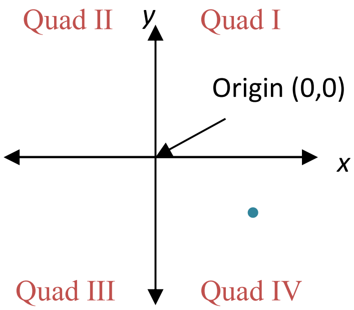
Chapter 1 – Functions & Graphs

Objective(s):

1. Students will be able to plot graphs
2. Find intercepts of equations using a graph and algebra.

1.1 Graphs and Graphing Utilities

The Cartesian Coordinate System or Rectangular Coordinate System is a system of graphing points on a plane using a set of coordinates (*x, y*), called an Ordered Pair.



The plane is broken up in to four areas called quadrants. The lines that divide the plane are called the *x*-axis and *y*-axis.

Each point in the plane is designated by a pair of numbers called an Ordered Pair.

Def: Given an ordered pair (x, y)

(a) The x-coordinate is called the abscissa

(b) The y-coordinate is called the ordinate.

The quadrants are numbered 1–4 starting in the top right quadrant in a counter clock-wise direction.

To plot a given point (a,b), simply move along x-axis starting at the origin (left negative, right positive) to a, and then up and down (+ vs. –) b units.

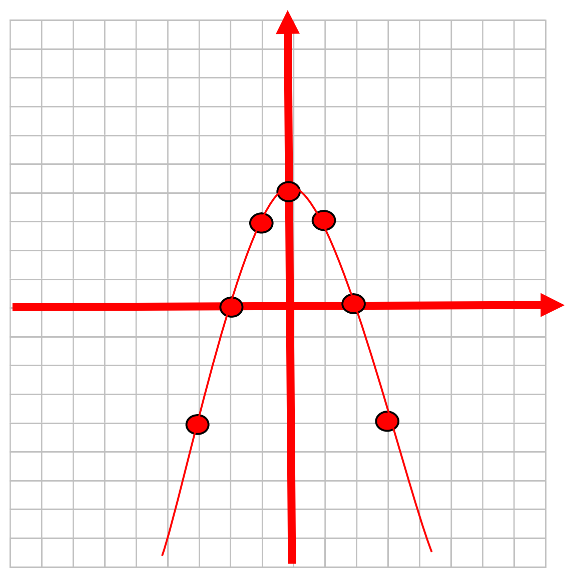
Graphing Equations:

To graph a equation: (***Point Plotting Method***)

1. Solve the equation for one of the variables (prefer to be y =)
2. Substitute different values of x into the equation and get the y-value. This makes a point (x,y)
3. Plot the point
4. Repeat for other values of x

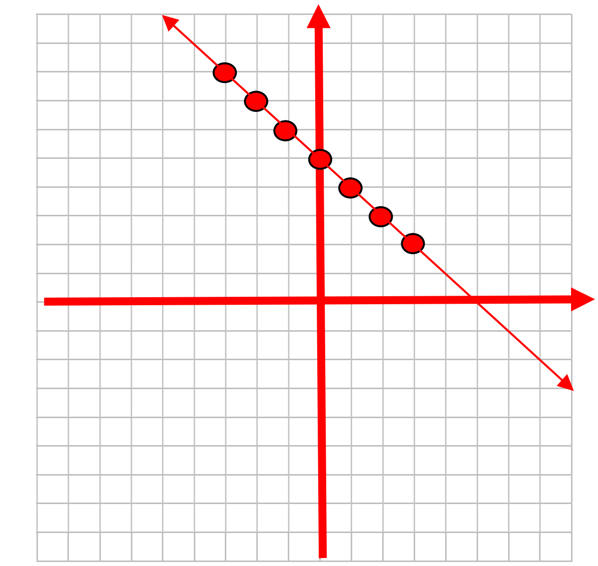
Ex. On graph paper, graph $y = 4 - x^2$ using selected values x from –3 to 3.

x	$4 - x^2$	y
–3	4 – 9	–5
–2	4 – 4	0
–1	4 – 1	3
0	4 – 0	4
1	4 – 1	3
2	4 – 4	0
3	4 – 9	–5



Ex. Graph $y = 5 - x$ for selected values of x from –3 to 3.

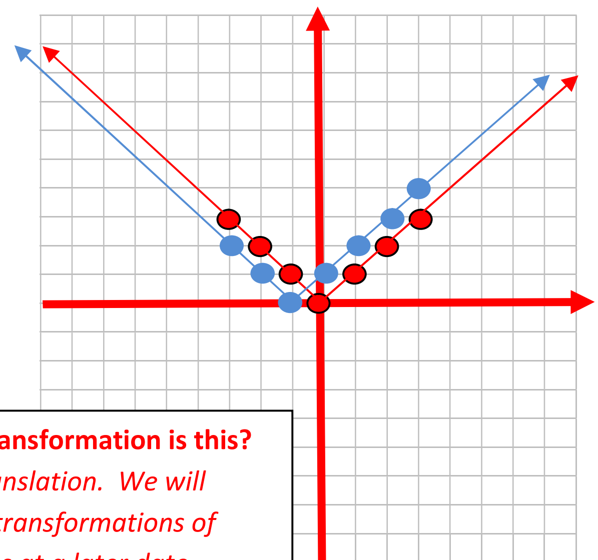
x	$5 - x$	y
-3	$5-(-3)$	8
-2	$5-(-2)$	7
-1	$5-(-1)$	6
0	$5-0$	5
1	$5-1$	4
2	$5-2$	3
3	$5-3$	2



Ex. (a) Using values of *x* from –3 to 3, graph $y = |x|$

(b) On the same set of axes, graph $y = |x + 1|$

x	$y = \lvert x \rvert$	$y = \lvert x+1 \rvert$
-3	3	2
-2	2	1
-1	1	0
0	0	1
1	1	2
2	2	3
3	3	4



What transformation is this?
It's a translation. We will discuss transformations of functions at a later date.

Intercepts:

Def: An intercept is where a graph crosses an axis.

(a) The x-intercept is where the graph crosses the *x*-axis.
- Has coordinates (*a*, 0)

(b) The y-intercept is where the graph crosses the *y*-axis
- Has coordinates (0, *b*)

To find the x-intercept: Set the y value equal to 0 and solve for x.

To find the y-intercept: Set the x value equal to 0 and solve for y.

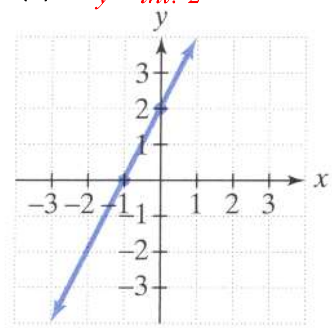
Ex. Find x and y intercepts for $y = 4 - x^2$

$x-$int: (Let $y = 0$)
$0 = 4 - x^2$
$x^2 = 4$
$x = \pm 2$ (2,0) & (−2,0)

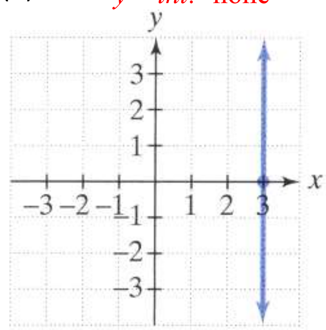
$y-$int: (Let $x = 0$)
$y = 4 - 0^2$
$y = 4$
(0,4)

Ex. Identify the x and y intercepts

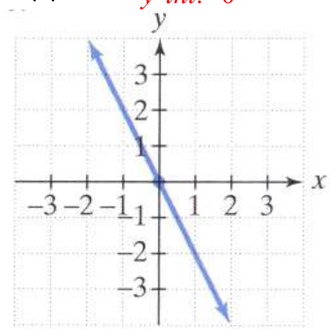
(a) x - *int*: -1
$y-$ *int*: 2



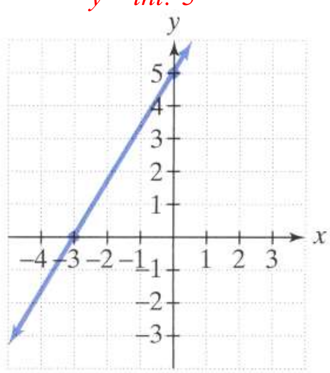
(b) $x-$ *int*: 3
$y-$ *int*: none



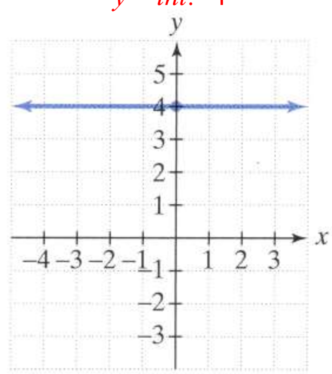
(c) $x-$ *int*: 0
y-int: 0



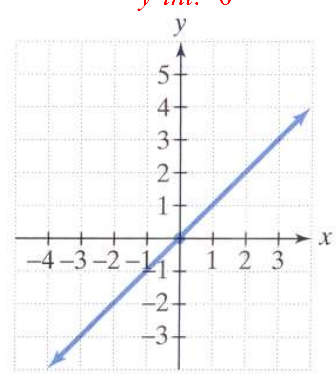
(d) x - *int*: −3
$y-$ *int*: 5



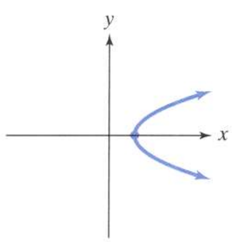
(e) $x-$ *int*: none
$y-$ *int*: 4



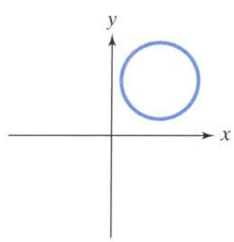
(f) $x-$ *int*: 0
y-int: 0



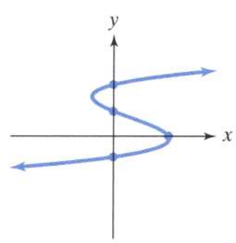
Graphs may also have no intercepts or several. Below are some examples:



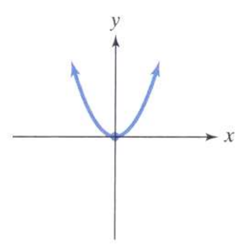
One x-intercept
No y-intercept



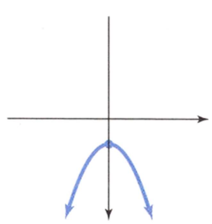
No intercepts



One x-intercept
Three y-intercepts



The same x-intercept
and y-intercept



No x-intercept
One y-intercept

Homework: Pg. 143–146 #1-12 (on same graph paper), 13-27odd, 41-46, 57-60, 71-82

b. Now let's use the line graph that shows the percentage of marriages ending in divorce when the wife is over 25 at the time of marriage. The graph is shown again in **Figure 1.12**. To find the percentage of marriages ending in divorce after 10 years:

- Locate 10 on the horizontal axis and locate the point above 10.
- Read across to the corresponding percent on the vertical axis.

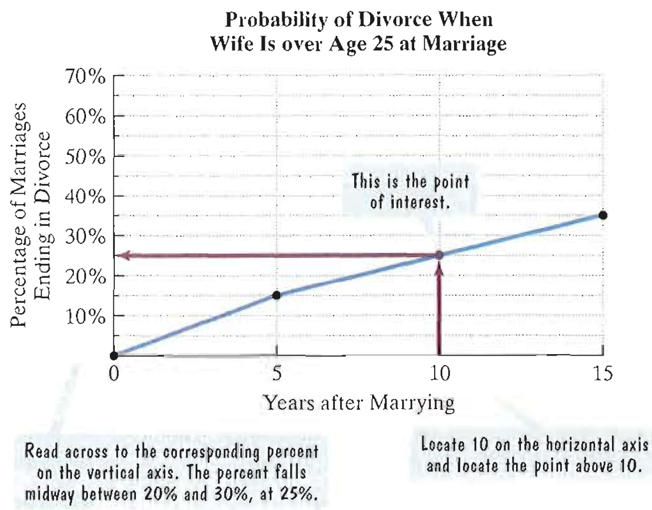


Figure 1.12

The actual data displayed by the graph indicate that 25% of these marriages end in divorce after 10 years.

c. The value obtained by evaluating the mathematical model, 24.5%, is close to, but slightly less than, the actual percentage of divorces, 25.0%. The difference between these percents is 25.0% − 24.5%, or 0.5%. The value given by the mathematical model, 24.5%, underestimates the actual percent, 25%, by only 0.5, providing a fairly accurate description of the data.

Check Point 6

a. Use the appropriate formula from Example 6 to determine the percentage of marriages ending in divorce after 15 years when the wife is under 18 at the time of marriage.

b. Use the appropriate line graph in **Figure 1.11** to determine the percentage of marriages ending in divorce after 15 years when the wife is under 18 at the time of marriage.

c. Does the value given by the mathematical model underestimate or overestimate the actual percentage of marriages ending in divorce after 15 years as shown by the graph? By how much?

Exercise Set 1.1

Practice Exercises

In Exercises 1–12, plot the given point in a rectangular coordinate system.

1. $(1, 4)$ **2.** $(2, 5)$ **3.** $(-2, 3)$
4. $(-1, 4)$ **5.** $(-3, -5)$ **6.** $(-4, -2)$
7. $(4, -1)$ **8.** $(3, -2)$ **9.** $(-4, 0)$
10. $(0, -3)$ **11.** $\left(\frac{7}{2}, -\frac{3}{2}\right)$ **12.** $\left(-\frac{5}{2}, \frac{3}{2}\right)$

Graph each equation in Exercises 13–28. Let $x = -3, -2, -1, 0, 1, 2,$ and 3.

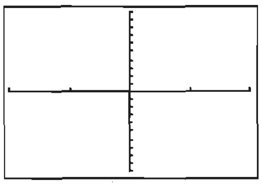
13. $y = x^2 - 2$ **14.** $y = x^2 + 2$ **15.** $y = x - 2$
16. $y = x + 2$ **17.** $y = 2x + 1$ **18.** $y = 2x - 4$
19. $y = -\frac{1}{2}x$ **20.** $y = -\frac{1}{2}x + 2$ **21.** $y = 2|x|$
22. $y = -2|x|$ **23.** $y = |x| + 1$ **24.** $y = |x| - 1$
25. $y = 9 - x^2$ **26.** $y = -x^2$ **27.** $y = x^3$
28. $y = x^3 - 1$

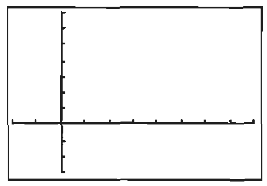
In Exercises 29–32, match the viewing rectangle with the correct figure. Then label the tick marks in the figure to illustrate this viewing rectangle.

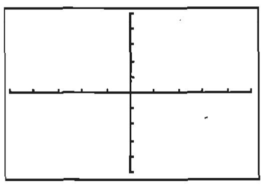
29. $[-5, 5, 1]$ by $[-5, 5, 1]$ **30.** $[-10, 10, 2]$ by $[-4, 4, 2]$

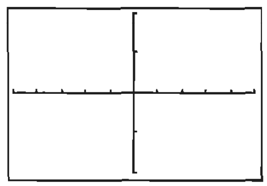
31. $[-20, 80, 10]$ by $[-30, 70, 10]$

32. $[-40, 40, 20]$ by $[-1000, 1000, 100]$

a.


b.


c.


d.


The table of values was generated by a graphing utility with a TABLE feature. Use the table to solve Exercises 33–40.

X	Y1	Y2
-3	9	5
-2	4	4
-1	1	3
0	0	2
1	1	1
2	4	0
3	9	-1

X= -3

33. Which equation corresponds to Y_2 in the table?

a. $y_2 = x + 8$ **b.** $y_2 = x - 2$

c. $y_2 = 2 - x$ **d.** $y_2 = 1 - 2x$

34. Which equation corresponds to Y_1 in the table?

a. $y_1 = -3x$ **b.** $y_1 = x^2$

c. $y_1 = -x^2$ **d.** $y_1 = 2 - x$

35. Does the graph of Y_2 pass through the origin?

36. Does the graph of Y_1 pass through the origin?

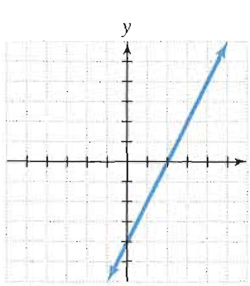
37. At which point does the graph of Y_2 cross the x-axis?

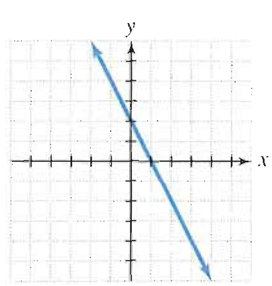
38. At which point does the graph of Y_2 cross the y-axis?

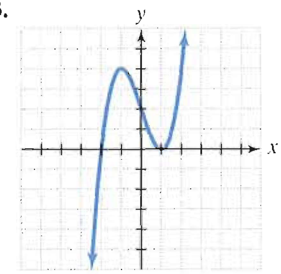
39. At which points do the graphs of Y_1 and Y_2 intersect?

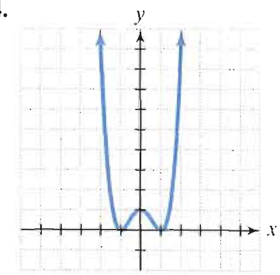
40. For which values of x is $Y_1 = Y_2$?

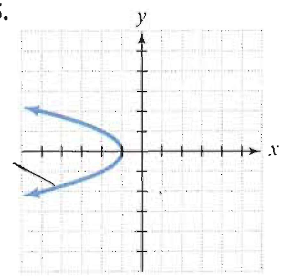
In Exercises 41–46, use the graph to **a.** *determine the x-intercepts, if any;* **b.** *determine the y-intercepts, if any. For each graph, tick marks along the axes represent one unit each.*

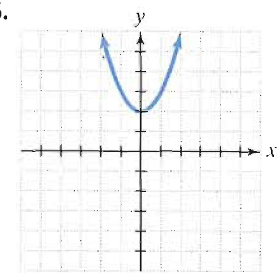
41.


42.


43.


44.


45.


46.


Practice Plus

In Exercises 47–50, write each English sentence as an equation in two variables. Then graph the equation.

47. The y-value is four more than twice the x-value.

48. The y-value is the difference between four and twice the x-value.

49. The y-value is three decreased by the square of the x-value.

50. The y-value is two more than the square of the x-value.

In Exercises 51–54, graph each equation.

51. $y = 5$ (Let $x = -3, -2, -1, 0, 1, 2,$ and 3.)

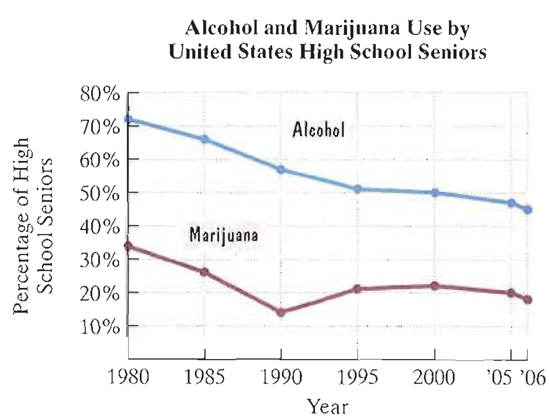
52. $y = -1$ (Let $x = -3, -2, -1, 0, 1, 2,$ and 3.)

53. $y = \frac{1}{x}$ (Let $x = -2, -1, -\frac{1}{2}, -\frac{1}{3}, \frac{1}{3}, \frac{1}{2}, 1,$ and 2.)

54. $y = -\frac{1}{x}$ (Let $x = -2, -1, -\frac{1}{2}, -\frac{1}{3}, \frac{1}{3}, \frac{1}{2}, 1,$ and 2.)

Application Exercises

The graphs show the percentage of high school seniors who used alcohol or marijuana during the 30 days prior to being surveyed for the University of Michigan's Monitoring the Future *study.*



Source: U.S. Department of Health and Human Services

The data can be described by the following mathematical models:

Percentage of seniors using alcohol $A = -n + 70$

Number of years after 1980

Percentage of seniors using marijuana $M = -0.4n + 28.$

Use this information to solve Exercises 55–56.

55. a. Use the appropriate line graph to determine the percentage of seniors who used marijuana in 2005.

b. Use the appropriate formula to determine the percentage of seniors who used marijuana in 2005. Does the formula underestimate or overestimate the actual percentage displayed by the graph? By how much?

c. Use the appropriate line graph to estimate the percentage of seniors who used alcohol in 2006.

d. Use the appropriate formula to determine the percentage of seniors who used alcohol in 2006. How does this compare with your estimate in part (c)?

e. For the period from 1980 through 2006, in which year was marijuana use by seniors at a minimum? Estimate the percentage of seniors who used marijuana in that year.

56. a. Use the appropriate line graph to determine the percentage of seniors who used alcohol in 2000.

b. Use the appropriate formula to determine the percentage of seniors who used alcohol in 2000. What do you observe?

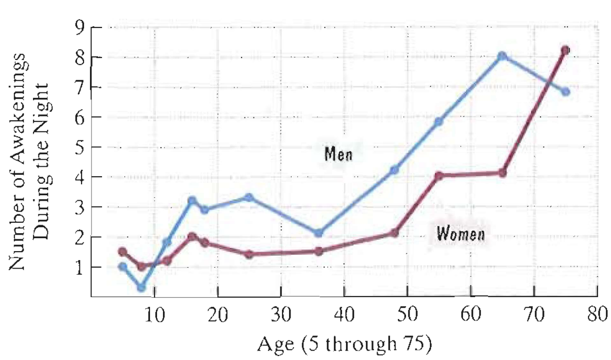
c. Use the appropriate line graph to estimate the percentage of seniors who used marijuana in 2000.

d. Use the appropriate formula to determine the percentage of seniors who used marijuana in 2000. How does this compare with your estimate in part (c)?

e. For the period from 1980 through 2006, in which year was alcohol use by seniors at a maximum? Estimate the percentage of seniors who used alcohol in that year.

