is vertical

Vertices: $(0, \pm a)$

Endpoints of Minor axis: $(\pm b, 0)$

Foci: $(0, \pm c)$

Proof: (Position 1 – works same for Position 2)



Ex. On the set of axes, graph and locate the foci:

1. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Center: (0,0)

$a^2 = 9 \rightarrow a = 3 \quad b^2 = 4 \rightarrow b = 2$

Position I

V: $(0 \pm 3,0) \rightarrow V(-3,0),(3,0)$

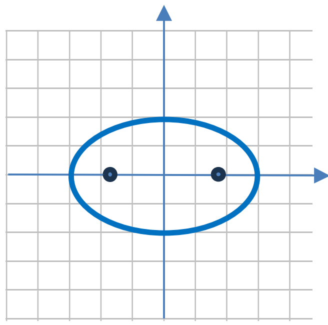
E: $(0,0 \pm 2) \rightarrow E(0,-2),(0,2)$

$c^2 = a^2 - b^2$

$c^2 = 9 - 4 = 5$

$c = \pm\sqrt{5}$

F: $(0 \pm \sqrt{5},0) \rightarrow (-\sqrt{5},0),(\sqrt{5},0)$



2. $\frac{x^2}{36} + \frac{y^2}{9} = 1$

Center: (0,0)

$a^2 = 36 \rightarrow a = 6 \quad b^2 = 9 \rightarrow b = 3$

Position I

V: $(0 \pm 6,0) \rightarrow V(-6,0),(6,0)$

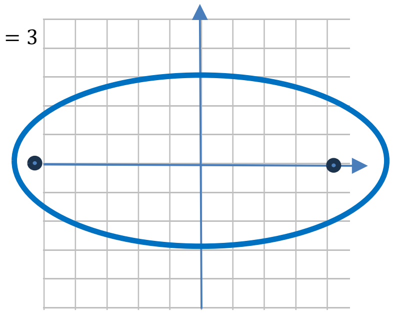
E: $(0,0 \pm 3) \rightarrow E(0,-3),(0,3)$

$c^2 = a^2 - b^2$

$c^2 = 36 - 9 = 27$

$c = \pm\sqrt{27}$

F: $(0 \pm \sqrt{27},0) \rightarrow (-\sqrt{27},0),(\sqrt{27},0)$



3. $25x^2 + 16y^2 = 400$

Divide by 400 to make it 1:

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

Center: (0,0)

$a^2 = 25 \rightarrow a = 5 \quad b^2 = 16 \rightarrow b = 4$

Position II

V: $(0,0 \pm 5) \rightarrow V(0,-5),(0,5)$

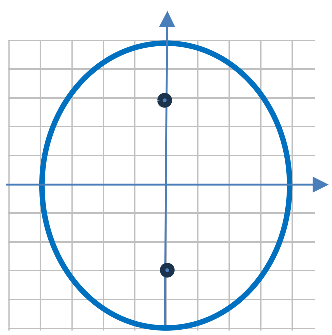
E: $(0 \pm 4,0) \rightarrow E(-4,0),(4,0)$

$c^2 = a^2 - b^2$

$c^2 = 9$

$c = \pm 3$

F: $(0,0 \pm 3) \rightarrow (0,-3),(0,3)$



4. $9x^2 + 16y^2 = 144$

Divide by 144 to make it 1:

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Center: (0,0)

$a^2 = 16 \rightarrow a = 4 \quad b^2 = 9 \rightarrow b = 3$

Position I

V: $(0 \pm 4,0) \rightarrow V(-4,0),(4,0)$

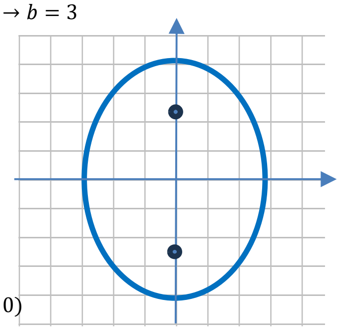
E: $(0,0 \pm 3) \rightarrow E(0,-3),(0,3)$

$c^2 = a^2 - b^2$

$c^2 = 16 - 9 = 7$

$c = \pm\sqrt{7}$

F: $(0 \pm \sqrt{7},0) \rightarrow (-\sqrt{7},0),(\sqrt{7},0)$



Ex. Find the equation, in standard form, of the ellipse whose foci at (-1,0) and (1,0) and whose vertices are (-2,0) and (2,0)

From the Foci: (1) Midpoint is center: (0,0) (2) They lie horizontally – Pos I (3) $2c = 2 \rightarrow c = 1 \rightarrow c^2 = 1$

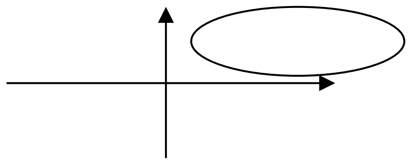
From the Vertices: (1) Midpoint is center (2) They lie horizontally – Pos I (3) $2a = 4 \rightarrow a = 2 \rightarrow a^2 = 4$

From above: $c^2 = a^2 - b^2 \rightarrow 1 = 4 - b^2 \rightarrow b^2 = 3$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

Standard Equation of an Ellipse whose Center is (h,k):

Position 1: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



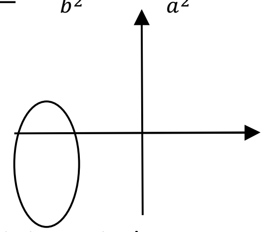
Major axis is horizontal

Vertices: $(h \pm a, k)$

Endpoints of Minor axis: $(h, k \pm b)$

Foci: $(h \pm c, k)$

Position 2: $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$



$c^2 = a^2 - b^2$

Major axis is vertical

Vertices: $(h, k \pm a)$

Endpoints of Minor axis: $(h \pm b, k)$

Foci: $(h, k \pm c)$

Ex. Graph $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$. What are the coordinates of the foci?

Position II: Center $(1, -2)$

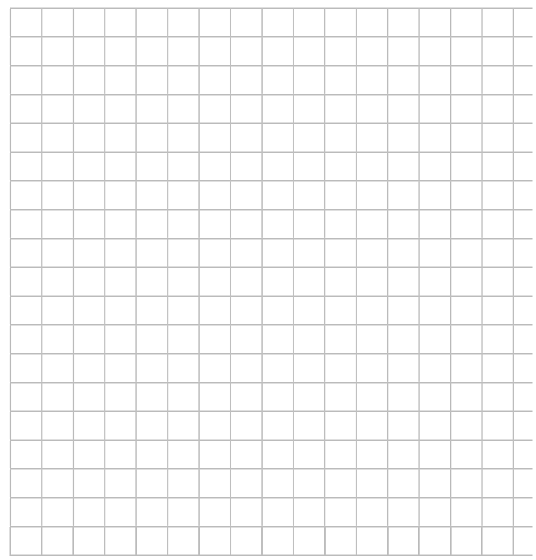
$a^2 = 9 \rightarrow \quad a = 3$

$b^2 = 4 \rightarrow b = 2$

$c^2 = a^2 - b^2 = 5 \rightarrow c = \sqrt{5}$

The coordinates of the foci are $(h, k \pm c)$ in Position II

So, the coordinates of the foci are $\left(1, -2 + \sqrt{5}\right)$ and $(1, -2 - \sqrt{5})$



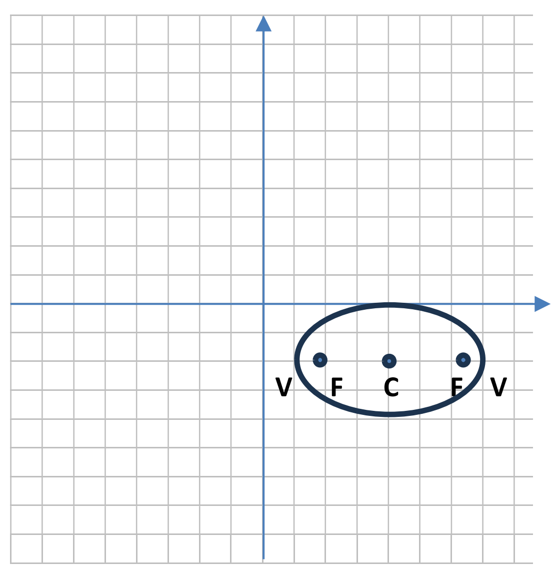
General Form of the Equation of an Ellipse: $Ax^2 + Cy^2 + Dx + Ey + F = 0$ with $A \neq C$

Ex. Graph and find the center, vertices, foci, and endpoints of the minor axis of the ellipse whose equation is

1. $4x^2 + 9y^2 - 32x + 36y + 64 = 0$

You have to complete the square:

$$4x^2 + 9y^2 - 32x + 36y + 64 = 0$$
$$4x^2 - 32x + 9y^2 + 36y = -64$$
$$4(x^2 - 8x + 16) + 9(y^2 + 4y + 4) = -64 + 64 + 36$$
$$4(x-4)^2 + 9(y+2)^2 = 36$$
$$\frac{(x-4)^2}{9} + \frac{(y+2)^2}{4} = 1 \rightarrow \quad Position\ I$$



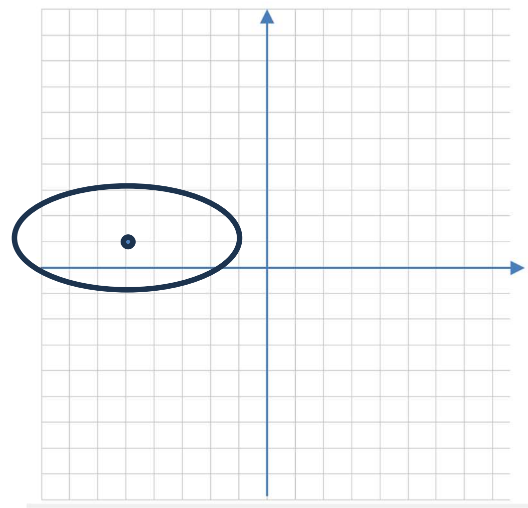
Center: (4,–2)

Vertices: $(4 \pm 3, -2) = (1, -2)$ and $(7, -2)$

Endpoints of Minor Axis: $(4, -2 \pm 2) = (4, 0)$ and $(4, 4)$

Foci: $c^2 = 9 - 4 = 5 \rightarrow c = \sqrt{5} \rightarrow (4 \pm \sqrt{5}, -2)$

2. $x^2 + 4y^2 + 10x - 8y + 13 = 0$



You have to complete the square:

$$x^2 + 4y^2 + 10x - 8y + 13 = 0$$
$$x^2 + 10x + 4y^2 - 8y = -13$$
$$(x^2 + 10x + 25) + 4(y^2 - 2y + 1) = -13 + 25 + 4$$
$$(x + 5)^2 + 4(y - 1)^2 = 16$$
$$\frac{(x + 5)^2}{16} + \frac{(y - 1)^2}{4} = 1 \rightarrow \ Position\ I$$

Center: $(-5, 1)$

Vertices: $(-5 \pm 4, 1) = (1, 1)$ and $(-9,\ 1)$

Endpoints of Minor Axis: $(-5, 1 \pm 2) = (-5, -1)$ and $(-5. 3)$

Foci: $c^2 = 16 - 4 = 12 \rightarrow c = \sqrt{12} \rightarrow (-5 \pm 2\sqrt{3}, 1)$

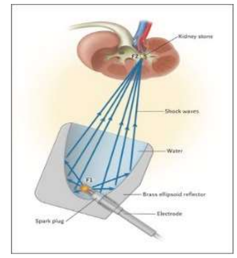
Homework

9.1b The Ellipse – Day 2

Applications involving Ellipses:

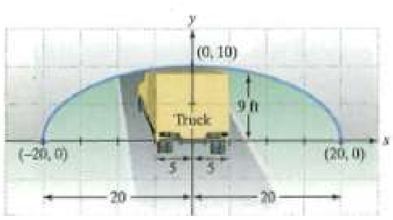
1. Planet orbits around the sun are elliptical
2. A whispering gallery is an elliptical room with an elliptical, dome-shaped ceiling. People standing at the foci can whisper and hear each other quite clearly, while people in different locations can not. The US Capitol Building has a room like this. John Quincy Adams understood the phenomenon and used this principal to eavesdrop on private conversations.
3. Medical equipment to disintegrate kidney stones are elliptical in shape.