Contrary to popular belief, older people do not need less sleep than younger adults. However, the line graphs show that they awaken more often during the night. The numerous awakenings are one reason why some elderly individuals report that sleep is less restful than it had been in the past. Use the line graphs to solve Exercises 57–60.

Average Number of Awakenings During the Night, by Age and Gender



Source: Stephen Davis and Joseph Palladino, *Psychology*, 5th Edition, Prentice Hall, 2007

57. At which age, estimated to the nearest year, do women have the least number of awakenings during the night? What is the average number of awakenings at that age?

58. At which age do men have the greatest number of awakenings during the night? What is the average number of awakenings at that age?

59. Estimate, to the nearest tenth, the difference between the average number of awakenings during the night for 25-year-old men and 25-year-old women.

60. Estimate, to the nearest tenth, the difference between the average number of awakenings during the night for 18-year-old men and 18-year-old women.

Writing in Mathematics

61. What is the rectangular coordinate system?

62. Explain how to plot a point in the rectangular coordinate system. Give an example with your explanation.

63. Explain why $(5, -2)$ and $(-2, 5)$ do not represent the same point.

64. Explain how to graph an equation in the rectangular coordinate system.

65. What does a $[-20, 2, 1]$ by $[-4, 5, 0.5]$ viewing rectangle mean?

Technology Exercise

66. Use a graphing utility to verify each of your hand-drawn graphs in Exercises 13–28. Experiment with the size of the viewing rectangle to make the graph displayed by the graphing utility resemble your hand-drawn graph as much as possible.

Critical Thinking Exercises

Make Sense? *In Exercises 67–70, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

67. The rectangular coordinate system provides a geometric picture of what an equation in two variables looks like.

68. There is something wrong with my graphing utility because it is not displaying numbers along the x- and y-axes.

69. I used the ordered pairs $(-2, 2)$, $(0, 0)$, and $(2, 2)$ to graph a straight line.

70. I used the ordered pairs

(time of day, calories that I burned)

to obtain a graph that is a horizontal line.

In Exercises 71–74, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

71. If the product of a point's coordinates is positive, the point must be in quadrant I.

72. If a point is on the x-axis, it is neither up nor down, so $x = 0$.

73. If a point is on the y-axis, its x-coordinate must be 0.

74. The ordered pair $(2, 5)$ satisfies $3y - 2x = -4$.

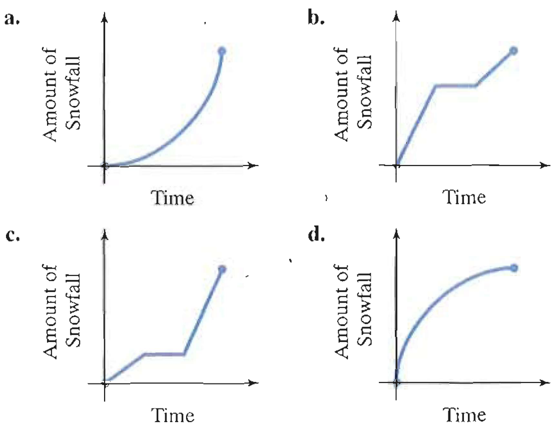
In Exercises 75–78, match the story with the correct figure. The figures are labeled (a), (b), (c), and (d).

75. As the blizzard got worse, the snow fell harder and harder.

76. The snow fell more and more softly.

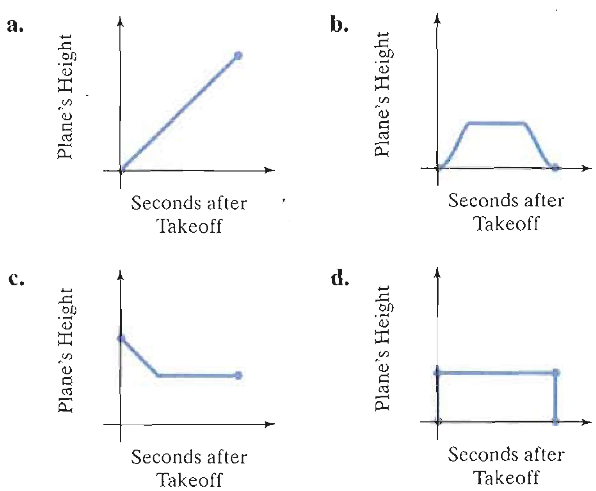
77. It snowed hard, but then it stopped. After a short time, the snow started falling softly.

78. It snowed softly, and then it stopped. After a short time, the snow started falling hard.

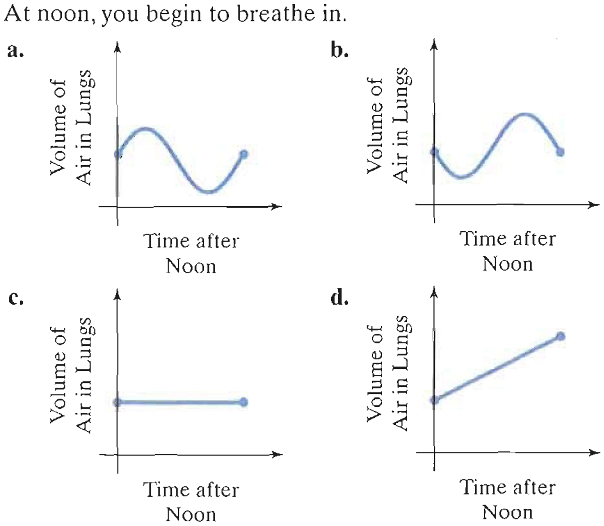


In Exercises 79–82, select the graph that best illustrates each story.

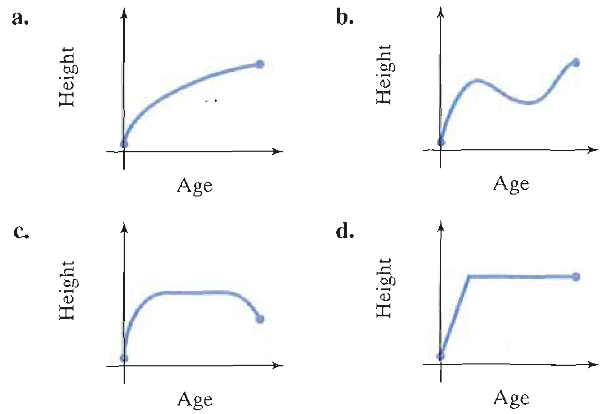
79. An airplane flew from Miami to San Francisco.



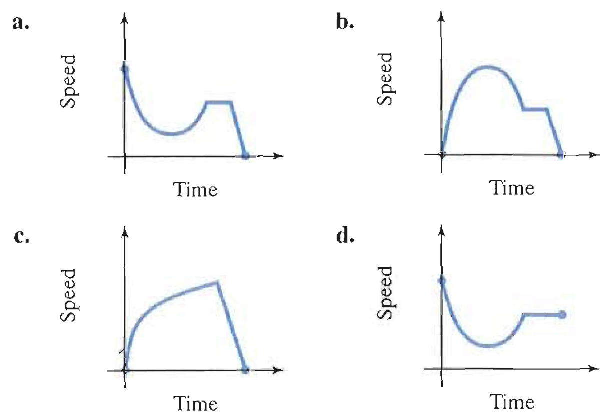
80. At noon, you begin to breathe in.



81. Measurements are taken of a person's height from birth to age 100.



82. You begin your bike ride by riding down a hill. Then you ride up another hill. Finally, you ride along a level surface before coming to a stop.



Preview Exercises

Exercises 83–85 will help you prepare for the material covered in the next section.

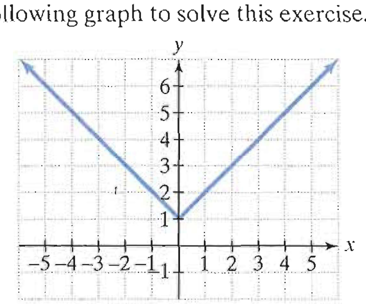
83. Here are two sets of ordered pairs:

$$\text{set 1: } \{(1, 5), (2, 5)\}$$
$$\text{set 2: } \{(5, 1), (5, 2)\}.$$

In which set is each x-coordinate paired with only one y-coordinate?

84. Graph $y = 2x$ and $y = 2x + 4$ in the same rectangular coordinate system. Select integers for x, starting with -2 and ending with 2.

85. Use the following graph to solve this exercise.



a. What is the y-coordinate when the x-coordinate is 2?

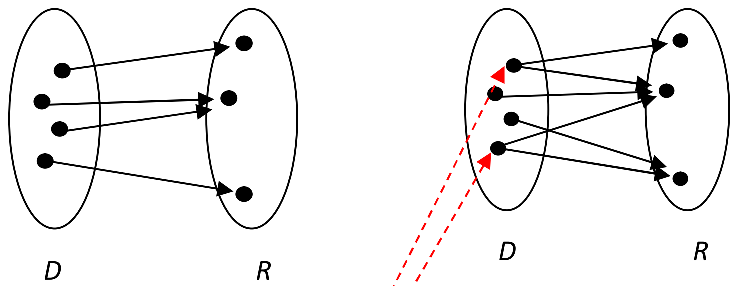
b. What are the x-coordinates when the y-coordinate is 4?

c. Describe the x-coordinates of all points on the graph.

d. Describe the y-coordinates of all points on the graph.

1.2 Basics of Functions and Their Graphs

Def: A relation is any set of order pairs. The set of all the first coordinates (x-coordinates) is called the domain *D* of the relation and the set of second coordinates (y-coordinates) is called the range *R* of the relation.



In the diagram above, the domain is the set of points in D and the range is the set of points in R. The arrows represent the mapping of points from D onto R.

Def: A function is a relation such that each element in the domain corresponds to exactly ONE element in the range.

In the diagram above, the left figure depicts a function while the right one does not. (Basically means that no one point on the left can have two arrows coming out of it – the right side can).

*** To determine if a relation is a function: if you can find matching x-coordinates and not matching y-coordinates, then it is not a function. Examples: (2, 4) and (2, 5) on the same graph – not a function

Functions as Equations:

When an equation is written in the form like: $y = x^2 - 6x + 6$, where the equation is solved for y, then we have y "in terms of x". x is called the **independent variable** because it can be assigned any value from its domain. y is called the **dependent variable** because its value is dependent on what the value of x is. *You can make x any value you want, and it will give you a y value based on it*

Ex. Solve for y and determine if the equation defines y as a function of x

1. $x^2 + y = 4$

$y = 4 - x^2$

Yes, it is a function

2. $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

No, it is not a function (because of the ±)

Function Notation: When y is a function of x, it is customary to use other letters (*f, g, h*,..) to represent it as a FUNCTION. It would be written as follows:

$$y = x^2 - 6x + 6 \quad \rightarrow \quad f(x) = x^2 - 6x + 6$$

$f(x)$ is pronounced "f of x" and means "the value of function f at x".

The term "function" was introduced by **Gottfried Wilhelm Leibniz (1646-1716)**

Function notation was created by **Leonhard Euler (***pronounced "OILER"***)**

Evaluating a function: Given function $y = f(x)$, then $f(2)$ means the value of function when x = 2.

Ex. If $f(x) = x^2 + 3x - 5$, then evaluate each of the following:

1. $f(2)$

$f(2) = (2)^2 + 3(2) - 5$
$f(2) = 4 + 6 - 5$
$f(2) = 5$

2. $f(-4)$

$f(-4) = (-4)^2 + 3(-4) - 5$
$f(-4) = 16 - 12 - 5$
$f(-4) = -1$

3. $f(a)$

$f(a) = a^2 + 3a - 5$

Ex. Using the function above, what would $f(x + 3)$?

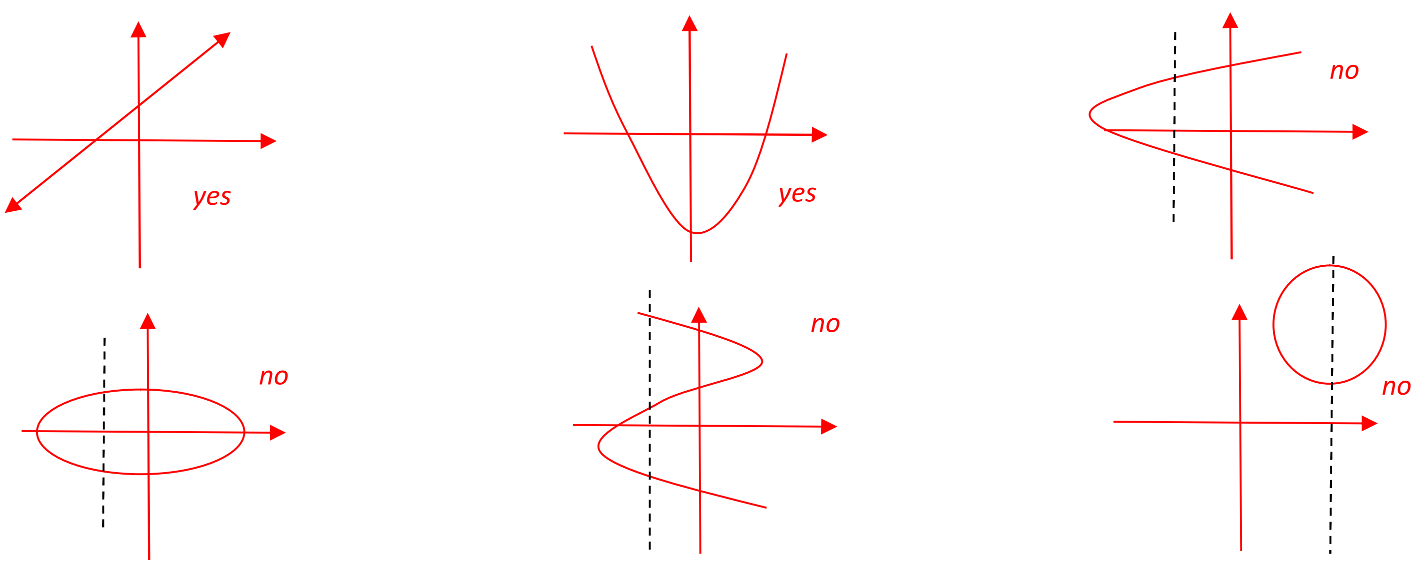
$f(x + 3) = (x + 3)^2 + 3(x + 3) - 5$
$f(x + 3) = x^2 + 6x + 9 + 3x + 9 - 5$
$f(x + 3) = x^2 + 9x + 13$

Graphs of functions:

- A graph is a function if it passes the ***Vertical Line Test (VLT)***
  - The vertical line test means that if you draw vertical lines through the graph, that it would only cross through the graph 1 time.
  - If the graph fails the VLT, then the relation is NOT a function.

If one line passes through more than one point → Not a function

Ex. Determine if each of the following graphs are graphs of functions:



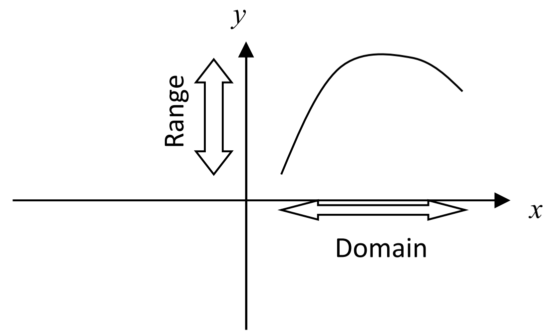
Homework: **Day 1: Pg. 159 – 161 #1–25odd, 27–63 (3's), 65–76**
Day 2: Pg. 161 – 163 #78-92even, 93, 94, 97, 98

1.2b Basics of Functions & Their Graphs

<u>Identifying Domain and Range Using a Function's Graph</u>: Suppose you have the graph on the right:

To find the domain – Look at the x-coordinates of the left most and right most points.

To find the range - Look at the y-coordinates of the lowest and highest points.

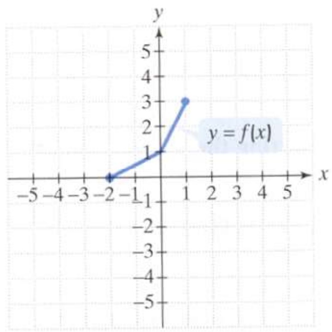


** The endpoints are not always the extreme points for domain and range!! **

Examples: Find the domain and range from the graphs below

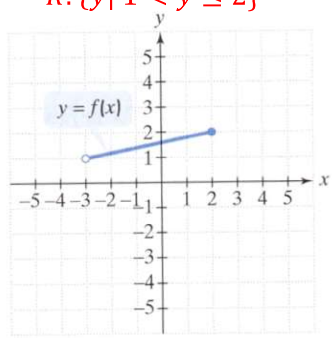
$D: \{x| -2 \le x \le 1\}$
$R: \{y|0 \le y \le 3\}$

a.



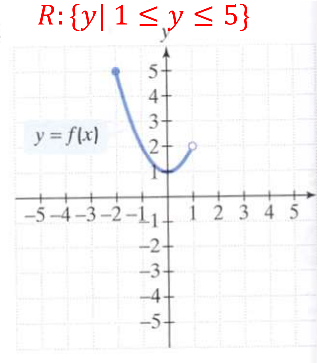
$D: \{x|-3 < x \le 2\}$
$R: \{y|\ 1 < y \le 2\}$

b.



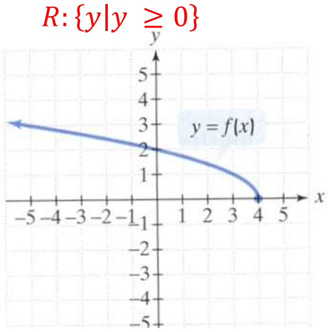
$D: \{x| -2 \le x < 1\}$
$R: \{y|\ 1 \le y \le 5\}$

c.



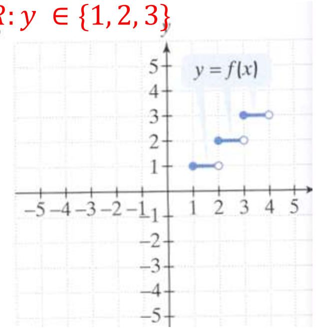
$D: \{x|\ x \le 4\}$
$R: \{y|y \ge 0\}$

d.



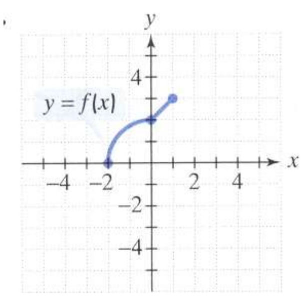
$D: \{x|1 \le x < 4\}$
$R: y \in \{1, 2, 3\}$

e.



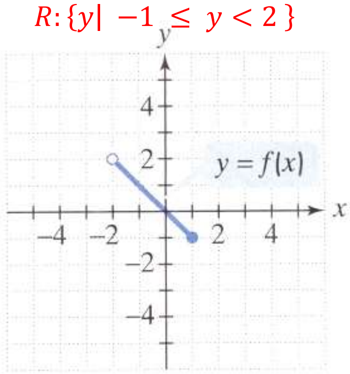
$D: \{x| -2 \le x < 1\}$
$R: \{y|\ 0 \le y \le 3\}$

f.



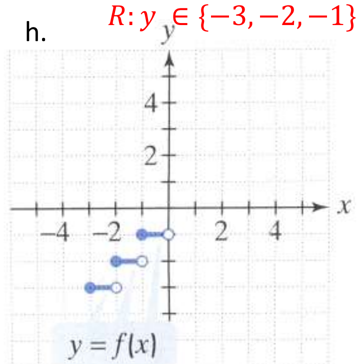
$D: \{x|\ -2 < x \le 1\}$
$R: \{y|\ -1 \le y < 2\}$

g.



$D: \{x|-3 \le x < 0\}$
$R: y \in \{-3, -2, -1\}$

h.

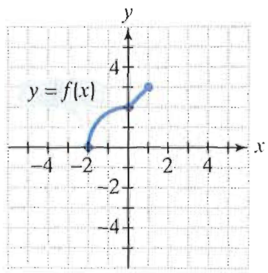


Homework:

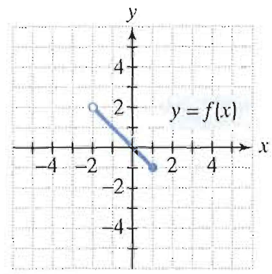
Pg. 161-163 #78-93even, 93, 94, 97, 98

Check Point 8 Use the graph of each function to identify its domain and its range.

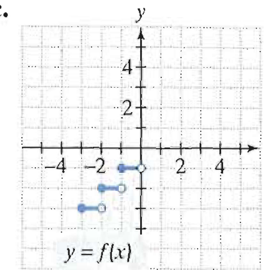
a.



b.



c.



9 Identify intercepts from a function's graph.

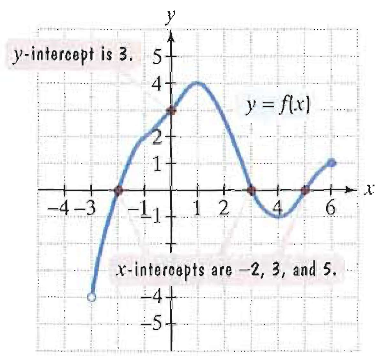


Figure 1.26 Identifying intercepts

Identifying Intercepts from a Function's Graph

Figure 1.26 illustrates how we can identify intercepts from a function's graph. To find the x-intercepts, look for the points at which the graph crosses the x-axis. There are three such points: $(-2, 0)$, $(3, 0)$, and $(5, 0)$. Thus, the x-intercepts are -2, 3, and 5. We express this in function notation by writing $f(-2) = 0$, $f(3) = 0$, and $f(5) = 0$. We say that -2, 3, and 5 are the *zeros of the function*. The **zeros of a function** f are the x-values for which $f(x) = 0$. Thus, the real zeros are the x-intercepts.

To find the y-intercept, look for the point at which the graph crosses the y-axis. This occurs at $(0, 3)$. Thus, the y-intercept is 3. We express this in function notation by writing $f(0) = 3$.

By the definition of a function, for each value of x we can have at most one value for y. What does this mean in terms of intercepts? **A function can have more than one x-intercept but at most one y-intercept.**

Exercise Set 1.2

Practice Exercises

In Exercises 1–10, determine whether each relation is a function. Give the domain and range for each relation.

1. $\{(1, 2), (3, 4), (5, 5)\}$
2. $\{(4, 5), (6, 7), (8, 8)\}$
3. $\{(3, 4), (3, 5), (4, 4), (4, 5)\}$
4. $\{(5, 6), (5, 7), (6, 6), (6, 7)\}$
5. $\{(3, -2), (5, -2), (7, 1), (4, 9)\}$
6. $\{(10, 4), (-2, 4), (-1, 1), (5, 6)\}$
7. $\{(-3, -3), (-2, -2), (-1, -1), (0, 0)\}$
8. $\{(-7, -7), (-5, -5), (-3, -3), (0, 0)\}$
9. $\{(1, 4), (1, 5), (1, 6)\}$
10. $\{(4, 1), (5, 1), (6, 1)\}$

In Exercises 11–26, determine whether each equation defines y as a function of x.

11. $x + y = 16$
12. $x + y = 25$
13. $x^2 + y = 16$
14. $x^2 + y = 25$
15. $x^2 + y^2 = 16$
16. $x^2 + y^2 = 25$
17. $x = y^2$
18. $4x = y^2$
19. $y = \sqrt{x + 4}$
20. $y = -\sqrt{x + 4}$
21. $x + y^3 = 8$
22. $x + y^3 = 27$
23. $xy + 2y = 1$
24. $xy - 5y = 1$
25. $|x| - y = 2$
26. $|x| - y = 5$

In Exercises 27–38, evaluate each function at the given values of the independent variable and simplify.

27. $f(x) = 4x + 5$
  a. $f(6)$ **b.** $f(x + 1)$ **c.** $f(-x)$

28. $f(x) = 3x + 7$
  a. $f(4)$ **b.** $f(x + 1)$ **c.** $f(-x)$

29. $g(x) = x^2 + 2x + 3$
  a. $g(-1)$ **b.** $g(x + 5)$ **c.** $g(-x)$

30. $g(x) = x^2 - 10x - 3$
  a. $g(-1)$ **b.** $g(x + 2)$ **c.** $g(-x)$

31. $h(x) = x^4 - x^2 + 1$
  a. $h(2)$ **b.** $h(-1)$
  c. $h(-x)$ **d.** $h(3a)$

32. $h(x) = x^3 - x + 1$
  a. $h(3)$ **b.** $h(-2)$
  c. $h(-x)$ **d.** $h(3a)$

33. $f(r) = \sqrt{r + 6} + 3$
  a. $f(-6)$ **b.** $f(10)$ **c.** $f(x - 6)$

34. $f(r) = \sqrt{25 - r} - 6$

a. $f(16)$ **b.** $f(-24)$ **c.** $f(25 - 2x)$

35. $f(x) = \dfrac{4x^2 - 1}{x^2}$

a. $f(2)$ **b.** $f(-2)$ **c.** $f(-x)$

36. $f(x) = \dfrac{4x^3 + 1}{x^3}$

a. $f(2)$ **b.** $f(-2)$ **c.** $f(-x)$

37. $f(x) = \dfrac{x}{|x|}$

a. $f(6)$ **b.** $f(-6)$ **c.** $f(r^2)$

38. $f(x) = \dfrac{|x + 3|}{x + 3}$

a. $f(5)$ **b.** $f(-5)$ **c.** $f(-9 - x)$

In Exercises 39–50, graph the given functions, f and g, in the same rectangular coordinate system. Select integers for x, starting with −2 and ending with 2. Once you have obtained your graphs, describe how the graph of g is related to the graph of f.

39. $f(x) = x, g(x) = x + 3$

40. $f(x) = x, g(x) = x - 4$

41. $f(x) = -2x, g(x) = -2x - 1$

42. $f(x) = -2x, g(x) = -2x + 3$

43. $f(x) = x^2, g(x) = x^2 + 1$

44. $f(x) = x^2, g(x) = x^2 - 2$

45. $f(x) = |x|, g(x) = |x| - 2$

46. $f(x) = |x|, g(x) = |x| + 1$

47. $f(x) = x^3, g(x) = x^3 + 2$

48. $f(x) = x^3, g(x) = x^3 - 1$

49. $f(x) = 3, g(x) = 5$

50. $f(x) = -1, g(x) = 4$

In Exercises 51–54, graph the given square root functions, f and g, in the same rectangular coordinate system. Use the integer values of x given to the right of each function to obtain ordered pairs. Because only nonnegative numbers have square roots that are real numbers, be sure that each graph appears only for values of x that cause the expression under the radical sign to be greater than or equal to zero. Once you have obtained your graphs, describe how the graph of g is related to the graph of f.

51. $f(x) = \sqrt{x}$ $(x = 0, 1, 4, 9)$ and
$g(x) = \sqrt{x} - 1$ $(x = 0, 1, 4, 9)$

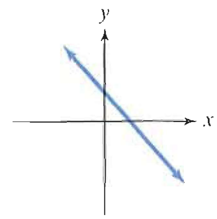
52. $f(x) = \sqrt{x}$ $(x = 0, 1, 4, 9)$ and
$g(x) = \sqrt{x} + 2$ $(x = 0, 1, 4, 9)$

53. $f(x) = \sqrt{x}$ $(x = 0, 1, 4, 9)$ and
$g(x) = \sqrt{x - 1}$ $(x = 1, 2, 5, 10)$

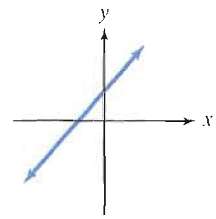
54. $f(x) = \sqrt{x}$ $(x = 0, 1, 4, 9)$ and
$g(x) = \sqrt{x + 2}$ $(x = -2, -1, 2, 7)$

In Exercises 55–64, use the vertical line test to identify graphs in which y is a function of x.

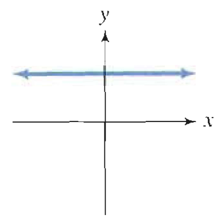
55.



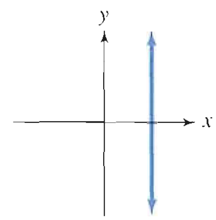
56.



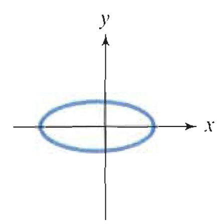
57.



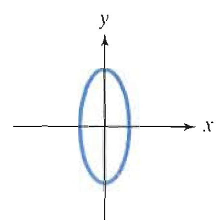
58.



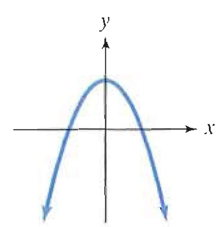
59.



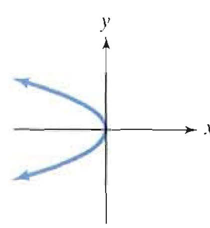
60.



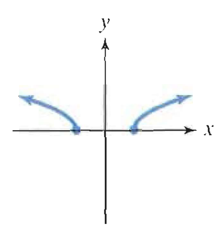
61.



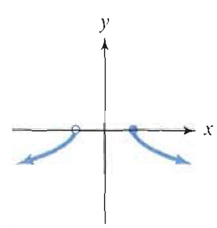
62.



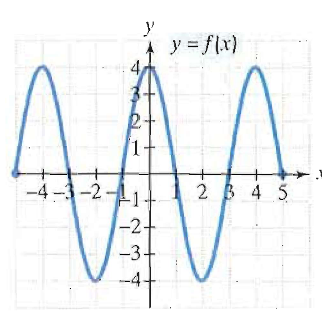
63.



64.



In Exercises 65–70, use the graph of f to find each indicated function value.



65. $f(-2)$

66. $f(2)$

67. $f(4)$

68. $f(-4)$

69. $f(-3)$

70. $f(-1)$

Use the graph of g to solve Exercises 71–76.

71. Find $g(-4)$.

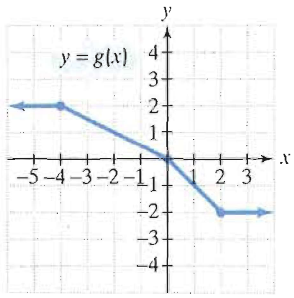
72. Find $g(2)$.

73. Find $g(-10)$.

74. Find $g(10)$.

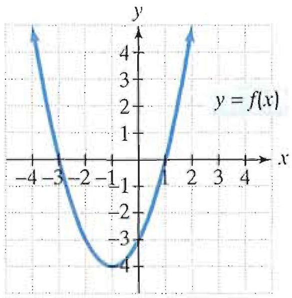
75. For what value of x is $g(x) = 1$?

76. For what value of x is $g(x) = -1$?



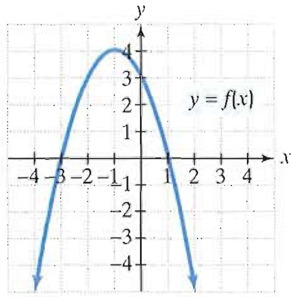
In Exercises 77–92, use the graph to determine **a.** *the function's domain;* **b.** *the function's range;* **c.** *the x-intercepts, if any;* **d.** *the y-intercept, if any; and* **e.** *the missing function values, indicated by question marks, below each graph.*

77.



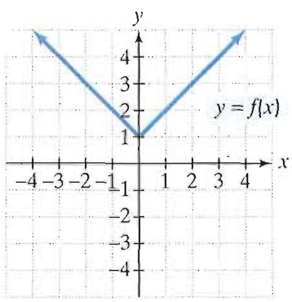
$f(-2) = ?$ $f(2) = ?$

78.



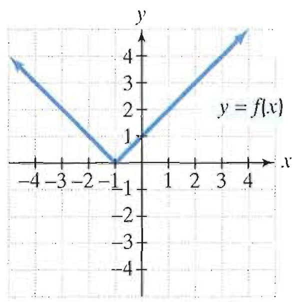
$f(-2) = ?$ $f(2) = ?$

79.



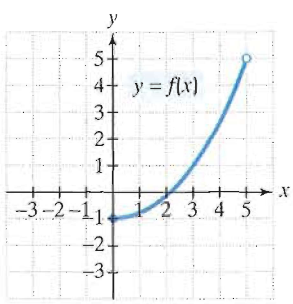
$f(-1) = ?$ $f(3) = ?$

80.



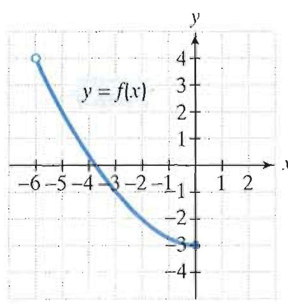
$f(-4) = ?$ $f(3) = ?$

81.



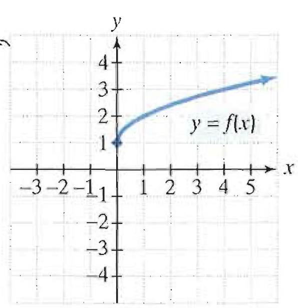
$f(3) = ?$

82.



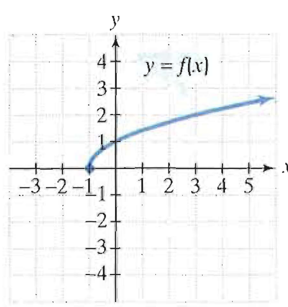
$f(-5) = ?$

83.



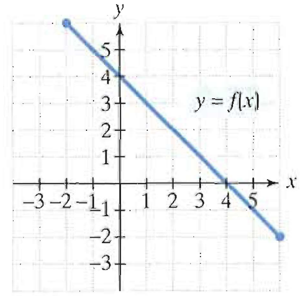
$f(4) = ?$

84.



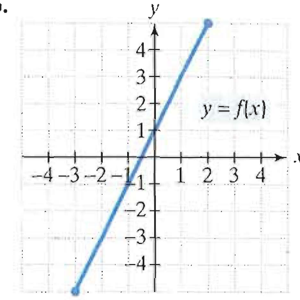
$f(3) = ?$

85.



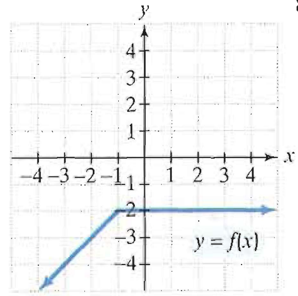
$f(-1) = ?$

86.



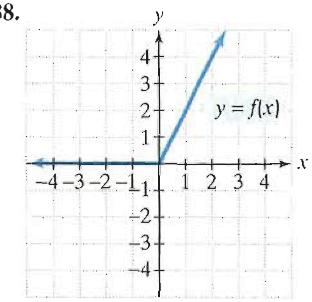
$f(-2) = ?$

87.



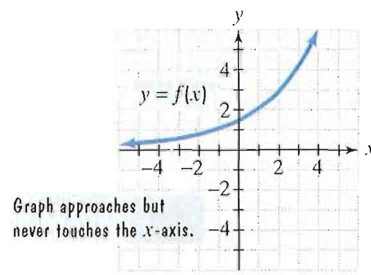
$f(-4) = ?$ $f(4) = ?$

88.



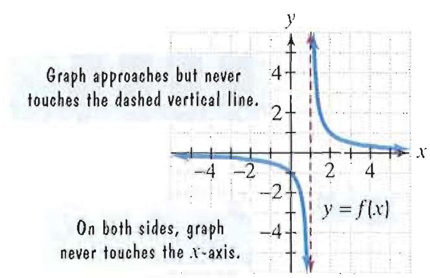
$f(-2) = ?$ $f(2) = ?$

89.



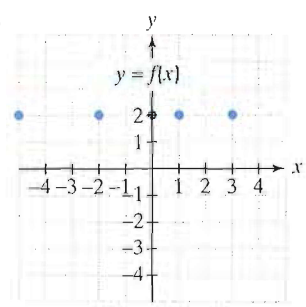
$f(4) = ?$

90.



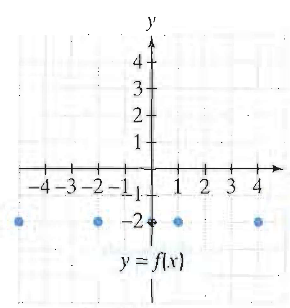
$f(2) = ?$

91.



$f(-5) + f(3) = ?$

92.



$f(-5) + f(4) = ?$

Practice Plus

In Exercises 93–94, let $f(x) = x^2 - x + 4$ *and* $g(x) = 3x - 5$.

93. Find $g(1)$ and $f(g(1))$. **94.** Find $g(-1)$ and $f(g(-1))$.

In Exercises 95–96, let f and g be defined by the following table:

x	$f(x)$	$g(x)$
−2	6	0
−1	3	4
0	−1	1
1	−4	−3
2	0	−6

95. Find $\sqrt{f(-1) - f(0)} - [g(2)]^2 + f(-2) \div g(2) \cdot g(-1)$.

96. Find $|f(1) - f(0)| - [g(1)]^2 + g(1) \div f(-1) \cdot g(2)$.

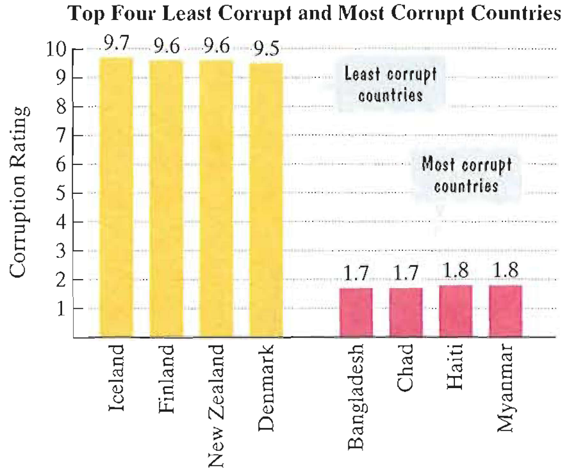
In Exercises 97–98, find $f(-x) - f(x)$ *for the given function f. Then simplify the expression.*

97. $f(x) = x^3 + x - 5$ **98.** $f(x) = x^2 - 3x + 7$

Application Exercises

The Corruption Perceptions Index uses perceptions of the general public, business people, and risk analysts to rate countries by how likely they are to accept bribes. The ratings are on a scale from 0 to 10, where higher scores represent less corruption. The graph shows the corruption ratings for the world's least corrupt and most corrupt countries. (The rating for the United States is 7.6.) Use the graph to solve Exercises 99–100.

Top Four Least Corrupt and Most Corrupt Countries



Source: Transparency International, *Corruption Perceptions Index*

99. Use the four least corrupt countries to solve this exercise.

a. Write a set of four ordered pairs in which countries correspond to corruption ratings. Each ordered pair should be in the form

(country, corruption rating).

b. Is the relation in part (a) a function? Explain your answer.

c. Write a set of four ordered pairs in which corruption ratings correspond to countries. Each ordered pair should be in the form

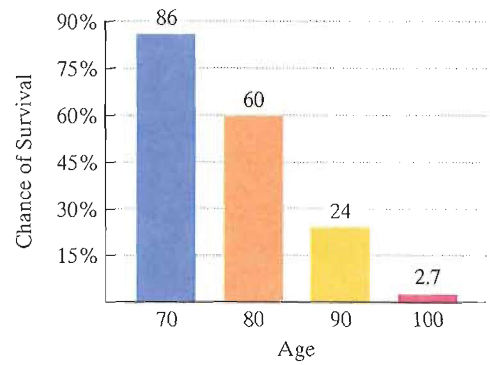
(corruption rating, country).

d. Is the relation in part (c) a function? Explain your answer.

100. Repeat parts (a) through (d) in Exercise 99 using the four most corrupt countries.

The bar graph shows your chances of surviving to various ages once you reach 60.

Chances of 60-Year-Olds Surviving to Various Ages



Source: National Center for Health Statistics

The functions

$$f(x) = -2.9x + 286$$
$$\text{and} \quad g(x) = 0.01x^2 - 4.9x + 370$$

model the chance, as a percent, that a 60-year-old will survive to age x. Use this information to solve Exercises 101–102.

101. a. Find and interpret $f(70)$. **b.** Find and interpret $g(70)$.

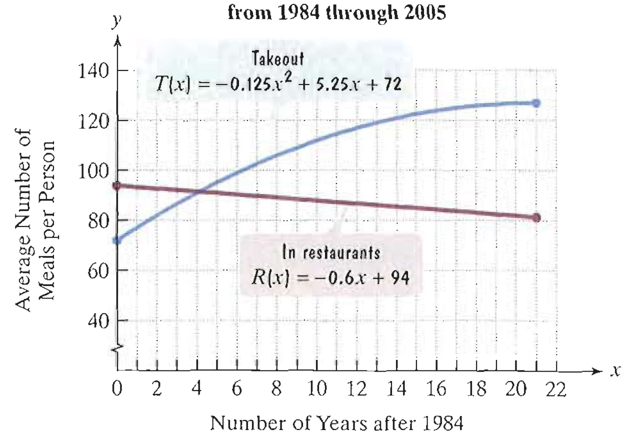
c. Which function serves as a better model for the chance of surviving to age 70?

102. a. Find and interpret $f(90)$. **b.** Find and interpret $g(90)$.

c. Which function serves as a better model for the chance of surviving to age 90?

To go, please. The graphs show that more Americans are ordering their food to go, instead of dining inside restaurants. A quadratic function models the average number of meals per person that Americans ordered for takeout and a linear function models the average number of meals ordered for eating in restaurants. In each model, x represents the number of years after 1984. Use the graphs and the displayed equations to solve Exercises 103–104.

Models for Average Number of Meals per Person Americans Ordered from 1984 through 2005



Source: NPD Group

(In Exercises 103–104, refer to the graphs and their equations at the bottom of the previous page.)

103. a. Use the equation for function T to find and interpret $T(20)$. How is this shown on the graph of T?

b. Use the equation for function R to find and interpret $R(0)$. How is this shown on the graph of R?

c. According to the graphs, in which year did the average number of takeout orders approximately equal the average number of in-restaurant orders? Use the equations for T and R to find the average number of meals per person for each kind of order in that year.

104. a. Use the equation for function T to find and interpret $T(18)$. How is this shown on the graph of T?

b. Use the equation for function R to find and interpret $R(20)$. How is this shown on the graph of R?

In Exercises 105–108, you will be developing functions that model given conditions.

105. A company that manufactures bicycles has a fixed cost of \$100,000. It costs \$100 to produce each bicycle. The total cost for the company is the sum of its fixed cost and variable costs. Write the total cost, C, as a function of the number of bicycles produced, x. Then find and interpret $C(90)$.