Whispering in an elliptical dome



And others

Ex. A semi-elliptical archway over a one-way road has a height of 10 ft and a width of 40 ft. Your truck has a width of 10 ft and a height of 9 ft. Will your truck clear the archway's opening?



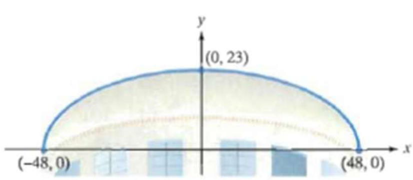
The arch is an ellipse in Position I. $a = 20$ and $b = 10$

Therefore, it's equation is: $\frac{x^2}{400} + \frac{y^2}{100} = 1$

The truck is 10 ft wide and 9 feet high, therefore the top corners will be at $(\pm 5, 9)$.

Testing the point: $\frac{5^2}{400} + \frac{9^2}{100} = \frac{25}{400} + \frac{81}{100} = \frac{1}{16} + \frac{81}{100} = \frac{50}{800} + \frac{648}{800} = \frac{698}{800} < 1 \rightarrow FITS$

Ex. John Quincy Adams eavesdropped on conversations in the Capitol using the foci of the ellipse that formed the ceiling in Statuary Hall (then the House of Representatives). If the Statuary Hall was 96 feet long and 23 feet tall, then how far from the center of the ellipse, along the major axis, did Adams place is desk? (Round to the nearest foot). Hint – put the ceiling and floor on a coordinate axes.



The major axis is 96', so $a = 48 \rightarrow \; a^2 = 2304$

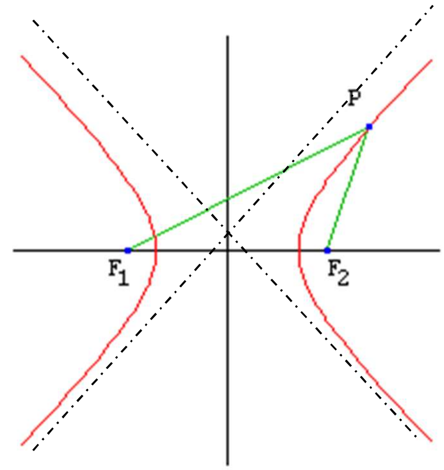
The minor axis is 23', so $b = 23 \rightarrow b^2 = 529$

Therefore, $c^2 = 2304 - 529 = 1775 \rightarrow \; \sqrt{1775} = 42.13\, ft \; \approx 42\, ft$

Place each desk 42 ft from the center of the room.

Homework:

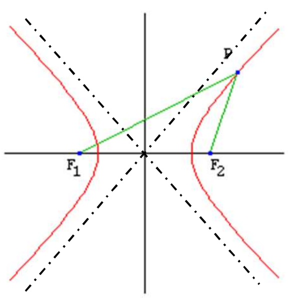
9.2 The Hyperbola – Day 1



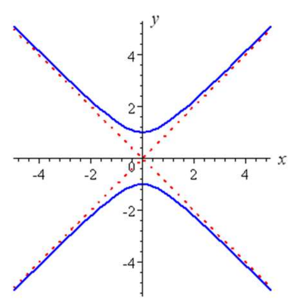
Def: A hyperbola is the locus of points in a plane the difference of whose distances from two fixed points, called foci, is constant.

- Hyperbolas are sometimes mistaken for 2 parabolas.
- The line that connects the two foci is called the transverse axis.
- The midpoint of the two foci is called the center.
- The two points on the hyperbola that intersect the transverse axis are called the vertices.
    - The center is also the midpoint of the vertices.
- The two branches of the hyperbola are asymptotic to two oblique lines (dashed lines)
    - The curve approaches the asymptotes but never touch
    - The two asymptotes intersect at the center.
- Similar to the ellipse, there are two positions for the hyperbola:

Position I – transverse axis horizontal



Position II – transverse axis vertical



Standard Form of the Equation of a Hyperbola: with center (0,0)

Position I: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Position II: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

- Note: a^2 is always with the positive fraction
    - If x is positive – position I ….. if y is positive – position II
- For either position, the distance between the vertices on the transverse axis is 2a
    - a units from the center along the transverse axis
- To locate the foci, use the formula: $c^2 = a^2 + b^2$ (or $b^2 = c^2 - a^2$)

Ex. Find the vertices, and foci of each of the following hyperbolas

1. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Pos I: C(0,0)

$c^2 = a^2 + b^2 = 16 + 9 = \sqrt{25} = 5$

Vertices: $(-4, 0), (4, 0)$

Foci: $(-5, 0), (5, 0)$

2. $\frac{y^2}{4} - \frac{x^2}{16} = 1$

Pos II: C(0,0)

$c^2 = a^2 + b^2 = 4 + 16 = \sqrt{20}$

Vertices: $(0, -2)$, $(0, 2)$

Foci: $(0, -\sqrt{20}), (0, \sqrt{20})$

Ex. Find the standard form of the equation of a hyperbola whose foci are (0,5) and (0,-5) and whose vertices are (0,-3) and (0, 3).

From the Foci: (1) Midpoint is center: (0,0) (2) They lie vertically – Pos I (3) $2c = 10 \rightarrow c = 5 \rightarrow c^2 = 25$

From the Vertices: (1) Midpoint is center (2) They lie vertically – Pos I (3) $2a = 6 \rightarrow a = 3 \rightarrow a^2 = 9$

From above: $c^2 = a^2 + b^2 \rightarrow 25 = 9 + b^2 \rightarrow b^2 = 16$

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

- As stated earlier, hyperbolas are asymptotic to two lines.