106. A car was purchased for \$22,500. The value of the car decreased by \$3200 per year for the first six years. Write a function that describes the value of the car, V, after x years, where $0 \le x \le 6$. Then find and interpret $V(3)$.

107. You commute to work a distance of 40 miles and return on the same route at the end of the day. Your average rate on the return trip is 30 miles per hour faster than your average rate on the outgoing trip. Write the total time, T, in hours, devoted to your outgoing and return trips as a function of your rate on the outgoing trip, x. Then find and interpret $T(30)$. Hint:

$$\text{Time traveled} = \frac{\text{Distance traveled}}{\text{Rate of travel}}.$$

108. A chemist working on a flu vaccine needs to mix a 10% sodium-iodine solution with a 60% sodium-iodine solution to obtain a 50-milliliter mixture. Write the amount of sodium iodine in the mixture, S, in milliliters, as a function of the number of milliliters of the 10% solution used, x. Then find and interpret $S(30)$.

Writing in Mathematics

109. What is a relation? Describe what is meant by its domain and its range.

110. Explain how to determine whether a relation is a function. What is a function?

111. How do you determine if an equation in x and y defines y as a function of x?

112. Does $f(x)$ mean f times x when referring to a function f? If not, what does $f(x)$ mean? Provide an example with your explanation.

113. What is the graph of a function?

114. Explain how the vertical line test is used to determine whether a graph represents a function.

115. Explain how to identify the domain and range of a function from its graph.

116. For people filing a single return, federal income tax is a function of adjusted gross income because for each value of adjusted gross income there is a specific tax to be paid. By contrast, the price of a house is not a function of the lot size on which the house sits because houses on same-sized lots can sell for many different prices.

a. Describe an everyday situation between variables that is a function.

b. Describe an everyday situation between variables that is not a function.

Technology Exercise

117. Use a graphing utility to verify any five pairs of graphs that you drew by hand in Exercises 39–54.

Critical Thinking Exercises

Make Sense? *In Exercises 118–121, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

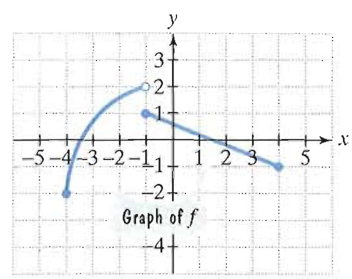
118. My body temperature is a function of the time of day.

119. Using $f(x) = 3x + 2$, I found $f(50)$ by applying the distributive property to $(3x + 2)50$.

120. I graphed a function showing how paid vacation days depend on the number of years a person works for a company. The domain was the number of paid vacation days.

121. I graphed a function showing how the average number of annual physician visits depends on a person's age. The domain was the average number of annual physician visits.

Use the graph of f to determine whether each statement in Exercises 122–125 is true or false.



122. The domain of f is $[-4, -1) \cup (-1, 4]$.

123. The range of f is $[-2, 2]$.

124. $f(-1) - f(4) = 2$

125. $f(0) = 2.1$

126. If $f(x) = 3x + 7$, find $\dfrac{f(a + h) - f(a)}{h}$.

127. Give an example of a relation with the following characteristics: The relation is a function containing two ordered pairs. Reversing the components in each ordered pair results in a relation that is not a function.

128. If $f(x + y) = f(x) + f(y)$ and $f(1) = 3$, find $f(2), f(3)$, and $f(4)$. Is $f(x + y) = f(x) + f(y)$ for all functions?

Preview Exercises

Exercises 129–131 will help you prepare for the material covered in the next section.

129. The function $C(t) = 20 + 0.40(t - 60)$ describes the monthly cost, $C(t)$, in dollars, for a cellular phone plan for t calling minutes, where $t > 60$. Find and interpret $C(100)$.

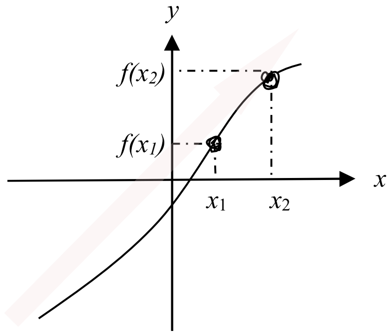
130. Use point plotting to graph $f(x) = x + 2$ if $x \le 1$.

131. Simplify: $2(x + h)^2 + 3(x + h) + 5 - (2x^2 + 3x + 5)$.

1.3 More on Functions & Their Graphs

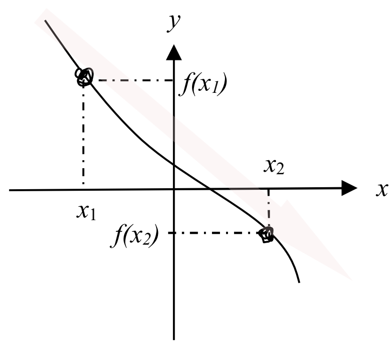
Def: A function is

1. increasing on an open interval *I*, if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.



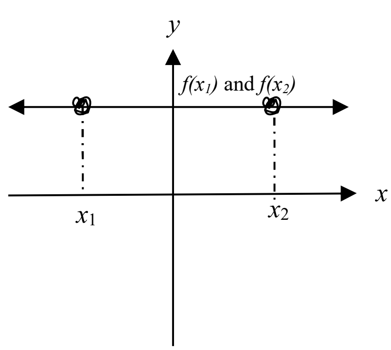
As you move along the x–axis from left to the right, the function goes in an upward direction

2. decreasing on an open interval *I*, if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.



As you move along the x–axis from left to the right, the function goes in a downward direction

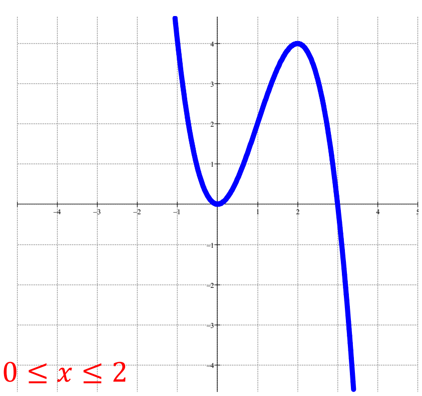
3. constant on an open interval *I*, if $f(x_1) = f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.



As you move along the x–axis from left to the right, the function does not rise or fall (it is horizontal)

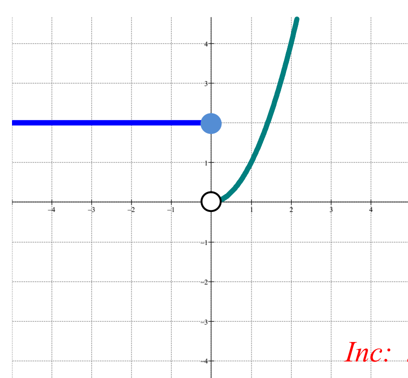
Ex. Using the graph, state the intervals in which the function is increasing, decreasing or constant

1.



Inc: $0 \leq x \leq 2$
Dec: $x < 0$ *or* $x > 2$
Constant: never
One interval gets the equality, the other does not

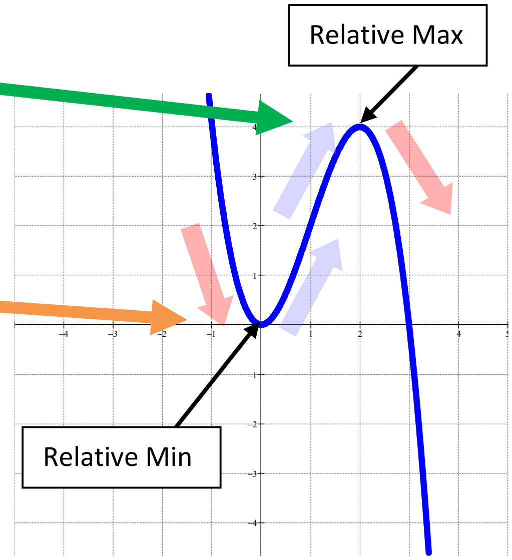
2.



Inc: $x > 0$
Dec: never
Constant: $x \leq 0$

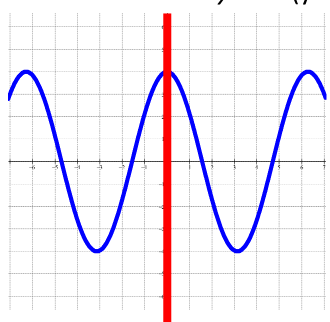
Relative Extrema: (Relative Maximums and Minimums)

1. A function has a relative maximum at $x=c$ if the function changes from an increasing function to a decreasing function at $x = c$.
   - For all x around c, $f(x) < f(c)$
2. A function has a relative minimum at $x=c$ if the function changes from a decreasing function to an increasing function at $x = c$.
   - For all x around c, $f(x) > f(c)$



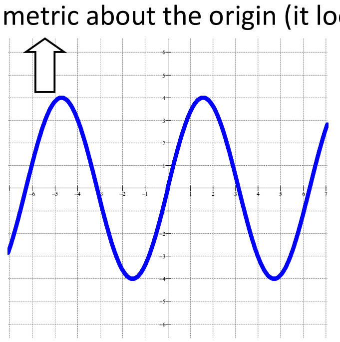
Def: A function $y = f(x)$ is

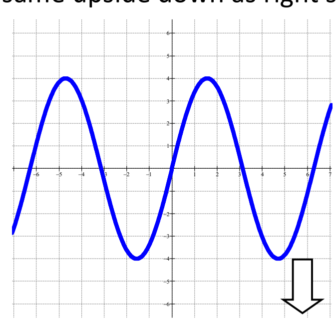
1. an even function if $f(-x) = f(x)$ for all x in the domain of f.
   - This means if you plug in $-x$ in for x and simplify, you get the original function $f(x)$
   - Even functions are symmetric about the y-axis (y-axis divides the function into mirror images)



2. an odd function if $f(-x) = -f(x)$ for all x in the domain of f.
   - This mean if you plug in $-x$ in for x and simplify, you get the original function $f(x)$ with all the signs changed.
   - Odd functions are symmetric about the origin (it looks the same upside down as right side up)

The arrow is a reference to show there the original top of the graph is.





Ex. Determine whether each of the following functions is even, odd, or neither.

1. $f(x) = x^3 - 6x$

$f(-x) = (-x)^3 - 6(-x)$
$f(-x) = -x^3 + 6x$
$f(-x) = -(x^3 - 6x) = -f(x)$
ODD

2. $f(x) = 3x^4 + 2x^2$

$f(-x) = 3(-x)^4 + 2(-x)^2$
$f(-x) = 3x^4 + 2x^2 = f(x)$

EVEN

3. $f(x) = x^2 - 1$

$f(-x) = (-x)^2 - 1$
$f(-x) = x^2 - 1 = f(x)$

EVEN

4. $f(x) = x^3 + 3x^2$

$f(-x) = (-x)^3 + 3(-x)^2$ Not even
$f(-x) = -x^3 - 3x^2 \neq f(x)$ Not odd
$f(-x) = -(x^3 + 3x^2) \neq -f(x)$
NEITHER

5. $f(x) = -x^2$

$f(-x) = -(-x)^2$
$f(-x) = -x^2 = f(x)$

EVEN

Homework: Pg. 172-174 #1-33odd, 34-36

1.3b More on Functions & Their Graphs

I refer to these numbers as **PIVOTS** (A personal term)
It's where the function pivots to the next one

Def: A function that is defined by two or more equations over a specified domain is called a piecewise function.

Ex.

$$y=\begin{cases} x^2-1, & x<1 \\ 2x-2, & x\geq 1 \end{cases}$$

For $x < 1$, you use the first expression, for all other values of x, use the 2[nd] expression ($2x-2$)

What is the value of y when $x = 0$, 1, and 2?

$f(0) = (0)^2 - 1 = -1$ *Use the top piece because* $0 < 1$
$f(1) = 2(1) - 2 = 2 - 2 = 0$ *Use the bottom piece because* $1 \geq 1$
$f(2) = 2(2) - 2 = 4 - 2 = 2$ *Again use the bottom piece because* $2 \geq 1$

Ex. Your cellular phone plan has a cost function shown below.

$$C(t)=\begin{cases} 20, 0\leq t\leq 60 \\ 20+(0.40(t-60)), t>60 \end{cases}$$

(a) Explain the meaning of the function above.
(b) Find and interpret each of the following:
1. C(30)
2. C(100)

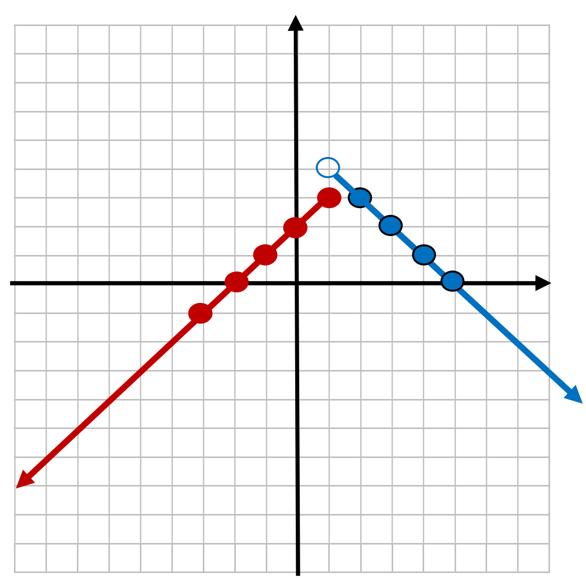
(a) For the first 60 units (maybe hours), the cost is a flat rate of \$20. For each unit over 60 (from the $t - 60$), the cost is the flat 20 plus 40 cents per unit

(b) (1) $C(30) = 20$
(2) $C(100) = 20 + 0.4(100 - 60) = 20 + 0.4(40) = 20 + 16 = 36$

Graphing a Piecewise :

Ex. On the set of axes to the right, graph the function below

$$f(x)=\begin{cases} x+2, & x\leq 1 \\ 5-x, & x>1 \end{cases}$$



$y = x - 2$

Line

Slope: $m = 1$
yint: $b = 2$

x	$x+2$	y
−3	−3+2	−1
−2	−2+2	0
−1	−1+2	1
0	0+2	2
1	1 + 2	3

$y = 5 - x$

Line

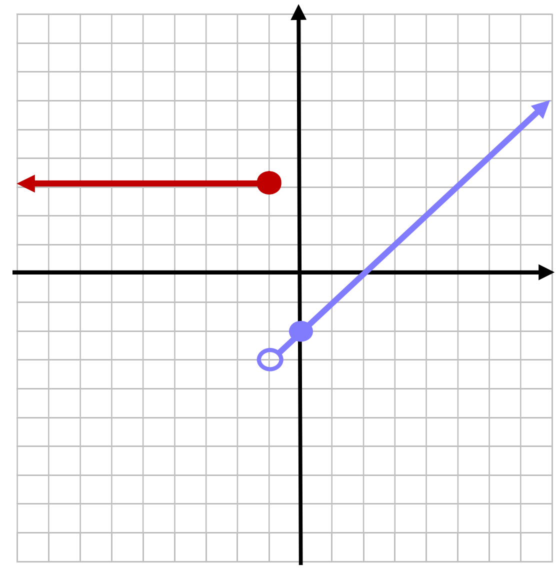
Slope: $m = -1$
y-int: $b = 5$

x	$5-x$	y
1	5 − 1	4
2	5 − 2	3
3	5 − 3	2
4	5 − 4	1
5	5 − 5	0

Make a table for each "piece" of the function.
Be sure you use solid or open dots correctly.

Ex. Graph the piecewise function on the axes to the right.

$$f(x)=\begin{cases}3, & x\le -1\\ x-2, & x>-1\end{cases}$$



$y = 3$

Horizontal Line

$y = x - 2$

x	$x-2$	y
-1	$-1-2$	-3
0	$0-2$	-2
1	$1-2$	-1
2	$2-2$	0
3	$3-2$	1

Show how to do this on a graphing calculator

rate of change of a function

Def: The Difference Quotient of a function is defined by

$$\frac{f(x+h)-f(x)}{h}\text{, provided } h \neq 0$$

Ex. For each of the following, find the difference quotient.

1. $f(x) = 2x^2 - x + 3$

$$DQ = \frac{f(x+h)-f(x)}{h}$$
$$DQ = \frac{2(x+h)^2 - (x+h) + 3 - (2x^2 - x + 3)}{h}$$
$$DQ = \frac{2x^2 + 4xh + 2h^2 - x - h + 3 - 2x^2 + x - 3}{h}$$
$$DQ = \frac{4xh + 2h^2 - h}{h}$$
$$DQ = \frac{h(4x + 2h - 1)}{h} = 4x + 2h - 1$$

2. $f(x) = -2x^2 + 4x + 5$

$$DQ = \frac{f(x+h)-f(x)}{h}$$
$$DQ = \frac{-2(x+h)^2 + 4(x+h) + 5 - (-2x^2 + 4x + 5)}{h}$$
$$DQ = \frac{-2x^2 - 4xh - 2h^2 + 4x + 4h + 5 + 2x^2 - 4x - 5}{h}$$
$$DQ = \frac{-4xh - 2h^2 + 4h}{h}$$
$$DQ = \frac{h(-4x - 2h + 4)}{h} = -4x - 2h + 4$$

Homework: Pg. 172-176 #18-28even, 37, 41, 42, 47, 51, 53, 57, 64, 83-89odd

Solution

a. We find $f(x + h)$ by replacing x with $x + h$ each time that x appears in the equation.

$$f(x) = 2x^2 \quad - \quad x \quad + \quad 3$$

Replace x with $x + h$. | Replace x with $x + h$. | Replace x with $x + h$. | Copy the 3. There is no x in this term.

$$\begin{aligned} f(x+h) &= 2(x+h)^2 - (x+h) + 3 \\ &= 2(x^2 + 2xh + h^2) - x - h + 3 \\ &= 2x^2 + 4xh + 2h^2 - x - h + 3 \end{aligned}$$

b. Using our result from part (a), we obtain the following:

This is $f(x + h)$ from part (a). This is $f(x)$ from the given equation.

$$\frac{f(x+h) - f(x)}{h} = \frac{\boxed{2x^2 + 4xh + 2h^2 - x - h + 3} - (2x^2 - x + 3)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - x - h + 3 - 2x^2 + x - 3}{h}$$ Remove parentheses and change the sign of each term in the parentheses.

$$= \frac{(2x^2 - 2x^2) + (-x + x) + (3 - 3) + 4xh + 2h^2 - h}{h}$$ Group like terms.

$$= \frac{4xh + 2h^2 - 1h}{h}$$ Simplify. (We wrote $-h$ as $-1h$ to avoid possible errors in the next factoring step.)

$$= \frac{h(4x + 2h - 1)}{h}$$ Factor h from the numerator.

$$= 4x + 2h - 1$$ Divide out identical factors of h in the numerator and denominator.

Check Point 5 If $f(x) = -2x^2 + x + 5$, find and simplify each expression:

a. $f(x + h)$ **b.** $\dfrac{f(x+h) - f(x)}{h}, h \neq 0.$

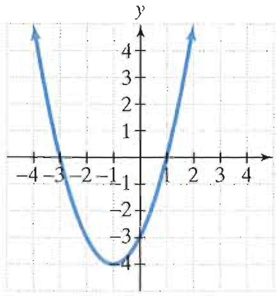
Exercise Set 1.3

Practice Exercises

In Exercises 1–12, use the graph to determine

a. *intervals on which the function is increasing, if any.*
b. *intervals on which the function is decreasing, if any.*
c. *intervals on which the function is constant, if any.*

1.



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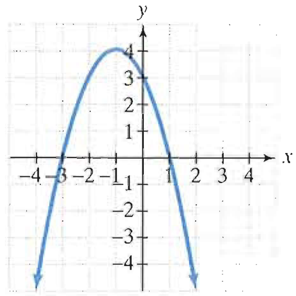

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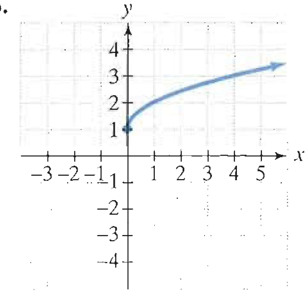

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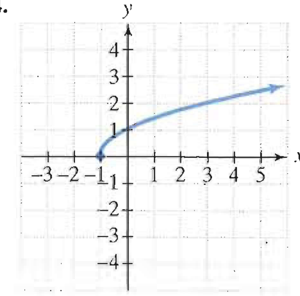

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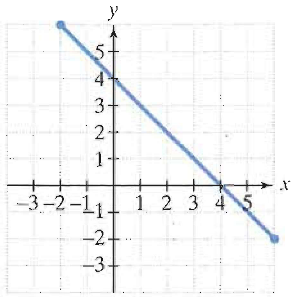

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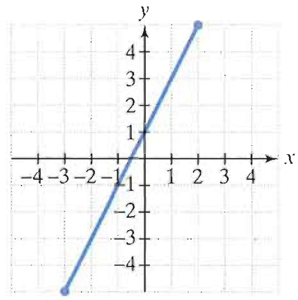

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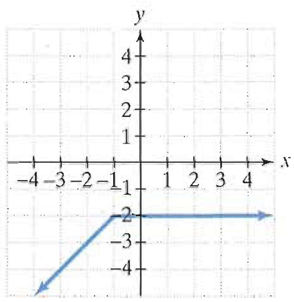

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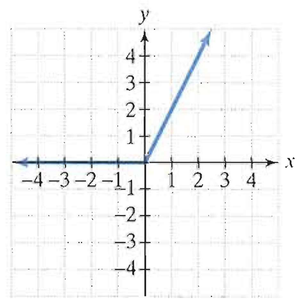
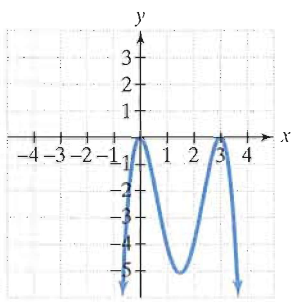
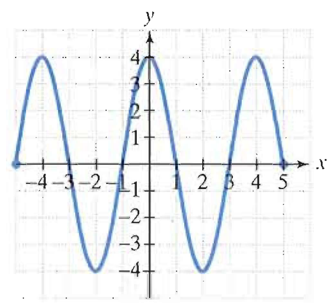


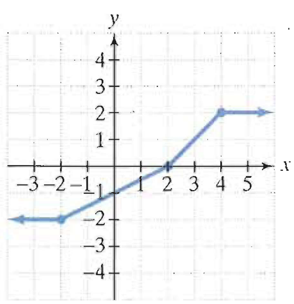
9.



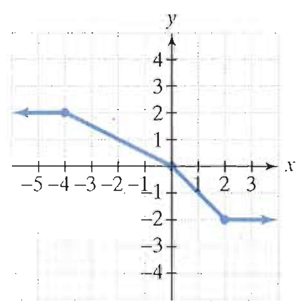
10.



11.



12.

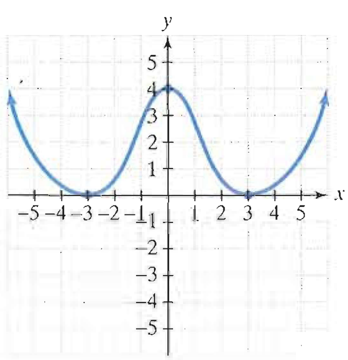


In Exercises 13–16, the graph of a function f is given. Use the graph to find each of the following:

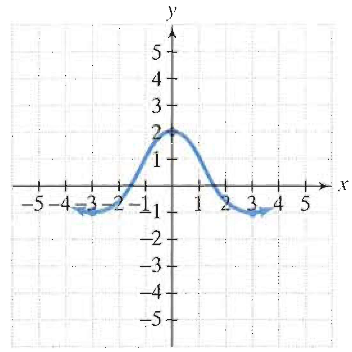
a. *The numbers, if any, at which f has a relative maximum. What are these relative maxima?*

b. *The numbers, if any, at which f has a relative minimum. What are these relative minima?*

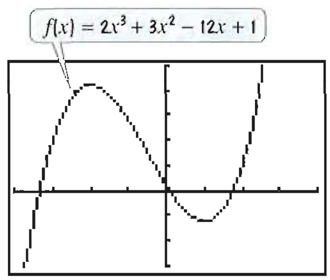
13.



14.

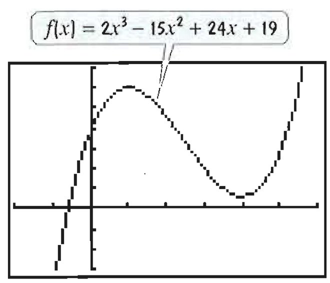


15.



[−4, 4, 1] by [−15, 25, 5]

16.



[−2, 6, 1] by [−15, 35, 5]

In Exercises 17–28, determine whether each function is even, odd, or neither.

17. $f(x) = x^3 + x$

18. $f(x) = x^3 - x$

19. $g(x) = x^2 + x$

20. $g(x) = x^2 - x$

21. $h(x) = x^2 - x^4$

22. $h(x) = 2x^2 + x^4$

23. $f(x) = x^2 - x^4 + 1$

24. $f(x) = 2x^2 + x^4 + 1$

25. $f(x) = \frac{1}{5}x^6 - 3x^2$

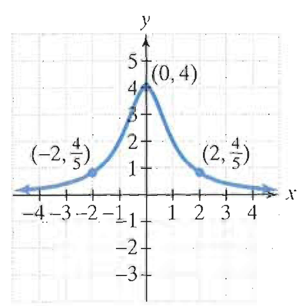
26. $f(x) = 2x^3 - 6x^5$

27. $f(x) = x\sqrt{1 - x^2}$

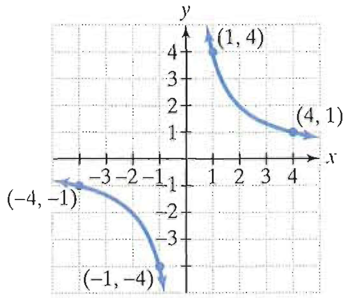
28. $f(x) = x^2\sqrt{1 - x^2}$

In Exercises 29–32, use possible symmetry to determine whether each graph is the graph of an even function, an odd function, or a function that is neither even nor odd.

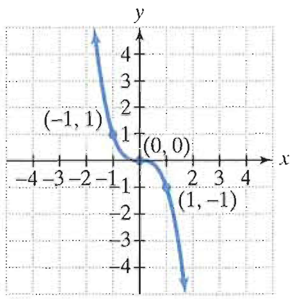
29.



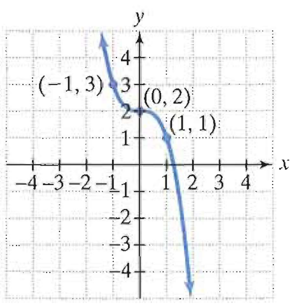
30.



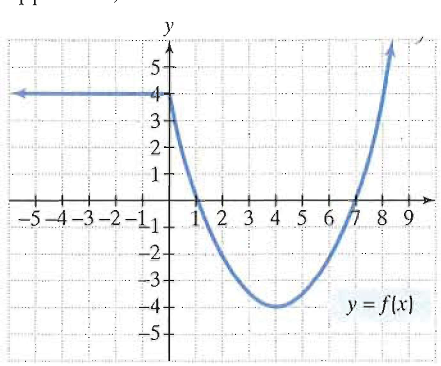
31.



32.



33. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the x-intercepts

d. the y-intercept

e. intervals on which f is increasing

f. intervals on which f is decreasing

g. intervals on which f is constant

h. the number at which f has a relative minimum

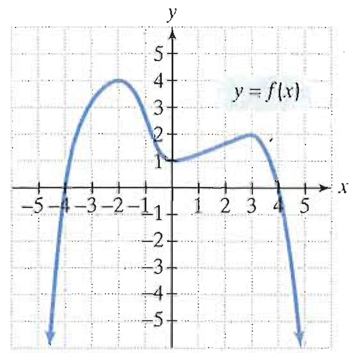
i. the relative minimum of f

j. $f(-3)$

k. the values of x for which $f(x) = -2$

l. Is f even, odd, or neither?

34. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the x-intercepts

d. the y-intercept

e. intervals on which f is increasing

f. intervals on which f is decreasing

g. values of x for which $f(x) \leq 0$

h. the numbers at which f has a relative maximum

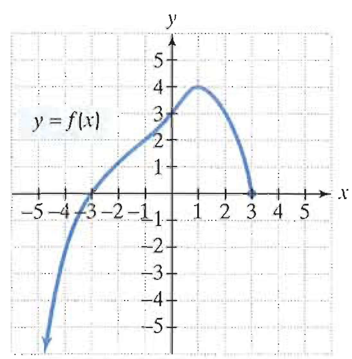
i. the relative maxima of f

j. $f(-2)$

k. the values of x for which $f(x) = 0$

l. Is f even, odd, or neither?

35. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the zeros of f

d. $f(0)$

e. intervals on which f is increasing

f. intervals on which f is decreasing

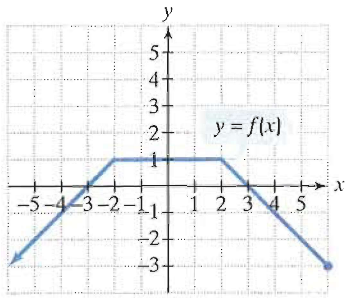
g. values of x for which $f(x) \leq 0$

h. any relative maxima and the numbers at which they occur

i. the value of x for which $f(x) = 4$

j. Is $f(-1)$ positive or negative?

36. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f
b. the range of f
c. the zeros of f
d. $f(0)$
e. intervals on which f is increasing
f. intervals on which f is decreasing
g. intervals on which f is constant
h. values of x for which $f(x) > 0$
i. values of x for which $f(x) = -2$
j. Is $f(4)$ positive or negative?
k. Is f even, odd, or neither?
l. Is $f(2)$ a relative maximum?

In Exercises 37–42, evaluate each piecewise function at the given values of the independent variable.

37. $f(x) = \begin{cases} 3x + 5 & \text{if } x < 0 \\ 4x + 7 & \text{if } x \geq 0 \end{cases}$

a. $f(-2)$ **b.** $f(0)$ **c.** $f(3)$

38. $f(x) = \begin{cases} 6x - 1 & \text{if } x < 0 \\ 7x + 3 & \text{if } x \geq 0 \end{cases}$

a. $f(-3)$ **b.** $f(0)$ **c.** $f(4)$

39. $g(x) = \begin{cases} x + 3 & \text{if } x \geq -3 \\ -(x + 3) & \text{if } x < -3 \end{cases}$

a. $g(0)$ **b.** $g(-6)$ **c.** $g(-3)$

40. $g(x) = \begin{cases} x + 5 & \text{if } x \geq -5 \\ -(x + 5) & \text{if } x < -5 \end{cases}$

a. $g(0)$ **b.** $g(-6)$ **c.** $g(-5)$

41. $h(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

a. $h(5)$ **b.** $h(0)$ **c.** $h(3)$

42. $h(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5} & \text{if } x \neq 5 \\ 10 & \text{if } x = 5 \end{cases}$

a. $h(7)$ **b.** $h(0)$ **c.** $h(5)$

In Exercises 43–54, the domain of each piecewise function is $(-\infty, \infty)$.

a. *Graph each function.*
b. *Use your graph to determine the function's range.*

43. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

44. $f(x) = \begin{cases} x & \text{if } x < 0 \\ -x & \text{if } x \geq 0 \end{cases}$

45. $f(x) = \begin{cases} 2x & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$

46. $f(x) = \begin{cases} \frac{1}{2}x & \text{if } x \leq 0 \\ 3 & \text{if } x > 0 \end{cases}$

47. $f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ x - 3 & \text{if } x \geq -2 \end{cases}$

48. $f(x) = \begin{cases} x + 2 & \text{if } x < -3 \\ x - 2 & \text{if } x \geq -3 \end{cases}$

49. $f(x) = \begin{cases} 3 & \text{if } x \leq -1 \\ -3 & \text{if } x > -1 \end{cases}$

50. $f(x) = \begin{cases} 4 & \text{if } x \leq -1 \\ -4 & \text{if } x > -1 \end{cases}$

51. $f(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x < 1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$

52. $f(x) = \begin{cases} -\frac{1}{2}x^2 & \text{if } x < 1 \\ 2x + 1 & \text{if } x \geq 1 \end{cases}$

53. $f(x) = \begin{cases} 0 & \text{if } x < -4 \\ -x & \text{if } -4 \leq x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

54. $f(x) = \begin{cases} 0 & \text{if } x < -3 \\ -x & \text{if } -3 \leq x < 0 \\ x^2 - 1 & \text{if } x \geq 0 \end{cases}$

In Exercises 55–76, find and simplify the difference quotient

$$\frac{f(x + h) - f(x)}{h}, h \neq 0$$

for the given function.

55. $f(x) = 4x$

56. $f(x) = 7x$

57. $f(x) = 3x + 7$

58. $f(x) = 6x + 1$

59. $f(x) = x^2$

60. $f(x) = 2x^2$

61. $f(x) = x^2 - 4x + 3$

62. $f(x) = x^2 - 5x + 8$

63. $f(x) = 2x^2 + x - 1$

64. $f(x) = 3x^2 + x + 5$

65. $f(x) = -x^2 + 2x + 4$

66. $f(x) = -x^2 - 3x + 1$

67. $f(x) = -2x^2 + 5x + 7$

68. $f(x) = -3x^2 + 2x - 1$

69. $f(x) = -2x^2 - x + 3$

70. $f(x) = -3x^2 + x - 1$

71. $f(x) = 6$

72. $f(x) = 7$

73. $f(x) = \frac{1}{x}$

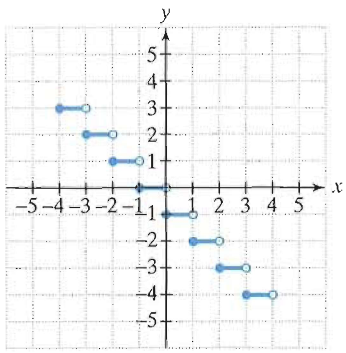
74. $f(x) = \frac{1}{2x}$

75. $f(x) = \sqrt{x}$

76. $f(x) = \sqrt{x - 1}$

Practice Plus

In Exercises 77–78, let f be defined by the following graph:



77. Find

$$\sqrt{f(-1.5) + f(-0.9)} - [f(\pi)]^2 + f(-3) \div f(1) \cdot f(-\pi).$$

78. Find

$$\sqrt{f(-2.5) - f(1.9)} - [f(-\pi)]^2 + f(-3) \div f(1) \cdot f(\pi).$$

A cellular phone company offers the following plans. Also given are the piecewise functions that model these plans. Use this information to solve Exercises 79–80.

Plan A

- *$30 per month buys 120 minutes.*
- *Additional time costs $0.30 per minute.*

$$C(t) = \begin{cases} 30 & \text{if } 0 \le t \le 120 \\ 30 + 0.30(t - 120) & \text{if } t > 120 \end{cases}$$

Plan B

- *$40 per month buys 200 minutes.*
- *Additional time costs $0.30 per minute.*

$$C(t) = \begin{cases} 40 & \text{if } 0 \le t \le 200 \\ 40 + 0.30(t - 200) & \text{if } t > 200 \end{cases}$$

79. Simplify the algebraic expression in the second line of the piecewise function for plan A. Then use point-plotting to graph the function.

80. Simplify the algebraic expression in the second line of the piecewise function for plan B. Then use point-plotting to graph the function.

In Exercises 81–82, write a piecewise function that models each cellular phone billing plan. Then graph the function.

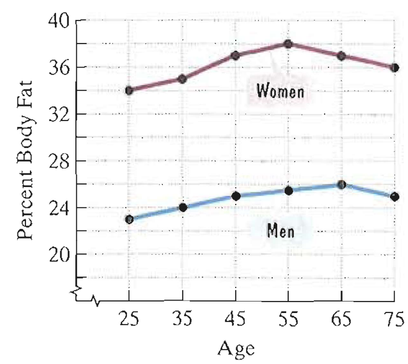
81. $50 per month buys 400 minutes. Additional time costs $0.30 per minute.

82. $60 per month buys 450 minutes. Additional time costs $0.35 per minute.

Application Exercises

With aging, body fat increases and muscle mass declines. The line graphs show the percent body fat in adult women and men as they age from 25 to 75 years. Use the graphs to solve Exercises 83–90.

Percent Body Fat in Adults



Source: Thompson et al., *The Science of Nutrition*, Benjamin Cummings, 2008

83. State the intervals on which the graph giving the percent body fat in women is increasing and decreasing.

84. State the intervals on which the graph giving the percent body fat in men is increasing and decreasing.

85. For what age does the percent body fat in women reach a maximum? What is the percent body fat for that age?

86. At what age does the percent body fat in men reach a maximum? What is the percent body fat for that age?

87. Use interval notation to give the domain and the range for the graph of the function for women.

88. Use interval notation to give the domain and the range for the graph of the function for men.

89. The function $p(x) = -0.002x^2 + 0.15x + 22.86$ models percent body fat, $p(x)$, where x is the number of years a person's age exceeds 25. Use the graphs to determine whether this model describes percent body fat in women or in men.

90. The function $p(x) = -0.004x^2 + 0.25x + 33.64$ models percent body fat, $p(x)$, where x is the number of years a person's age exceeds 25. Use the graphs to determine whether this model describes percent body fat in women or in men.

Here is the 2007 Federal Tax Rate Schedule X that specifies the tax owed by a single taxpayer.

If Your Taxable Income Is Over	But Not Over	The Tax You Owe Is	Of the Amount Over
$ 0	$ 7825	10%	$ 0
$ 7825	$ 31,850	$ 782.50 + 15%	$ 7825
$ 31,850	$ 77,100	$ 4386.25 + 25%	$ 31,850
$ 77,100	$160,850	$ 15,698.75 + 28%	$ 77,100
$160,850	$349,700	$ 39,148.75 + 33%	$160,850
$349,700	—	$101,469.25 + 35%	$349,700

The tax table on the previous page can be modeled by a piecewise function, where x represents the taxable income of a single taxpayer and $T(x)$ is the tax owed:

$$T(x) = \begin{cases} 0.10x & \text{if} \quad 0 < x \le 7825 \\ 782.50 + 0.15(x - 7825) & \text{if} \quad 7825 < x \le 31{,}850 \\ 4386.25 + 0.25(x - 31{,}850) & \text{if} \quad 31{,}850 < x \le 77{,}100 \\ 15{,}698.75 + 0.28(x - 77{,}100) & \text{if} \quad 77{,}100 < x \le 160{,}850 \\ \underline{\quad ? \quad} & \text{if} \quad 160{,}850 < x \le 349{,}700 \\ \underline{\quad ? \quad} & \text{if} \quad x > 349{,}700. \end{cases}$$

Use this information to solve Exercises 91–94.

91. Find and interpret $T(20{,}000)$.

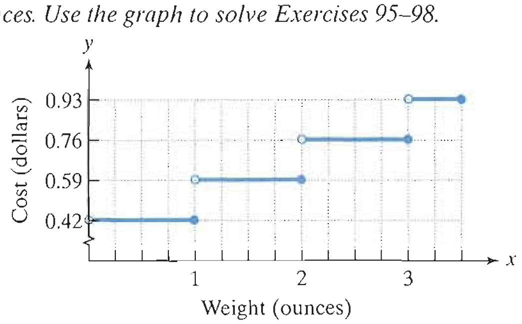
92. Find and interpret $T(50{,}000)$.

In Exercises 93–94, refer to the tax table on the previous page.

93. Find the algebraic expression for the missing piece of $T(x)$ that models tax owed for the domain $(160{,}850, 349{,}700]$.

94. Find the algebraic expression for the missing piece of $T(x)$ that models tax owed for the domain $(349{,}700, \infty)$.

The figure shows the cost of mailing a first-class letter, $f(x)$, as a function of its weight, x, in ounces, for weights not exceeding 3.5 ounces. Use the graph to solve Exercises 95–98.



Source: Lynn E. Baring, Postmaster, Inverness, CA

95. Find $f(3)$. What does this mean in terms of the variables in this situation?

96. Find $f(3.5)$. What does this mean in terms of the variables in this situation?

97. What is the cost of mailing a letter that weighs 1.5 ounces?

98. What is the cost of mailing a letter that weighs 1.8 ounces?

99. If $3.5 < x \le 4$, the cost of mailing a first-class letter jumps to \$1.34. The cost then increases by \$0.17 per ounce for weights not exceeding 13 ounces:

Weight Not Exceeding	Cost
5 ounces	\$1.51
6 ounces	\$1.68

etc.

Use this information to extend the graph shown above so that the function's domain is $(0, 13]$.

Writing in Mathematics

100. What does it mean if a function f is increasing on an interval?

101. Suppose that a function f whose graph contains no breaks or gaps on (a, c) is increasing on (a, b), decreasing on (b, c), and defined at b. Describe what occurs at $x = b$. What does the function value $f(b)$ represent?

102. If you are given a function's equation, how do you determine if the function is even, odd, or neither?

103. If you are given a function's graph, how do you determine if the function is even, odd, or neither?

104. What is a piecewise function?

105. Explain how to find the difference quotient of a function f, $\dfrac{f(x + h) - f(x)}{h}$, if an equation for f is given.

Technology Exercises

106. The function

$$f(x) = -0.00002x^3 + 0.008x^2 - 0.3x + 6.95$$

models the number of annual physician visits, $f(x)$, by a person of age x. Graph the function in a $[0, 100, 5]$ by $[0, 40, 2]$ viewing rectangle. What does the shape of the graph indicate about the relationship between one's age and the number of annual physician visits? Use the TABLE or minimum function capability to find the coordinates of the minimum point on the graph of the function. What does this mean?

In Exercises 107–112, use a graphing utility to graph each function. Use a $[-5, 5, 1]$ by $[-5, 5, 1]$ viewing rectangle. Then find the intervals on which the function is increasing, decreasing, or constant.

108. $f(x) = x^3 - 6x^2 + 9x + 1$

108. $g(x) = |4 - x^2|$

109. $h(x) = |x - 2| + |x + 2|$

110. $f(x) = x^{\frac{1}{3}}(x - 4)$

111. $g(x) = x^{\frac{2}{3}}$

112. $h(x) = 2 - x^{\frac{2}{5}}$

113. **a.** Graph the functions $f(x) = x^n$ for $n = 2, 4$, and 6 in a $[-2, 2, 1]$ by $[-1, 3, 1]$ viewing rectangle.

b. Graph the functions $f(x) = x^n$ for $n = 1, 3$, and 5 in a $[-2, 2, 1]$ by $[-2, 2, 1]$ viewing rectangle.

c. If n is positive and even, where is the graph of $f(x) = x^n$ increasing and where is it decreasing?

d. If n is positive and odd, what can you conclude about the graph of $f(x) = x^n$ in terms of increasing or decreasing behavior?