  The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has two asymptotes: $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$

  The hyperbola $\frac{y}{a^2} - \frac{x^2}{b^2} = 1$ has two asymptotes: $y = \frac{a}{b}x$ and $y = -\frac{a}{b}x$

Ex. What are the equations of the asymptotes of the following hyperbolas?

1. $\frac{x^2}{25} - \frac{y^2}{9} = 1$

   $a^2 = 25 \rightarrow a = 5 \quad b^2 = 9 \rightarrow b = 3$

   $y = \pm\frac{3}{5}x$

2. $\frac{y^2}{4} - \frac{x^2}{16} = 1$

   $a^2 = 4 \rightarrow a = 2 \quad b^2 = 16 \rightarrow b = 4$

   $y = \pm\frac{1}{2}x$

Ex. On the set of axes below, graph $\frac{x^2}{25} - \frac{y^2}{16} = 1$

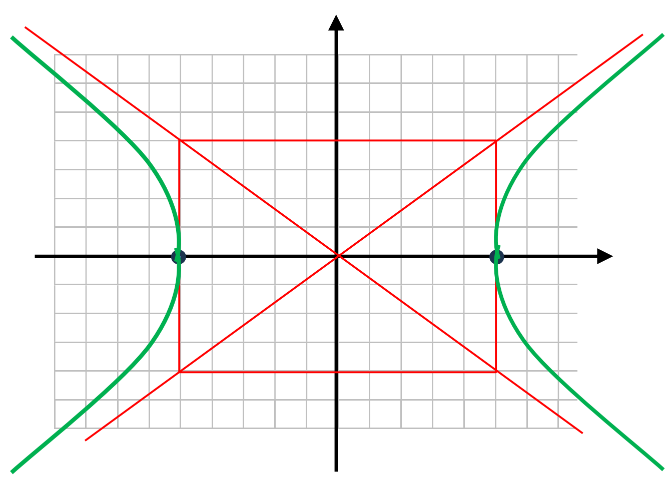
$a^2 = 25 \rightarrow a = 5 \quad b^2 = 16 \rightarrow b = 4$

$c^2 = a^2 + b^2 = 25 + 16 \rightarrow c = \sqrt{41}$

$y = \pm\frac{4}{5}x$

<u>To graph a hyperbola</u>:

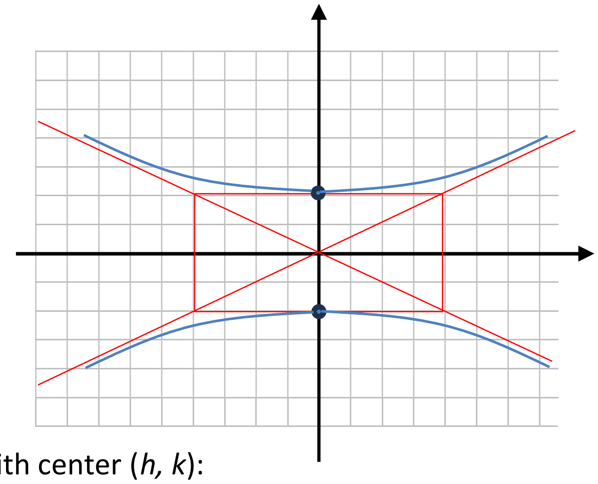
1. Locate the center & vertices
2. Use dashed lines to draw a rectangle centered at the origin using sides of 2a and 2b
3. Use dashed line to draw the diagonals of this rectangle – these are the asymptotes
4. Draw the two branches of the hyperbola by starting at each vertex and approaching the asymptotes



Homework:

9.2 The Hyperbola – Day 2

Ex. On the set of axes, graph $\frac{y^2}{4} - \frac{x^2}{16} = 1$



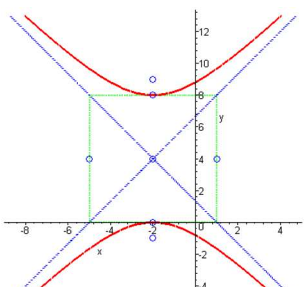
$a^2 = 4 \rightarrow a = 2 \quad b^2 = 16 \rightarrow b = 4 \quad \text{Pos II}$

$c^2 = a^2 + b^2 = 4 + 16 \rightarrow c = \sqrt{20}$

$y = \pm\frac{1}{2}x$

<u>Def</u>: The Standard Form of the Equation of a Hyperbola with center (*h, k*):

<u>Position I</u>: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ <u>Position II</u>: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$



Position II

- a^2 is always with the positive fraction
- Equations of the Asymptotes:

  <u>Position I</u>: $y - k = \pm\frac{b}{a}(x - h)$

  <u>Position II</u>: $y - k = \pm\frac{a}{b}(x - h)$

- Graphing is done the same way as before

<u>To graph a hyperbola</u>:

1. Locate the center & vertices
2. Use dashed lines to draw a rectangle centered at the origin using sides of 2a and 2b
3. Use dashed line to draw the diagonals of this rectangle – these are the asymptotes
4. Draw the two branches of the hyperbola by starting at each vertex and approaching the asymptotes

Ex. Graph $\frac{(x-2)^2}{16} - \frac{(y-3)^2}{9} = 1$. What are the equations of the asymptotes? Where are the foci?

Position I – Center (2, 3)

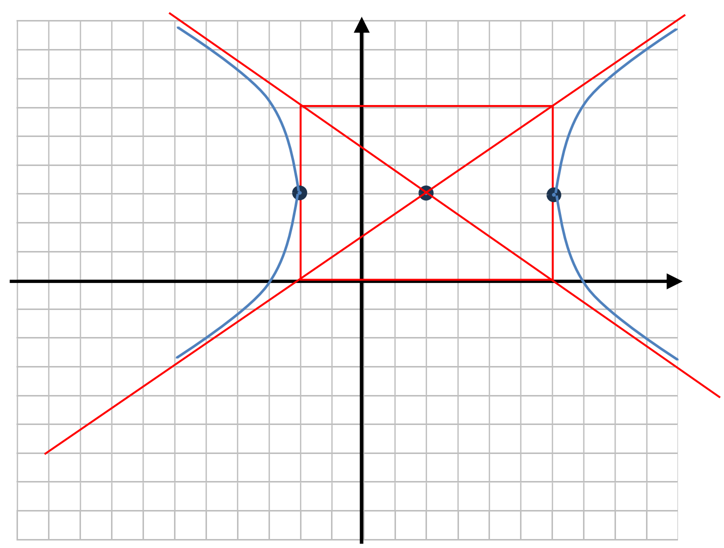
$a^2 = 16 \rightarrow a = 4 \quad b^2 = 9 \rightarrow b = 3$