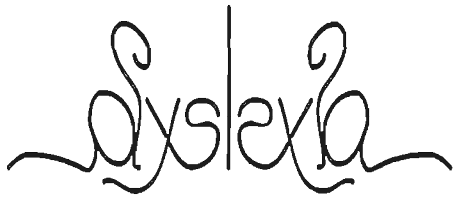
e. Graph all six functions in a $[-1, 3, 1]$ by $[-1, 3, 1]$ viewing rectangle. What do you observe about the graphs in terms of how flat or how steep they are?

Critical Thinking Exercises

Make Sense? *In Exercises 114–117, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

114. My graph is decreasing on $(-\infty, a)$ and increasing on (a, ∞), so $f(a)$ must be a relative maximum.

115. This work by artist Scott Kim (1955–) has the same kind of symmetry as an even function.



"DYSLEXIA," 1981

116. I graphed

$$f(x) = \begin{cases} 2 & \text{if} \quad x \neq 4 \\ 3 & \text{if} \quad x = 4 \end{cases}$$

and one piece of my graph is a single point.

117. I noticed that the difference quotient is always zero if $f(x) = c$, where c is any constant.

118. Sketch the graph of f using the following properties. (More than one correct graph is possible.) f is a piecewise function that is decreasing on $(-\infty, 2)$, $f(2) = 0$, f is increasing on $(2, \infty)$, and the range of f is $[0, \infty)$.

119. Define a piecewise function on the intervals $(-\infty, 2]$, $(2, 5)$, and $[5, \infty)$ that does not "jump" at 2 or 5 such that one piece is a constant function, another piece is an increasing function, and the third piece is a decreasing function.

120. Suppose that $h(x) = \dfrac{f(x)}{g(x)}$. The function f can be even, odd, or neither. The same is true for the function g.

a. Under what conditions is h definitely an even function?

b. Under what conditions is h definitely an odd function?

Group Exercise

121. (For assistance with this exercise, refer to the discussion of piecewise functions beginning on page 169, as well as to Exercises 79–80.) Group members who have cellular phone plans should describe the total monthly cost of the plan as follows:

$_____ per month buys _____ minutes. Additional time costs $_____ per minute.

(For simplicity, ignore other charges.) The group should select any three plans, from "basic" to "premier." For each plan selected, write a piecewise function that describes the plan and graph the function. Graph the three functions in the same rectangular coordinate system. Now examine the graphs. For any given number of calling minutes, the best plan is the one whose graph is lowest at that point. Compare the three calling plans. Is one plan always a better deal than the other two? If not, determine the interval of calling minutes for which each plan is the best deal. (You can check out cellular phone plans by visiting www.point.com.)

Preview Exercises

Exercises 122–124 will help you prepare for the material covered in the next section.

122. If $(x_1, y_1) = (-3, 1)$ and $(x_2, y_2) = (-2, 4)$, find $\dfrac{y_2 - y_1}{x_2 - x_1}$.

123. Find the ordered pairs (_____, 0) and (0, _____) satisfying $4x - 3y - 6 = 0$.

124. Solve for y: $3x + 2y - 4 = 0$.

Section 1.4 Linear Functions and Slope

Objectives

1. Calculate a line's slope.
2. Write the point-slope form of the equation of a line.
3. Write and graph the slope-intercept form of the equation of a line.
4. Graph horizontal or vertical lines.
5. Recognize and use the general form of a line's equation.
6. Use intercepts to graph the general form of a line's equation.
7. Model data with linear functions and make predictions.

Is there a relationship between literacy and child mortality? As the percentage of adult females who are literate increases, does the mortality of children under five decrease? **Figure 1.35** on the next page indicates that this is, indeed, the case. Each point in the figure represents one country.

1.4 Linear Functions & Slope

Def: The slope of a line through distinct points (x_1, y_1) and (x_2, y_2) is

$$\frac{\Delta y}{\Delta x} = \frac{change in\ y}{change in\ x} = \frac{y_2 - y_1}{x_2 - x_1} \text{ this is also referred to as } \frac{\text{" rise "}}{\text{" run "}}$$

provided that $x_1 \neq x_2$ *If the x coordinates are the same, the line has NO SLOPE and is vertical*

ex. Find the slope of the line passing through each of the given points.

1. (-3, -1) and (-2, 4)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-1)}{-2-(-3)} = \frac{5}{1} = 5$$

2. (-3,4) and (2, -2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-4}{2-(-3)} = \frac{-6}{5} = -\frac{6}{5}$$

3. (-3,4) and (-4, -2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-4}{-4-(-3)} = \frac{-6}{-1} = 6$$

4. (4, -2) and (-1,5)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{-1-4} = \frac{7}{-5} = -\frac{7}{5}$$

Equations of Lines: There are several different forms of equations for lines.

1. Point-Slope Form: For a non-vertical line, whose slope is *m* and passes through a point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$ *Note the signs of the point are opposite in the eq.*

Ex. Write an equation of the line, in Point-Slope Form,

1. with slope 4 and passes through (-1,3) $y - 3 = 4(x+1)$
2. with slope -2 and passes through (4,-1) $y + 1 = -2(x-4)$
3. passing through points (3,2) and (-1, 10)

Find the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-3}{1-3} = \frac{7}{-2} = -\frac{7}{2}$$

Plug the slope and one of the points (and it does not matter which pt.)

$$y - 2 = -\frac{7}{2}(x-3)$$

$$y - 10 = -\frac{7}{2}(x+1)$$

*** ALWAYS USE THIS FORM IF YOU HAVE THE SLOPE AND A POINT ***

2. Slope-Intercept Form: For a non-vertical line with slope *m* and *y*-intercept *b*

$$y = mx + b$$

Ex. Write the equation of the line, in Slope-Intercept form,

1. slope 6 and y-intercept 3 $y = 6x + 3$
2. slope -3 and y-intercept -1 $y = -3x - 1$
3. slope -2 and passes through (1,4)

$$y - 4 = -2(x-1)$$
$$y - 4 = -2x + 2$$
$$y = -2x + 6$$

4. Passes through (1,1) and (4, -2)

Does not matter which point you use

$$m = \frac{\Delta y}{\Delta x}$$
$$m = \frac{-2-1}{4-1} = -\frac{3}{3}$$
$$m = -1$$

$$y - 1 = -1(x-1)$$
$$y - 1 = -x + 1$$
$$y = -x + 2$$

To graph a line using the Slope and *y*-intercept:

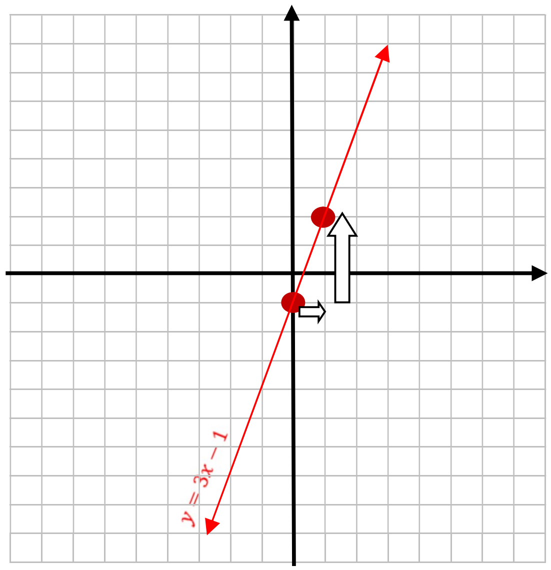
1. Plot the y-intercept on the y-axis.
2. Using the slope, in fraction form, move right the bottom number of places, and up/down the top number of places (up – positive, down – negative). Plot this point
3. Using a straight edge, draw the line through both these points.

Ex. On the set of axes graph the line whose equation is

1. $y = 3x - 1$

$m = 3 = \frac{3}{1}$

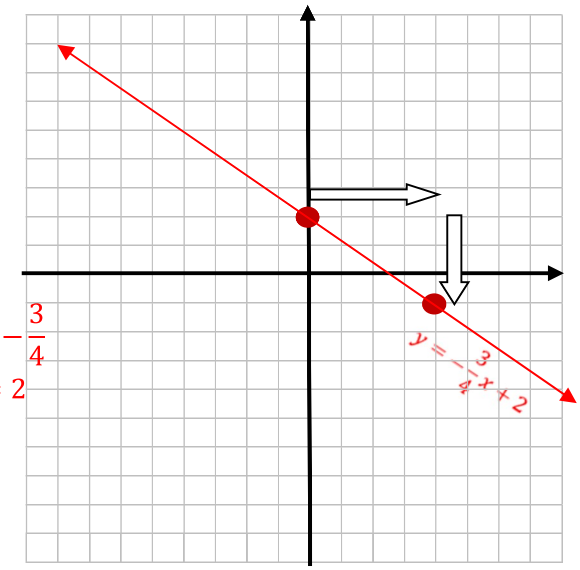
$b = -1$



2. $y = -\frac{3}{4}x + 2$

$m = -\frac{3}{4}$

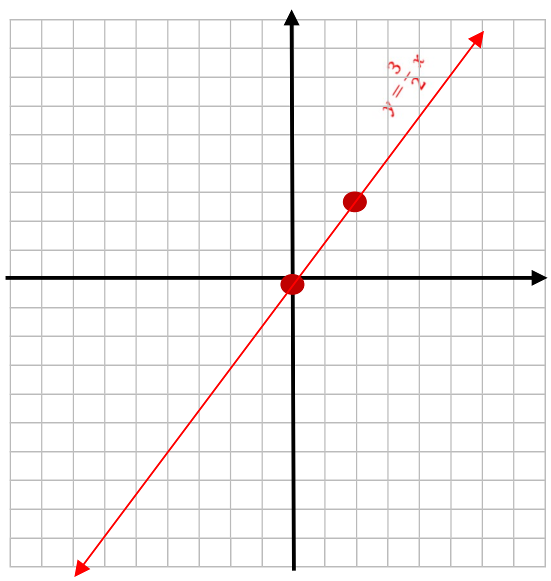
$b = 2$



3. $y = \frac{3}{2}x$

$m = \frac{3}{2}$

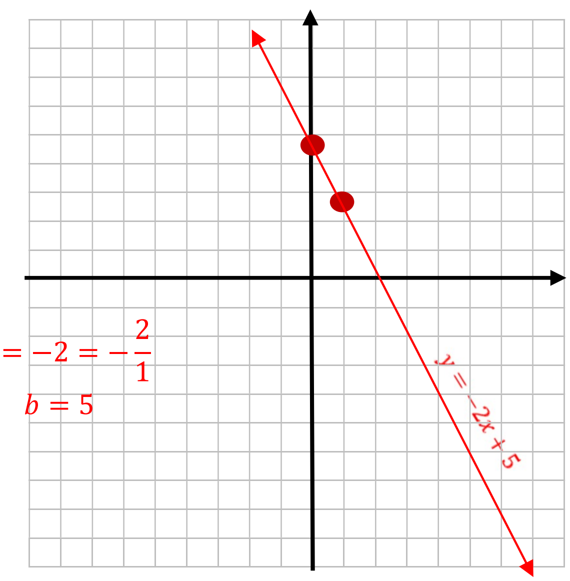
$b = 0$



4. $y = -2x + 5$

$m = -2 = -\frac{2}{1}$

$b = 5$



Homework: Pg. 188-189 #1-17o,25-47o, 59, 60

1.4b Linear Functions & Slope

First two: Point-Slope and Slope-Intercept

Equations of Lines: There are several different forms of equations for lines. (con't from yesterday)

3. Equation of a Horizontal Line: All horizontal line are of the form $y = k$, where k is any real number

   Ex. $y = 4$ $\quad$ $y = -2$ $\quad$ $y = 0$ ← this is the x-axis

   No x in the equation

4. Equation of a Vertical Line: All vertical line are of the form $x = k$, where k is any real number

   Ex. $x = 8$ $\quad$ $x = -5$ $\quad$ $x = 0$ ← this is the y-axis

   No y in the equation

5. General Form of a Line: Every line has an equation that can be written in the general form:

$$Ax + By + C = 0$$

A, B, and C are integers (NOT FRACTIONS)

Ex. $3x + 2y - 6 = 0$ $\quad$ $-2x + 5y - 1 = 0$ $\quad$ $2x + 0y - 8 = 0$

Ex. Consider the line whose General Equation is $3x - 2y - 6 = 0$.

(a) Find the slope

(b) Find the y-intercept

(c) Graph on the set of axes to the right.

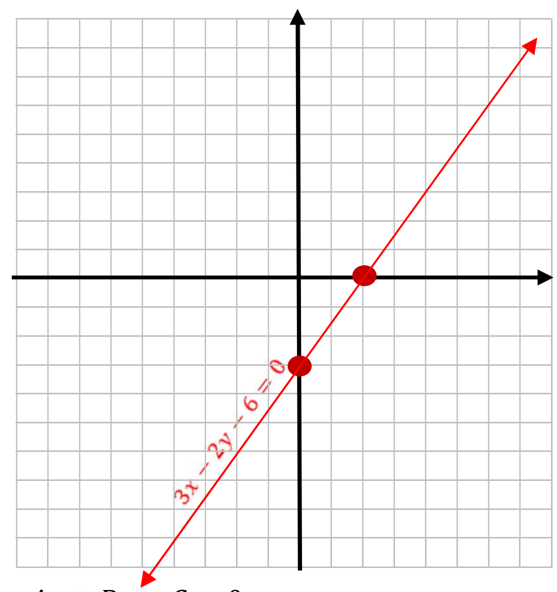
$3x - 2y - 6 = 0$

$2y = 3x - 6$

$y = \frac{3}{2}x - 3$

(a) Slope $m = \frac{3}{2}$

(b) *y-int*: $b = -3$



- Another way to graph a line is to use the Intercepts to $Ax + By + C = 0$
  - Graph the x-intercept: Set y = 0, solve for x and plot on the x-axis
  - Graph the y-intercept: Set x =0, solve for y and plot on the y-axis
  - Draw a line through the two intercepts using a straight edge.

A sneaky way is to simply remove the opposite term of the intercept you are looking for and solve

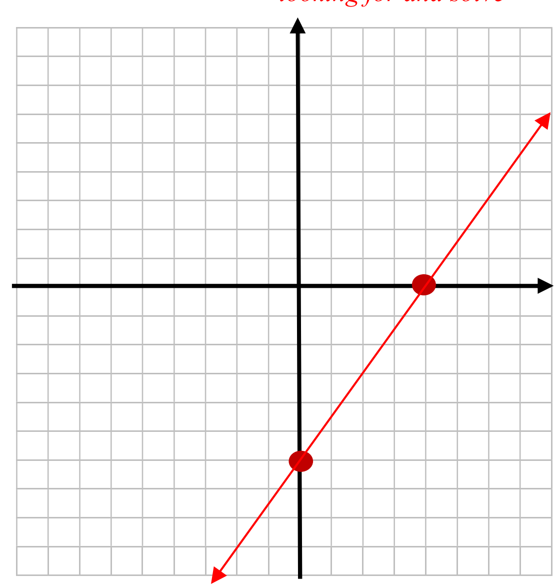
Ex. Using intercepts, graph the line whose equation is

$3x - 2y - 12 = 0$

$3x - 2y - 12 = 0$

$x - int{:}\ \ 3x - 12 = 0 \qquad x = 4$

$y - int{:}\ -2y - 12 = 0 \qquad y = -6$



Ex On the set of axes, graph using intercepts.

1. $4x - 3y - 6 = 0$

$4x - 3y - 6 = 0$

$x - int{:}\ \ 4x - 6 = 0 \qquad x = \frac{6}{4} = \frac{3}{2}$

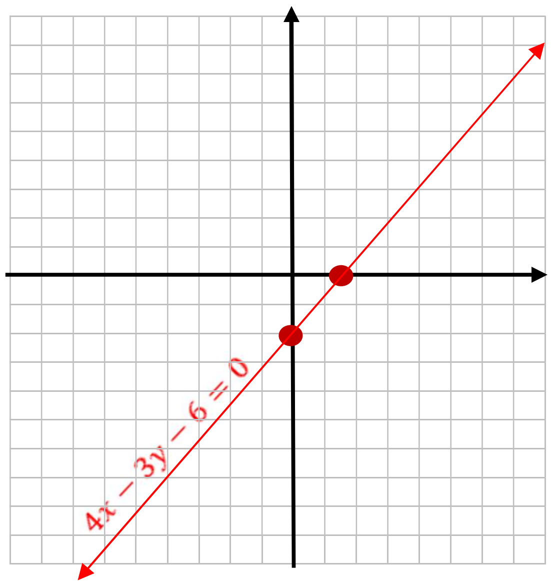
$y - int{:}\ -3y - 6 = 0 \qquad y = -2$

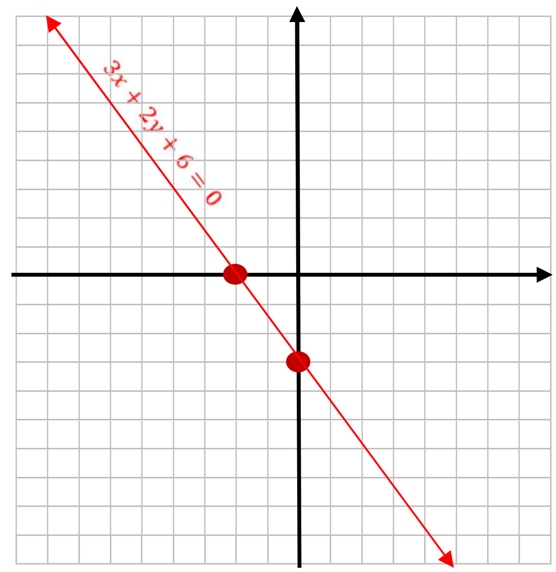
2. $3x + 2y + 6 = 0$

$3x + 2y + 6 = 0$

$x - int{:}\ \ 3x + 6 = 0 \qquad x = -2$

$y - int{:}\ 2y + 6 = 0 \qquad y = -3$





Homework: Pg. 188-189 #61-72, 79, 80, 83

The slope, approximately 0.02, indicates that for each increase of one part per million in carbon dioxide concentration, the average global temperature is increasing by approximately 0.02°F.

Now we write the line's equation in slope-intercept form.

$y - y_1 = m(x - x_1)$ — Begin with the point-slope form.

$y - 57.06 = 0.02(x - 326)$ — Either ordered pair, (326, 57.06) or (377, 58.11), can be (x_1, y_1). Let $(x_1, y_1) = (326, 57.06)$. From above, $m \approx 0.02$.

$y - 57.06 = 0.02x - 6.52$ — Apply the distributive property: $0.02(326) = 6.52$.

$y = 0.02x + 50.54$ — Add 57.06 to both sides and solve for y.

A linear function that models average global temperature, $f(x)$, for an atmospheric carbon dioxide concentration of x parts per million is

$$f(x) = 0.02x + 50.54.$$

b. If carbon dioxide concentration doubles from its preindustrial level of 280 parts per million, which many experts deem very likely, the concentration will reach 280×2, or 560 parts per million. We use the linear function to predict average global temperature at this concentration.

$f(x) = 0.02x + 50.54$ — Use the function from part (a).

$f(560) = 0.02(560) + 50.54$ — Substitute 560 for x.

$= 11.2 + 50.54 = 61.74$

Our model projects an average global temperature of 61.74°F for a carbon dioxide concentration of 560 parts per million. Compared to the average global temperature of 58.11° for 2005 shown in **Figure 1.45(a)** on the previous page, this is an increase of

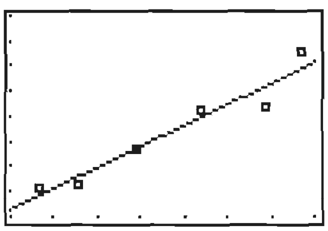
$$61.74°F - 58.11°F = 3.63°F.$$

This is consistent with a rise between 2°F and 5°F as predicted by the Intergovernmental Panel on Climate Change.

Check Point 9 Use the data points (317, 57.04) and (354, 57.64), shown, but not labeled, in **Figure 1.45(b)** on the previous page to obtain a linear function that models average global temperature, $f(x)$, for an atmospheric carbon dioxide concentration of x parts per million. Round m to three decimal places and b to one decimal place. Then use the function to project average global temperature at a concentration of 600 parts per million.

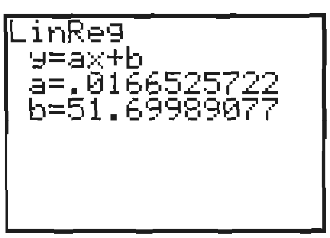
Technology

You can use a graphing utility to obtain a model for a scatter plot in which the data points fall on or near a straight line. After entering the data in **Figure 1.45(a)** on the previous page, a graphing utility displays a scatter plot of the data and the regression line, that is, the line that best fits the data.



[310, 380, 10] by [56.8, 58.4, 0.2]

Also displayed is the regression line's equation.



Exercise Set 1.4

Practice Exercises

In Exercises 1–10, find the slope of the line passing through each pair of points or state that the slope is undefined. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.

1. (4, 7) and (8, 10)
2. (2, 1) and (3, 4)
3. (−2, 1) and (2, 2)
4. (−1, 3) and (2, 4)
5. (4, −2) and (3, −2)
6. (4, −1) and (3, −1)
7. (−2, 4) and (−1, −1)
8. (6, −4) and (4, −2)
9. (5, 3) and (5, −2)
10. (3, −4) and (3, 5)

In Exercises 11–38, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

11. Slope = 2, passing through (3, 5)
12. Slope = 4, passing through (1, 3)
13. Slope = 6, passing through (−2, 5)
14. Slope = 8, passing through (4, −1)
15. Slope = −3, passing through (−2, −3)
16. Slope = −5, passing through (−4, −2)
17. Slope = −4, passing through (−4, 0)
18. Slope = −2, passing through (0, −3)

19. Slope $= -1$, passing through $\left(-\frac{1}{2}, -2\right)$

20. Slope $= -1$, passing through $\left(-4, -\frac{1}{4}\right)$

21. Slope $= \frac{1}{2}$, passing through the origin

22. Slope $= \frac{1}{3}$, passing through the origin

23. Slope $= -\frac{2}{3}$, passing through $(6, -2)$

24. Slope $= -\frac{3}{5}$, passing through $(10, -4)$

25. Passing through $(1, 2)$ and $(5, 10)$

26. Passing through $(3, 5)$ and $(8, 15)$

27. Passing through $(-3, 0)$ and $(0, 3)$

28. Passing through $(-2, 0)$ and $(0, 2)$

29. Passing through $(-3, -1)$ and $(2, 4)$

30. Passing through $(-2, -4)$ and $(1, -1)$

31. Passing through $(-3, -2)$ and $(3, 6)$

32. Passing through $(-3, 6)$ and $(3, -2)$

33. Passing through $(-3, -1)$ and $(4, -1)$

34. Passing through $(-2, -5)$ and $(6, -5)$

35. Passing through $(2, 4)$ with x-intercept $= -2$

36. Passing through $(1, -3)$ with x-intercept $= -1$

37. x-intercept $= -\frac{1}{2}$ and y-intercept $= 4$

38. x-intercept $= 4$ and y-intercept $= -2$

In Exercises 39–48, give the slope and y-intercept of each line whose equation is given. Then graph the linear function.

39. $y = 2x + 1$

40. $y = 3x + 2$

41. $f(x) = -2x + 1$

42. $f(x) = -3x + 2$

43. $f(x) = \frac{3}{4}x - 2$

44. $f(x) = \frac{3}{4}x - 3$

45. $y = -\frac{3}{5}x + 7$

46. $y = -\frac{2}{5}x + 6$

47. $g(x) = -\frac{1}{2}x$

48. $g(x) = -\frac{1}{3}x$

In Exercises 49–58, graph each equation in a rectangular coordinate system.

49. $y = -2$

50. $y = 4$

51. $x = -3$

52. $x = 5$

53. $y = 0$

54. $x = 0$

55. $f(x) = 1$

56. $f(x) = 3$

57. $3x - 18 = 0$

58. $3x + 12 = 0$

In Exercises 59–66,

a. *Rewrite the given equation in slope-intercept form.*

b. *Give the slope and y-intercept.*

c. *Use the slope and y-intercept to graph the linear function.*

59. $3x + y - 5 = 0$

60. $4x + y - 6 = 0$

61. $2x + 3y - 18 = 0$

62. $4x + 6y + 12 = 0$

63. $8x - 4y - 12 = 0$

64. $6x - 5y - 20 = 0$

65. $3y - 9 = 0$

66. $4y + 28 = 0$

In Exercises 67–72, use intercepts to graph each equation.

67. $6x - 2y - 12 = 0$

68. $6x - 9y - 18 = 0$

69. $2x + 3y + 6 = 0$

70. $3x + 5y + 15 = 0$

71. $8x - 2y + 12 = 0$

72. $6x - 3y + 15 = 0$

Practice Plus

In Exercises 73–76, find the slope of the line passing through each pair of points or state that the slope is undefined. Assume that all variables represent positive real numbers. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.

73. $(0, a)$ and $(b, 0)$

74. $(-a, 0)$ and $(0, -b)$

75. (a, b) and $(a, b + c)$

76. $(a - b, c)$ and $(a, a + c)$

In Exercises 77–78, give the slope and y-intercept of each line whose equation is given. Assume that $B \neq 0$.

77. $Ax + By = C$

78. $Ax = By - C$

In Exercises 79–80, find the value of y if the line through the two given points is to have the indicated slope.

79. $(3, y)$ and $(1, 4)$, $m = -3$

80. $(-2, y)$ and $(4, -4)$, $m = \frac{1}{3}$

In Exercises 81–82, graph each linear function.

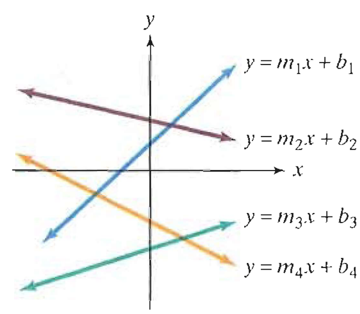
81. $3x - 4f(x) - 6 = 0$

82. $6x - 5f(x) - 20 = 0$

83. If one point on a line is $(3, -1)$ and the line's slope is -2, find the y-intercept.

84. If one point on a line is $(2, -6)$ and the line's slope is $-\frac{3}{2}$, find the y-intercept.

Use the figure to make the lists in Exercises 85–86.



85. List the slopes m_1, m_2, m_3, and m_4 in order of decreasing size.

86. List the y-intercepts b_1, b_2, b_3, and b_4 in order of decreasing size.

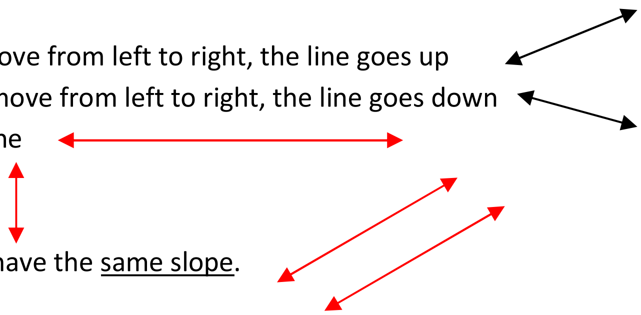
Application Exercises

Americans are getting married later in life, or not getting married at all. In 2006, nearly half of Americans ages 25 through 29 were unmarried. The bar graph at the top of the next page shows the percentage of never-married men and women in this age group. The data are displayed on the next page as two sets of four points each, one scatter plot for the percentage of never-married American men and one for the percentage of never-married American women. Also shown for each scatter plot is a line that passes through or near the four points. Use these lines to solve Exercises 87–88.

1.5 More on Slope

There are 4 Kinds of Sloped Lines:

1. Positive Slope – as you move from left to right, the line goes up
2. Negative Slope – as you move from left to right, the line goes down
3. Zero Slope – horizontal line
4. No Slope – vertical line

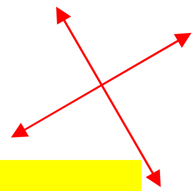


Property: 2 Lines are parallel if they have the same slope.

$$m_1 = m_2$$

Property: 2 lines are perpendicular if their slopes are negative reciprocals.

$$m_1 = -\frac{1}{m_2}$$



ex.

$y = 5x - 6 \quad m = 5$

$y = -\frac{1}{5}x - 3 \quad m = -\frac{1}{5}$

Since they are negative reciprocals, they are perpendicular

Ex. Write the equation of the line

1. that passes through (4, 3) and is parallel to $y = -2x - 2$

$m = -2 \rightarrow m_{||} = -2$

$y - 3 = -2(x - 4)$

$y - 3 = -2x + 8 \rightarrow y = -2x + 11$

2. that passes through (1,2) and is parallel to $4x - 5y - 3 = 0$

$4x - 5y - 3 = 0 \rightarrow 4x - 3 = 5y \rightarrow y = \frac{4}{5}x - \frac{3}{5}$

$m = \frac{4}{5} \rightarrow m_{||} = \frac{4}{5}$

$y - 2 = \frac{4}{5}(x - 1)$

SHORTCUT
The slope of
$\boldsymbol{Ax + By + C = 0}$
is
$\boldsymbol{m = -\frac{A}{B}}$
ALWAYS!

3. that passes through (–3, 5) and is perpendicular to $y = \frac{2}{3}x - 3$

$m = \frac{2}{3} \rightarrow m_{\perp} = -\frac{3}{2}$

$y - 5 = -\frac{3}{2}(x + 3)$

4. that passes through (3, 1) and is perpendicular to $2x - y + 1 = 9$

$m = -\frac{A}{B} \rightarrow m = -\frac{2}{-1} = 2 \rightarrow m_{\perp} = -\frac{1}{2}$

$y - 1 = -\frac{1}{2}(x - 3)$

SHORTCUT
The slope of the $\perp$ to
$\boldsymbol{Ax + By + C = 0}$
is
$\boldsymbol{m = \frac{B}{A}}$
ALWAYS!

Slope as a Rate of Change

Def: The Average Rate of Change of a Function

Let $(x_1, f(x_1))$ and $(x_2, f(x_2))$ be two distinct points on the graph of $y = f(x)$. The **average rate of change** of f from x_1 to x_2, denoted by $\frac{\Delta y}{\Delta x}$ ("delta y over delta x" = "change in y over the change in x") is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

← variation of the slope formula

Ex. Find the average rate of change of $f(x) = x^2$ on the interval

(a) [–3,4]

$$\frac{4^2 - (-3)^2}{4 - (-3)} = \frac{16 - 9}{7}$$

$$\frac{7}{7} = 1$$

(b) [1, 2]

$$\frac{2^2 - 1^2}{2 - 1} = \frac{4 - 1}{1}$$

$$\frac{3}{1} = 3$$

(c) [–2,0]

$$\frac{0^2 - (-2)^2}{0 - (-2)} = \frac{0 - 4}{2}$$

$$\frac{-4}{2} = -2$$

Ex. Find the average rate of change of $f(x) = x^3 - 2x + 1$ on the interval

(a) [3, 6]

$$\frac{(6^3 - 2(6) + 1) - (3^3 - 2(3) + 1)}{6 - 3}$$

$$\frac{205 - 22}{3}$$

$$\frac{183}{3} = 61$$

(b) [–1, 2]

$$\frac{(2^3 - 2(2) + 1) - ((-1)^3 - 2(-1) + 1)}{2 - (-1)}$$

$$\frac{(8 - 4 + 1) - (-1 + 2 + 1)}{3} = \frac{5 - 2}{3}$$

$$\frac{3}{3} = 1$$

(c) [–2,1]

$$\frac{(1^3 - 2(1) + 1) - ((-2)^3 - 2(-2) + 1)}{1 - (-2)}$$

$$\frac{(1 - 2 + 1) - (-8 + 4 + 1)}{3}$$

$$\frac{0 - (-3)}{3} = \frac{3}{3} = 1$$

Def: Suppose an object's position is expressed by the function $s(t)$, where t is time. The **average velocity** of the object from time t_1 to t_2 is

$$\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Ex. Consider the position of a ball rolling down a ramp is given by $s(t) = 5t^2$, find the ball's average velocity from

(a) t=2 to t=3

$$\frac{s(3) - s(2)}{3 - 2} = \frac{45 - 20}{3 - 2}$$

25

(b) t =2 t0 2.5

$$\frac{s(2.5) - s(2)}{2.5 - 2} = \frac{31.25 - 20}{0.5}$$

$$\frac{11.25}{0.5} = 22.5$$

(c) t = 2 to t=2.01

$$\frac{s(2.01) - s(2)}{2.01 - 2} = \frac{20.2005 - 20}{0.01}$$

$$\frac{0.2005}{0.01} = 20.05$$

Homework: pg. 200 – 201 #1-25o

b. The ball's average velocity between 2 and 2.5 seconds is

$$\frac{\Delta s}{\Delta t}=\frac{s(2.5)-s(2)}{2.5\text{ sec}-2\text{ sec}}=\frac{5(2.5)^2-5\cdot 2^2}{0.5\text{ sec}}=\frac{31.25\text{ ft}-20\text{ ft}}{0.5\text{ sec}}=22.5\text{ ft/sec.}$$

c. The ball's average velocity between 2 and 2.01 seconds is

$$\frac{\Delta s}{\Delta t}=\frac{s(2.01)-s(2)}{2.01\text{ sec}-2\text{ sec}}=\frac{5(2.01)^2-5\cdot 2^2}{0.01\text{ sec}}=\frac{20.2005\text{ ft}-20\text{ ft}}{0.01\text{ sec}}=20.05\text{ ft/sec.}$$

In Example 6, observe that each calculation begins at 2 seconds and involves shorter and shorter time intervals. In calculus, this procedure leads to the concept of *instantaneous*, as opposed to *average*, velocity. Instantaneous velocity is discussed in the introduction to calculus in Chapter 11.

Check Point 6 The distance, $s(t)$, in feet, traveled by a ball rolling down a ramp is given by the function

$$s(t)=4t^2,$$

where t is the time, in seconds, after the ball is released. Find the ball's average velocity from

a. $t_1=1$ second to $t_2=2$ seconds.

b. $t_1=1$ second to $t_2=1.5$ seconds.

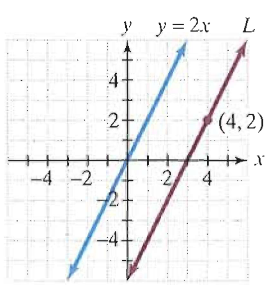
a. $t_1=1$ second to $t_2=1.01$ seconds.

Exercise Set 1.5

Practice Exercises

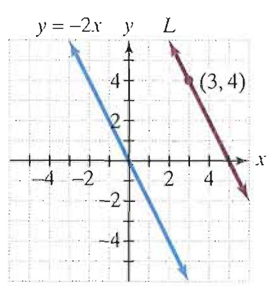
In Exercises 1–4, write an equation for line L in point-slope form and slope-intercept form.

1.



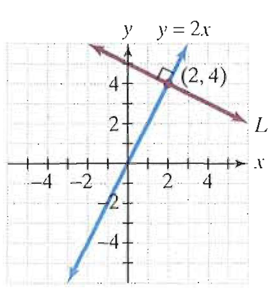
L is parallel to $y=2x$.

2.



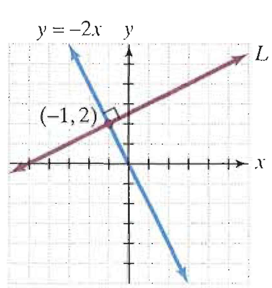
L is parallel to $y=-2x$.

3.



L is perpendicular to $y=2x$.

4.



L is perpendicular to $y=-2x$.

In Exercises 5–8, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

5. Passing through $(-8,-10)$ and parallel to the line whose equation is $y=-4x+3$

6. Passing through $(-2,-7)$ and parallel to the line whose equation is $y=-5x+4$

7. Passing through $(2,-3)$ and perpendicular to the line whose equation is $y=\frac{1}{5}x+6$

8. Passing through $(-4,2)$ and perpendicular to the line whose equation is $y=\frac{1}{3}x+7$

In Exercises 9–12, use the given conditions to write an equation for each line in point-slope form and general form.

9. Passing through $(-2,2)$ and parallel to the line whose equation is $2x-3y-7=0$

10. Passing through $(-1,3)$ and parallel to the line whose equation is $3x-2y-5=0$

11. Passing through $(4,-7)$ and perpendicular to the line whose equation is $x-2y-3=0$

12. Passing through $(5,-9)$ and perpendicular to the line whose equation is $x+7y-12=0$

In Exercises 13–18, find the average rate of change of the function from x_1 to x_2.

13. $f(x)=3x$ from $x_1=0$ to $x_2=5$

14. $f(x)=6x$ from $x_1=0$ to $x_2=4$

15. $f(x)=x^2+2x$ from $x_1=3$ to $x_2=5$

16. $f(x)=x^2-2x$ from $x_2=3$ to $x_2=6$

17. $f(x)=\sqrt{x}$ from $x_1=4$ to $x_2=9$

18. $f(x)=\sqrt{x}$ from $x_1=9$ to $x_2=16$

In Exercises 19–20, suppose that a ball is rolling down a ramp. The distance traveled by the ball is given by the function in each exercise, where t is the time, in seconds, after the ball is released, and s(t) is measured in feet. For each given function, find the ball's average velocity from

a. $t_1 = 3$ to $t_2 = 4$. **b.** $t_1 = 3$ to $t_2 = 3.5$.

c. $t_1 = 3$ to $t_2 = 3.01$. **d.** $t_1 = 3$ to $t_2 = 3.001$.

19. $s(t) = 10t^2$ **20.** $s(t) = 12t^2$

Practice Plus

In Exercises 21–26, write an equation in slope-intercept form of a linear function f whose graph satisfies the given conditions.

21. The graph of f passes through $(-1, 5)$ and is perpendicular to the line whose equation is $x = 6$.

22. The graph of f passes through $(-2, 6)$ and is perpendicular to the line whose equation is $x = -4$.

23. The graph of f passes through $(-6, 4)$ and is perpendicular to the line that has an x-intercept of 2 and a y-intercept of -4.

24. The graph of f passes through $(-5, 6)$ and is perpendicular to the line that has an x-intercept of 3 and a y-intercept of -9.

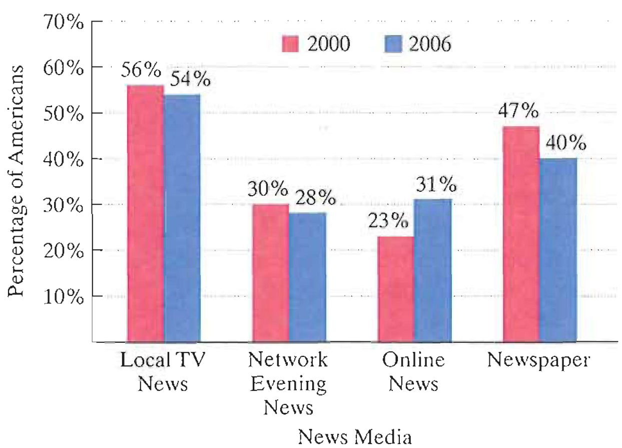
25. The graph of f is perpendicular to the line whose equation is $3x - 2y - 4 = 0$ and has the same y-intercept as this line.

26. The graph of f is perpendicular to the line whose equation is $4x - y - 6 = 0$ and has the same y-intercept as this line.

Application Exercises

The bar graph shows that as online news has grown, traditional news media have slipped.

Percentage of Americans Who Regularly Use Various News Media



Source: Pew Research Center

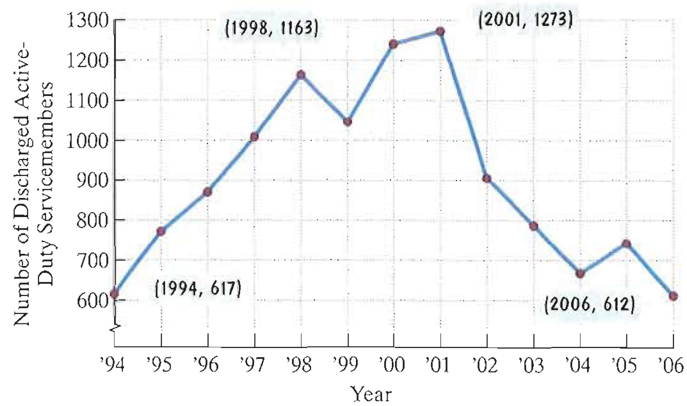
In Exercises 27–28, find a linear function in slope-intercept form that models the given description. Each function should model the percentage of Americans, P(x), who regularly used the news outlet x years after 2000.

27. In 2000, 47% of Americans regularly used newspapers for getting news and this percentage has decreased at an average rate of approximately 1.2 per year since then.

28. In 2000, 23% of Americans regularly used online news for getting news and this percentage has increased at an average rate of approximately 1.3 per year since then.

The stated intent of the 1994 "don't ask, don't tell" policy was to reduce the number of discharges of gay men and lesbians from the military. Nearly 12,000 active-duty gay servicemembers have been dismissed under the policy. The line graph shows the number of discharges under "don't ask, don't tell" from 1994 through 2006. Use the data displayed by the graph to solve Exercises 29–30.

Number of Active-Duty Gay Servicemembers Discharged from the Military for Homosexuality



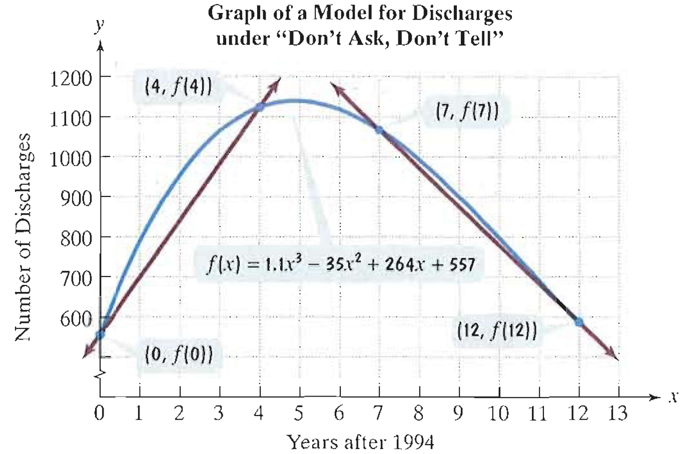
Source: General Accountability Office

29. Find the average rate of change, rounded to the nearest whole number, from 1994 through 1998. Describe what this means.

30. Find the average rate of change, rounded to the nearest whole number, from 2001 through 2006. Describe what this means.

The function $f(x) = 1.1x^3 - 35x^2 + 264x + 557$ models the number of discharges, $f(x)$, under "don't ask, don't tell" x years after 1994. Use this model and its graph, shown on the domain [0, 12] to solve Exercises 31–32.

Graph of a Model for Discharges under "Don't Ask, Don't Tell"



31. a. Find the slope of the secant line, rounded to the nearest whole number, from $x_1 = 0$ to $x_2 = 4$.

b. Does the slope from part (a) underestimate or overestimate the average yearly increase that you determined in Exercise 29? By how much?