$c^2 = a^2 + b^2 = 25 \rightarrow c = 5$

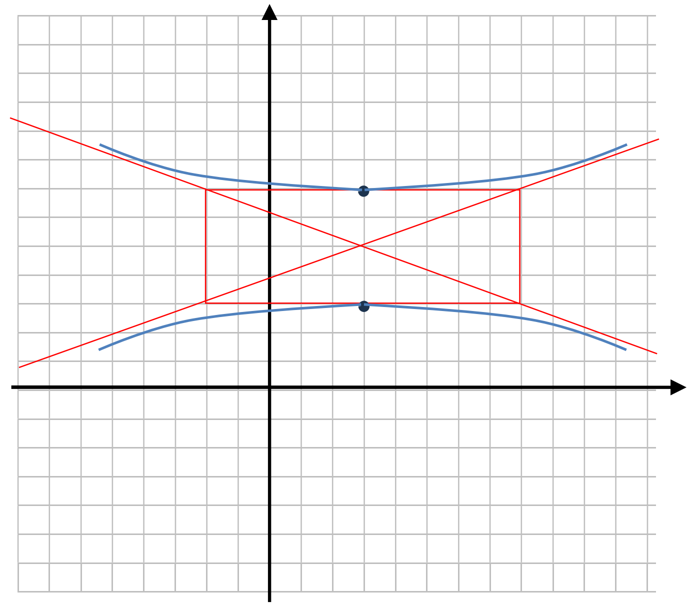
Vertices: $V(2 \pm 4) = (-2, 3), (6, 3)$

Foci: $F(2 \pm 5,) = (-3, 3), (7, 3)$

$y - 3 = \pm\frac{3}{4}(x - 2)$



Ex. Graph $4x^2 - 25y^2 - 24x + 250y - 489 = 0$. What are the equations of the asymptotes? Where are the foci?



Completing the square:

$4x^2 - 24x - 25y^2 + 250y = 489$

$4(x^2 - 6x + 9) - 25(y^2 - 10y + 25) = 489 + 36 - 625$

$4(x-3)^2 - 25(y-5)^2 = -100$

$$\frac{(y-5)^2}{4} - \frac{(x-3)^2}{25} = 1$$

Position II – Center (3, 5)

$a^2 = 4 \rightarrow a = 2 \quad b^2 = 25 \rightarrow b = 5$

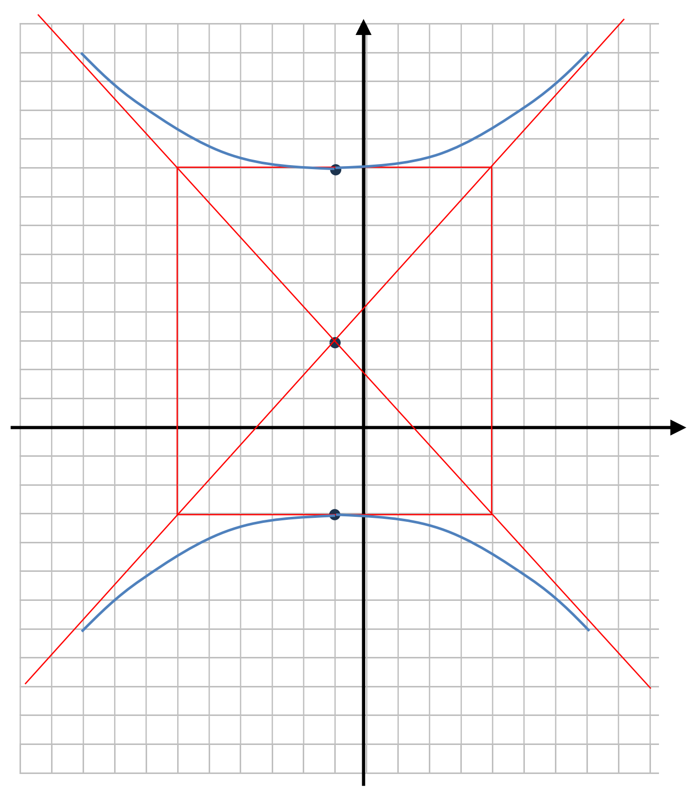
$c^2 = a^2 + b^2 = 4 + 25 = 29 \rightarrow c = \sqrt{29}$

Vertices: (3, 3), (3,7)

Foci: $(3, 5 \pm \sqrt{29})$

Asymptotes: $y - 5 = \pm\frac{2}{5}(x-3)$

Ex. Graph $25y^2 - 36x^2 - 150y - 72x - 711 = 0$. What are the equations of the asymptotes? Where are the foci?



Completing the square:

$25y^2 - 150y - 36x^2 - 72x = 711$

$25(y^2 - 6y + 9) - 36(x^2 + 2x + 1) = 711 + 225 - 36$

$25(y-3)^2 - 36(x+1)^2 = 900$

$$\frac{(y-3)^2}{36} - \frac{(x+1)^2}{25} = 1$$

Position II – Center (−1, 3)

$a^2 = 36 \rightarrow a = 6 \quad b^2 = 25 \rightarrow b = 5$

$c^2 = a^2 + b^2 = 36 + 25 = 61 \rightarrow c = \sqrt{61}$

Vertices: (−1, 9), (−1,−3)

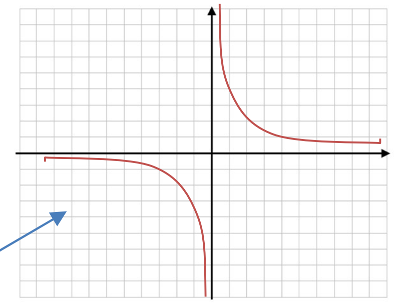
Foci: $(-1, 3 \pm \sqrt{61})$

Asymptotes: $y - 3 = \pm\frac{6}{5}(x+1)$

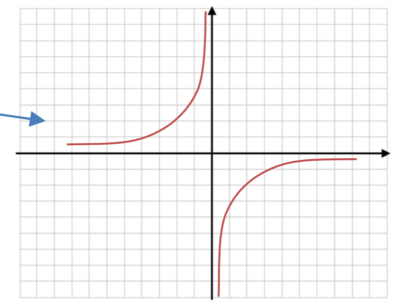
Def: A **rectangular hyperbola**, centered at the origin, is a hyperbola whose equation is of the form:

$$xy = N$$

where N is any real value not equal to 0.



- Based on the equation, what are the asymptotes?

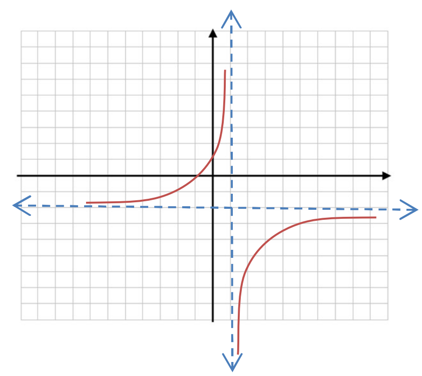


- If $N > 0$, then the curves of the hyperbola will be in Quadrants I & III
- If $N < 0$, then the curves of the hyperbola will be in Quadrants II & IV
- A rectangular hyperbola can be centered at a point (h,k) not at the origin as well.

Def: A **rectangular hyperbola**, centered at the point (h,k), is a hyperbola whose equation is of the form:

$$(x - h)(y - k) = N$$

where N is a real number not equal to 0.



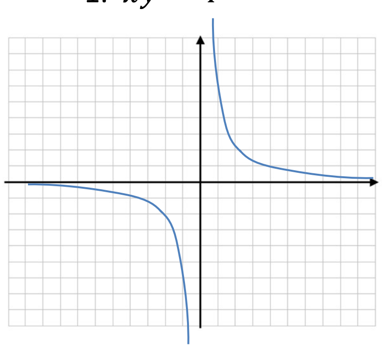
- The asymptotes for the rectangular hyperbola are:

$$x = h \text{ and } y = k$$

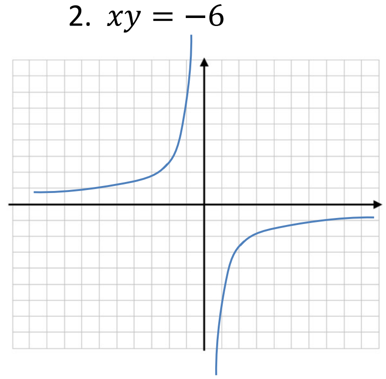
- This is a transformation (translation or slide) of $xy = k$, h units along the x-axis and k units along the y-axis.

Ex. On the set of axes, graph each of the following:

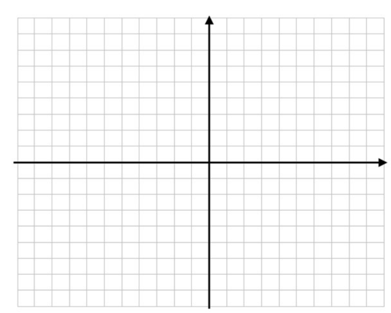
1. $xy = 4$



2. $xy = -6$

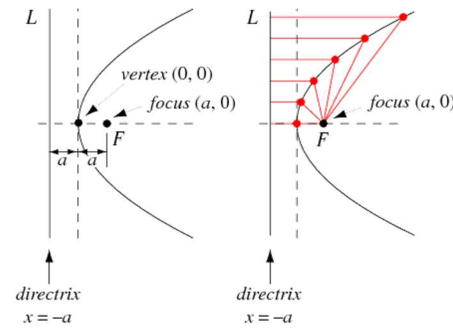


3. $(x - 1)(y + 2) = 8$



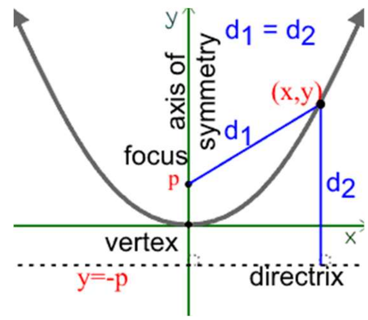
Homework:

9.3 The Parabola – Day 1



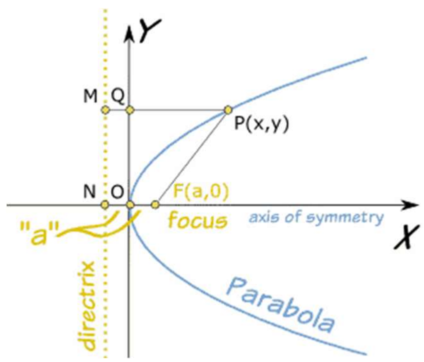
Def: A parabola is the set of all points in a plane that are equidistant from a fixed line, called the directrix, and a fixed point, the focus, that is not on the directrix.

- The line that passes through the focus and is perpendicular to the directrix is called the axis of symmetry.
- The point on the parabola that intersects the axis of symmetry is called the vertex.
  - The vertex is the midway between the focus and the directrix.



Standard Form of the Equation of the Parabola:

Suppose we have a parabola as shown in the graph below:



- The distance from point P to M (on the directrix), d_1, is equal to the distance from P to F, d_2.

$$d_1 = d_2$$

$$\sqrt{(x-(-a))^2 + (y-y)^2} = \sqrt{(x-a)^2 + (y-0)^2}$$

$$(x+a)^2 = (x-a)^2 + y^2$$

$$x^2 + 2ax + a^2 = x^2 - 2ax + a^2 + y^2$$

$$2ax = -2ax + y^2$$

$$\boxed{y^2 = 4ax}$$

This is the standard equation of a parabola with vertex (0,0) and opens up or down.

- Up if $a > 0$
- Down if $a < 0$

There are two positions for a parabola: I – opens left or right and II – opens up/down

The Standard Form of the Equation of a Parabola with vertex at the origin is:

Position I: $y^2 = 4ax$ and Position II: $x^2 = 4ay$

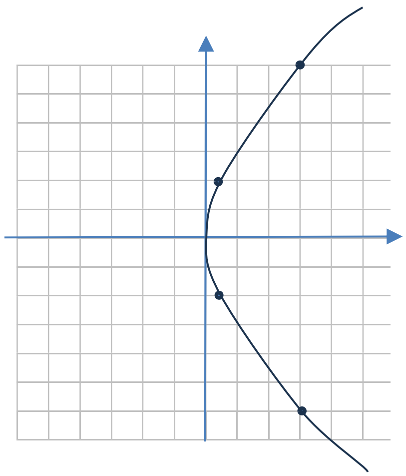
- For any position, the vertex is (0,0) and the focus lies on the axis (direction determined by *a*) *a* units from the vertex.
  - The coordinates of the focus:
    - Position I: (a,0)
    - Position II: (0,a)
- The directrix has an equation:
  - Position I: $x = -a$
  - Position II: $y = -a$
- The distance between the focus and the directrix is $2a$

Def: The latus rectum (or right chord) of a parabola is a line segment that passes through the focus, parallel to the directrix, and has it's endpoints on the parabola.

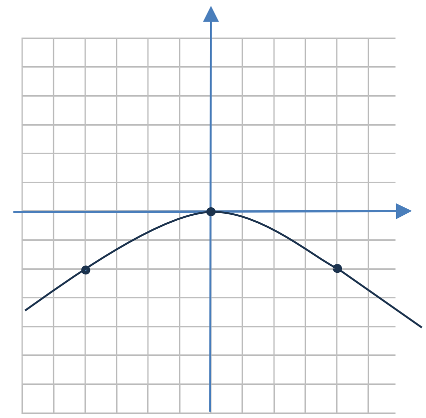
- The length of the latus rectum for either $y^2 = 4ax$ and $x^2 = 4ay$ is $|4a|$.
  - You can the endpoints of the latus rectum by moving $2a$ units parallel to the directrix (perpendicular to the axis of symmetry) in both directions.
    - This gives you three points to sketch the graph.

Ex. Consider the parabola whose equation is given. Find the focus, directrix and the endpoints of the latus rectum. Sketch the graph on the axes provided.

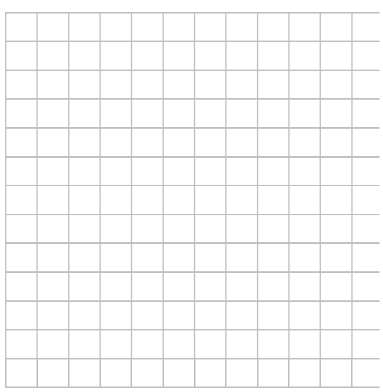
1. $y^2 = 12x$



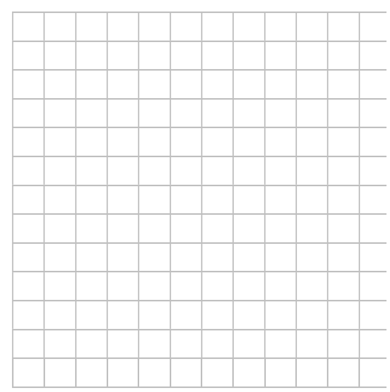
2. $x^2 = -8y$



3. $x^2 = 10x$



4. $y^2 = -4x$



Ex. Find the equation of the parabola whose vertex is (0,0) and whose focus is (-2,0). What are the coordinates of the endpoints of the latus rectum and what is the equation of the directrix?

$p = 2$ $\quad$ *Position II – opens left* $\quad$ $y^2 = -8x$

Directrix: $x = 2$ $\quad$ Endpoints of L.R.: $(-2,2)$ and $(-2,-2)$

Ex. Find the equation of the parabola whose focus is (0,8) and whose directrix has equation $y = -8$

$2p = 8$ $\quad$ *Position I – opens up* $\quad$ $x^2 = 16y$

Homework

9.3 The Parabola – Day 2

The Standard Form of a Parabola with Vertex (h,k):

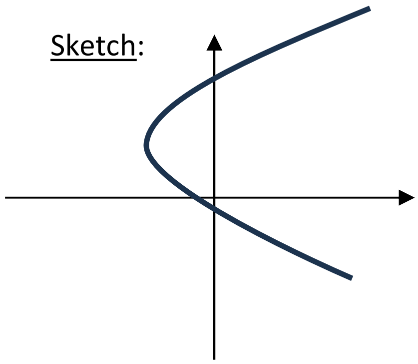
Position I: $(y-k)^2 = 4a(x-h)$

Directrix: $x = h - a$

Axis of Symmetry: $y = k$

Focus: $(h+a, k)$

Sketch:



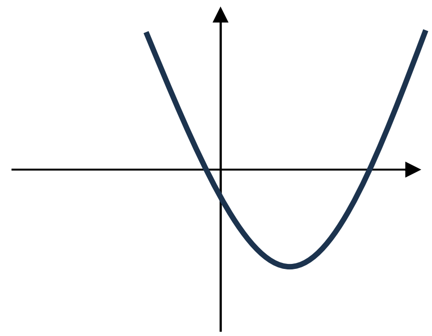
Position II: $(x-h)^2 = 4a(y-k)$

Directrix: $x = k - a$

Axis of Symmetry: $x = h$

Focus: $(h, k+a)$

Sketch:



Ex. Find the vertex, focus, directrix, and the endpoints of the latus rectum of the parabola given. Graph the parabola:

1. $(x-3)^2 = 8(y+1)$

V(3, −1)

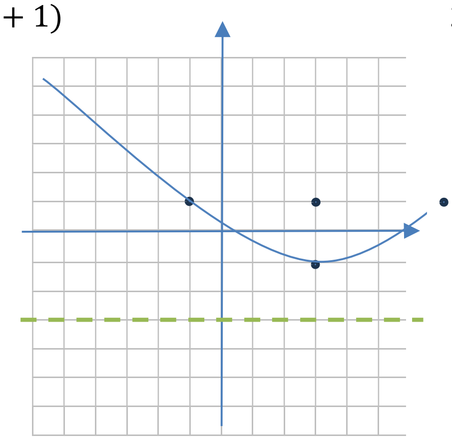
$4p = 8 \rightarrow p = 2$

Focus: (3, 1)

Directrix: $y = -3$

Endpoints of LR:

(−1, 1) and (7,1)



2. $(y-2)^2 = 4(x+1)$

V(−1, 2)

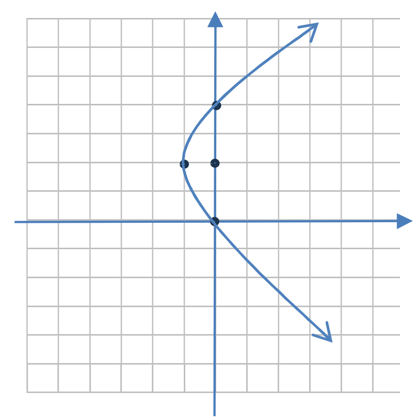
$4p = 4 \rightarrow p = 1$

Focus: (0, 2)

Directrix: $x = -2$

Endpoints of LR:

(0, 4) and (0,0)



Ex. Find the vertex, focus, directrix, and endpoints of the latus rectum of the given parabola. Sketch the parabola.

1. $y^2 + 2y + 12x - 23 = 0$

$y^2 + 2y + 1 = -12x + 23 + 1$

$(y+1)^2 = -12(x-2)$

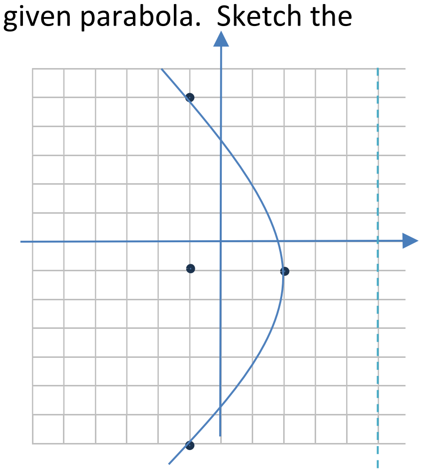
Vertex: (2, −1)

$4p = -12 \rightarrow p = -3 \rightarrow Opens\ LEFT$

Focus: (−1, −1)

Directrix: $x = 5$

Endpts of LR: (−1, 5) and (−1, −7)



2. $x^2 + 4x + 8y - 20 = 0$

$x^2 + 4x + 4 = -8y + 20 + 4$

$(x+2)^2 = -8(y-3)$

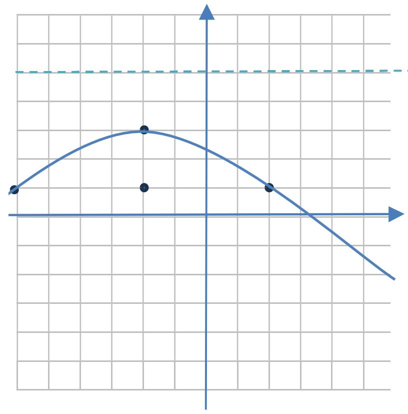
Vertex: $(-2, 3)$

$4p = -8 \rightarrow p = -2 \rightarrow Opens\ DOWN$

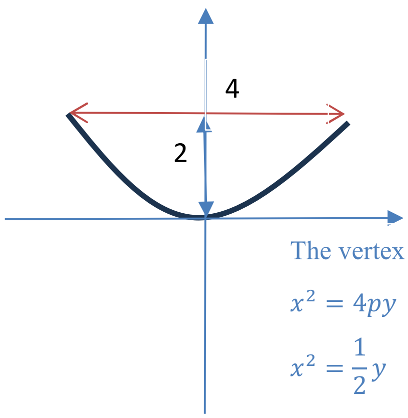
Focus: $(-2, 1)$

Directrix: $y = 5$

Endpts of LR: $(-6, 1)$ and $(2, 1)$



Ex. An engineer is designing a flashlight using a parabolic reflecting mirror and a light source. The casting has a diameter of 4 inches and a depth of 2 inches. What is the equation of the parabola used to shape the mirror? At what point should the light source be place relative to the mirror's vertex?



Hints:

1. Light rays will strike the mirror and always reflect to the focus.
2. Place the vertex at (0,0) on a graph, opening up or right.

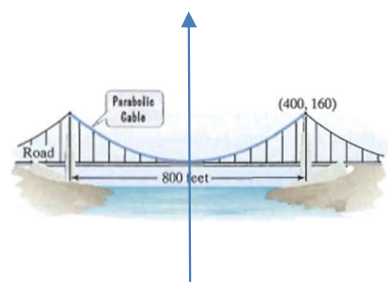
The vertex is at (0,0) and there is a point at (2, 2):

$x^2 = 4py \rightarrow 2^2 = 4p(2) \rightarrow 4 = 8p \rightarrow p = \frac{1}{2}$

$x^2 = \frac{1}{2}y$

The light source should be placed at the focus: (0, ½) or ½ inch above the vertex.

Ex. The towers of a suspension bridge are 800 ft apart and rise 160 ft above the road. The cable between the towers has the shape of a parabola and the cable touches the sides of the road midway between the towers. What is the height of the cable 100 ft from a tower?



Place a set of axes with the origin at the middle of the bridge.

That places a point on the parabola at (400,160)

$x^2 = 4py \rightarrow 400^2 = 4p(160) \rightarrow 160000 = 640p \rightarrow p = 250$

$x^2 = 1000y$

100 feet from the tower puts the x-coordinate at 300

$300^2 = 1000y \rightarrow \frac{90000}{1000} = y \rightarrow y = 90\ feet$

Homework