32. a. Find the slope of the secant line, rounded to the nearest whole number, from $x_1 = 7$ to $x_2 = 12$.

b. Does the slope from part (b) underestimate or overestimate the average yearly decrease that you determined in Exercise 30? By how much?

Writing in Mathematics

33. If two lines are parallel, describe the relationship between their slopes.
34. If two lines are perpendicular, describe the relationship between their slopes.
35. If you know a point on a line and you know the equation of a line perpendicular to this line, explain how to write the line's equation.
36. A formula in the form $y = mx + b$ models the average retail price, y, of a new car x years after 2000. Would you expect m to be positive, negative, or zero? Explain your answer.
37. What is a secant line?
38. What is the average rate of change of a function?

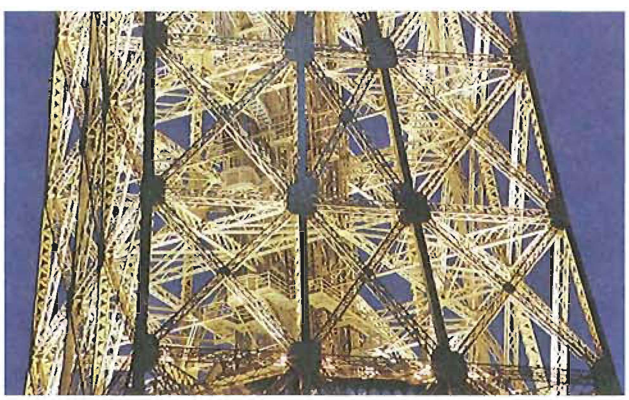
Technology Exercise

39. **a.** Why are the lines whose equations are $y = \frac{1}{3}x + 1$ and $y = -3x - 2$ perpendicular?
 b. Use a graphing utility to graph the equations in a $[-10, 10, 1]$ by $[-10, 10, 1]$ viewing rectangle. Do the lines appear to be perpendicular?
 c. Now use the zoom square feature of your utility. Describe what happens to the graphs. Explain why this is so.

Critical Thinking Exercises

Make Sense? *In Exercises 40–43, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

40. Some of the steel girders in this photo of the Eiffel Tower appear to have slopes that are negative reciprocals of each other.



41. I have linear functions that model changes for men and women over the same time period. The functions have the same slope, so their graphs are parallel lines, indicating that the rate of change for men is the same as the rate of change for women.
42. The graph of my function is not a straight line, so I cannot use slope to analyze its rates of change.
43. According to the essay on page 199, calculus studies change by analyzing slopes of secant lines over successively shorter intervals.
44. What is the slope of a line that is perpendicular to the line whose equation is $Ax + By + C = 0$, $A \neq 0$ and $B \neq 0$?
45. Determine the value of A so that the line whose equation is $Ax + y - 2 = 0$ is perpendicular to the line containing the points $(1, -3)$ and $(-2, 4)$.

Preview Exercises

Exercises 46–48 will help you prepare for the material covered in the next section. In each exercise, graph the functions in parts (a) and (b) in the same rectangular coordinate system.

46. **a.** Graph $f(x) = |x|$ using the ordered pairs $(-3, f(-3))$, $(-2, f(-2))$, $(-1, f(-1))$, $(0, f(0))$, $(1, f(1))$, $(2, f(2))$, and $(3, f(3))$.
 b. Subtract 4 from each y-coordinate of the ordered pairs in part (a). Then graph the ordered pairs and connect them with two linear pieces.
 c. Describe the relationship between the graph in part (b) and the graph in part (a).
47. **a.** Graph $f(x) = x^2$ using the ordered pairs $(-3, f(-3))$, $(-2, f(-2))$, $(-1, f(-1))$, $(0, f(0))$, $(1, f(1))$, $(2, f(2))$, and $(3, f(3))$.
 b. Add 2 to each x-coordinate of the ordered pairs in part (a). Then graph the ordered pairs and connect them with a smooth curve.
 c. Describe the relationship between the graph in part (b) and the graph in part (a).
48. **a.** Graph $f(x) = x^3$ using the ordered pairs $(-2, f(-2))$, $(-1, f(-1))$, $(0, f(0))$, $(1, f(1))$, and $(2, f(2))$.
 b. Replace each x-coordinate of the ordered pairs in part (a) with its opposite, or additive inverse. Then graph the ordered pairs and connect them with a smooth curve.
 c. Describe the relationship between the graph in part (b) and the graph in part (a).

Chapter 1 Mid-Chapter Check Point

What You Know: We learned that a function is a relation in which no two ordered pairs have the same first component and different second components. We represented functions as equations and used function notation. We graphed functions and applied the vertical line test to identify graphs of functions. We determined the domain and range of a function from its graph, using inputs on the x-axis for the domain and outputs on the y-axis for the range. We used graphs to identify intervals on which functions increase, decrease, or are constant, as well as to locate relative maxima or minima. We identified even functions [$f(-x) = f(x)$: y-axis symmetry] and odd functions [$f(-x) = -f(x)$: origin symmetry]. Finally, we studied linear functions and slope, using slope (change in y divided by change in x) to develop various forms for equations of lines:

Point-slope form	Slope-intercept form	Horizontal line	Vertical line	General form
$y - y_1 = m(x - x_1)$	$y = f(x) = mx + b$	$y = f(x) = b$	$x = a$	$Ax + By + C = 0.$

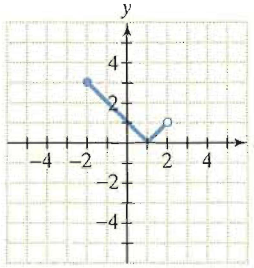
We saw that parallel lines have the same slope and that perpendicular lines have slopes that are negative reciprocals. For linear functions, slope was interpreted as the rate of change of the dependent variable per unit change in the independent variable. For nonlinear functions, the slope of the secant line between $(x_1, f(x_1))$ and $(x_2, f(x_2))$ described the average rate of change of f from x_1 to x_2: $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$.

In Exercises 1–6, determine whether each relation is a function. Give the domain and range for each relation.

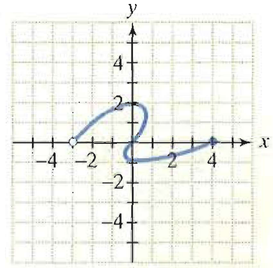
1. $\{(2, 6), (1, 4), (2, -6)\}$

2. $\{(0, 1), (2, 1), (3, 4)\}$

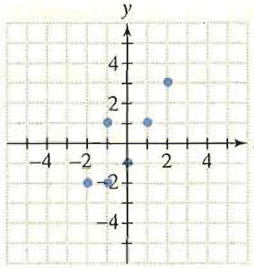
3.



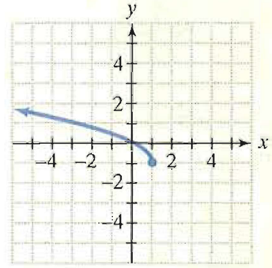
4.



5.



6.

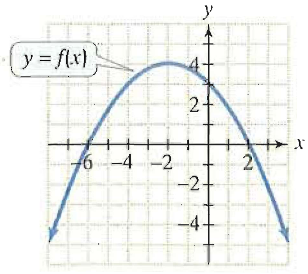


In Exercises 7–8, determine whether each equation defines y as a function of x.

7. $x^2 + y = 5$

8. $x + y^2 = 5$

Use the graph of f to solve Exercises 9–24. Where applicable, use interval notation.



9. Explain why f represents the graph of a function.

10. Find the domain of f.

11. Find the range of f.

12. Find the x-intercept(s).

13. Find the y-intercept.

14. Find the interval(s) on which f is increasing.

15. Find the interval(s) on which f is decreasing.

16. At what number does f have a relative maximum?

17. What is the relative maximum of f?

18. Find $f(-4)$.

19. For what value or values of x is $f(x) = -2$?

20. For what value or values of x is $f(x) = 0$?

21. For what values of x is $f(x) > 0$?

22. Is $f(100)$ positive or negative?

23. Is f even, odd, or neither?

24. Find the average rate of change of f from $x_1 = -4$ to $x_2 = 4$.

In Exercises 25–36, graph each equation in a rectangular coordinate system.

25. $y = -2x$

26. $y = -2$

27. $x + y = -2$

28. $y = \frac{1}{3}x - 2$

29. $x = 3.5$

30. $4x - 2y = 8$

31. $f(x) = x^2 - 4$

32. $f(x) = x - 4$

33. $f(x) = |x| - 4$

34. $5y = -3x$

35. $5y = 20$

36. $f(x) = \begin{cases} -1 & \text{if} \quad x \le 0 \\ 2x + 1 & \text{if} \quad x > 0 \end{cases}$

37. Let $f(x) = -2x^2 + x - 5$.

a. Find $f(-x)$. Is f even, odd, or neither?

b. Find $\dfrac{f(x + h) - f(x)}{h}$, $h \neq 0$.

38. Let $C(x) = \begin{cases} 30 & \text{if} \quad 0 \le t \le 200 \\ 30 + 0.40(t - 200) & \text{if} \quad t > 200 \end{cases}$.

a. Find $C(150)$. **b.** Find $C(250)$.

In Exercises 39–42, write a linear function in slope-intercept form whose graph satisfies the given conditions.

39. Slope $= -2$, passing through $(-4, 3)$

40. Passing through $(-1, -5)$ and $(2, 1)$

41. Passing through $(3, -4)$ and parallel to the line whose equation is $3x - y - 5 = 0$

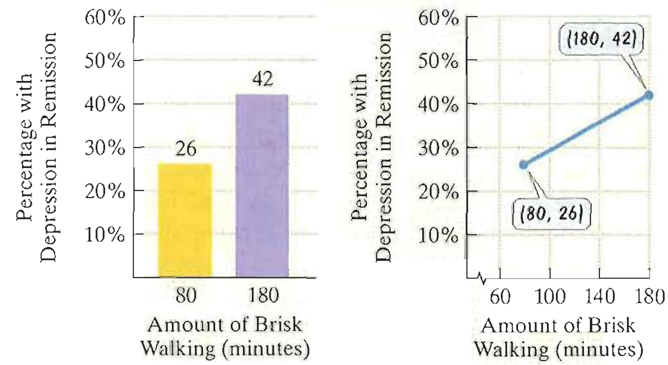
42. Passing through $(-4, -3)$ and perpendicular to the line whose equation is $2x - 5y - 10 = 0$

43. Determine whether the line through $(2, -4)$ and $(7, 0)$ is parallel to a second line through $(-4, 2)$ and $(1, 6)$.

44. Exercise is useful not only in preventing depression, but also as a treatment. The graphs in the next column show the percentage of patients with depression in remission when exercise (brisk walking) was used as a treatment. (The control group that engaged in no exercise had 11% of the patients in remission.)

a. Find the slope of the line passing through the two points shown by the voice balloons. Express the slope as a decimal.

Exercise and Percentage of Patients with Depression in Remission



Source: Newsweek, March 26, 2007

b. Use your answer from part (a) to complete this statement:

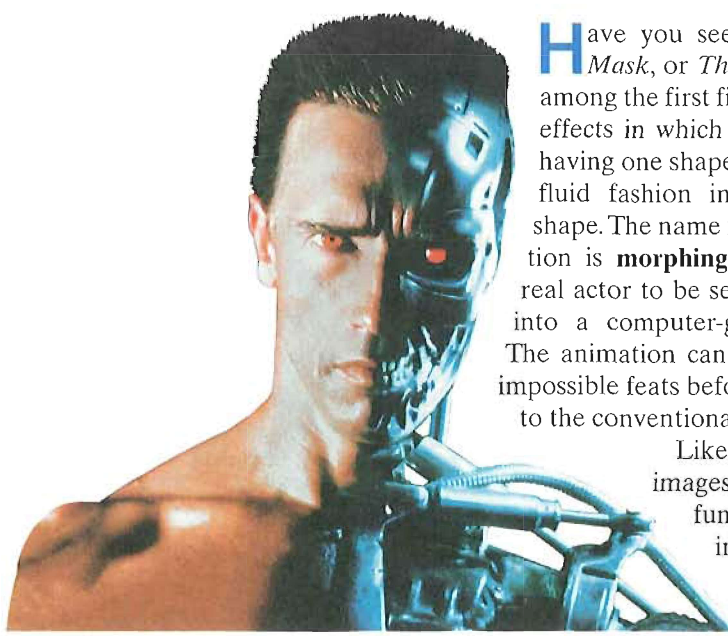
For each minute of brisk walking, the percentage of patients with depression in remission increased by _____ %. The rate of change is _____ % per _____.

45. Find the average rate of change of $f(x) = 3x^2 - x$ from $x_1 = -1$ to $x_2 = 2$.

Section 1.6 Transformations of Functions

Objectives

1. Recognize graphs of common functions.
2. Use vertical shifts to graph functions.
3. Use horizontal shifts to graph functions.
4. Use reflections to graph functions.
5. Use vertical stretching and shrinking to graph functions.
6. Use horizontal stretching and shrinking to graph functions.
7. Graph functions involving a sequence of transformations.



Have you seen *Terminator 2, The Mask*, or *The Matrix?* These were among the first films to use spectacular effects in which a character or object having one shape was transformed in a fluid fashion into a quite different shape. The name for such a transformation is **morphing**. The effect allows a real actor to be seamlessly transformed into a computer-generated animation. The animation can be made to perform impossible feats before it is morphed back to the conventionally filmed image.

Like transformed movie images, the graph of one function can be turned into the graph of a different function. To do this, we need to rely on a function's equation. Knowing that a graph is a transformation of a familiar graph makes graphing easier.

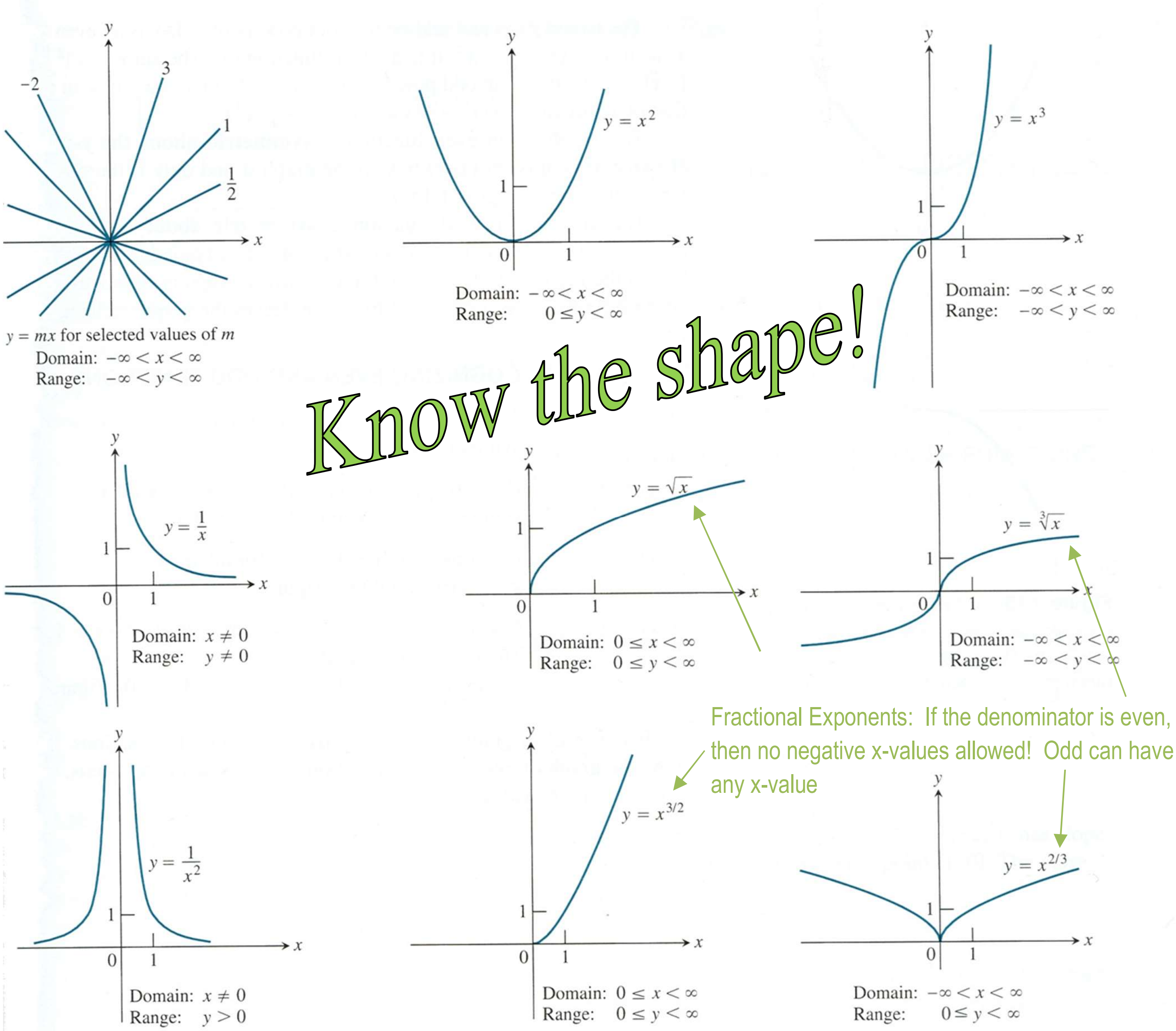
1 Recognize graphs of common functions.

Graphs of Common Functions

Table 1.3 on the next page gives names to seven frequently encountered functions in algebra. The table shows each function's graph and lists characteristics of the function. Study the shape of each graph and take a few minutes to verify the function's characteristics from its graph. Knowing these graphs is essential for analyzing their transformations into more complicated graphs.

1.6 Transformations of Functions

There are functions whose graphs you should know when you see it:



These are "standard" graphs. Many other graphs are simply transformations of these standard graphs.

<u>Vertical Shift</u>

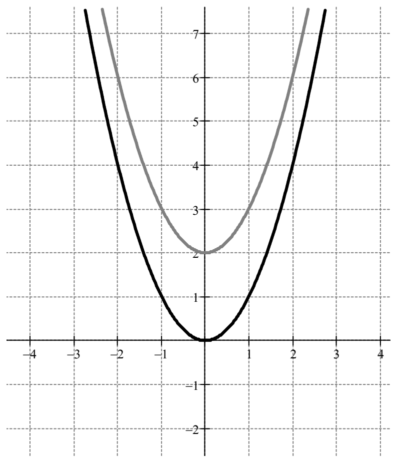
- A <u>vertical shift</u> occurs to a standard graph, when the graph "shifts" up or down.

<u>Def</u>: A function $f(x)$ will have a vertical shift into a new function $g(x)$ if for any real number k:

$$g(x) = f(x) + k$$

- The curve will have the exact same shape, it will just be higher or lower than its original form.
- The shift will be upward if $k > 0$
- The shift will be downward if $k < 0$

Ex. $y = x^2 + 2$

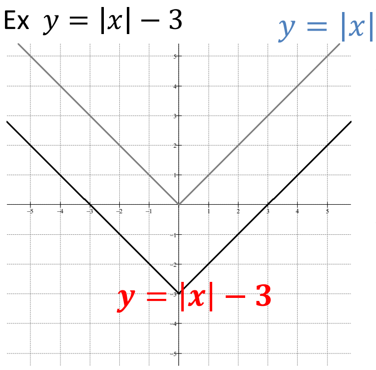


As shown in the diagram, the darker curve is the graph of $y = x^2$, the lighter curve is $y = x^2$ raised up two units.

This results in the curve $y = x^2 + 2$

A shift downward occurs when the *k* value is negative

Ex $y = |x| - 3$



As shown in the diagram, the lighter curve is the graph of $y = |x|$, the darker curve is 3 units lower than the lighter curve.

So a vertical shift occurs when a real number *k* is added to/subtracted from the base function.

Horizontal Shift

- A horizontal shift to a standard graph when the graph "shifts" left or right.

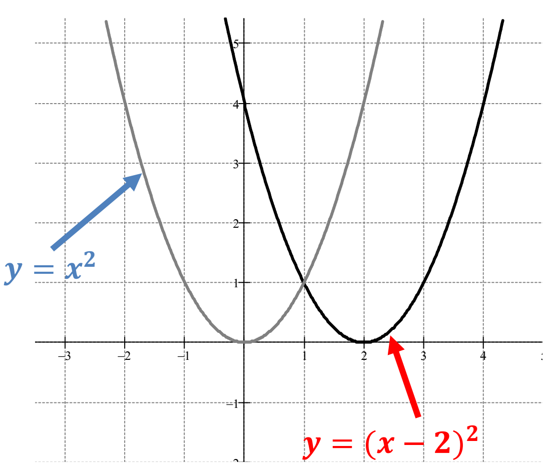
Def: A function $f(x)$ will have a horizontal shift into a new function $g(x)$ if for any real number *k*:

$$g(x) = f(x + k)$$

- The curve will have the exact same shape, it will just be left or right than its original form.
- The shift will be to the right *k* units if $k < 0$
- The shift will be to the left *k* units if $k > 0$

This is opposite of what you might think!

Ex. $y = (x - 2)^2$ **So, this is a horizontal shift of 2 places to the RIGHT!**



This is referred to as the base function

As you see in the graph, the lighter curve is $y = x^2$.

Observe what the table of values would be for both curves:

x	$y = x^2$	$y = (x - 2)^2$
-2	4	$(-4)^2=16$
-1	1	$(-3)^2=9$
0	0	$(-2)^2=4$
1	1	$(-1)^2=1$
2	4	$0^2=0$
3	9	$1^2=1$
4	16	$2^2=4$

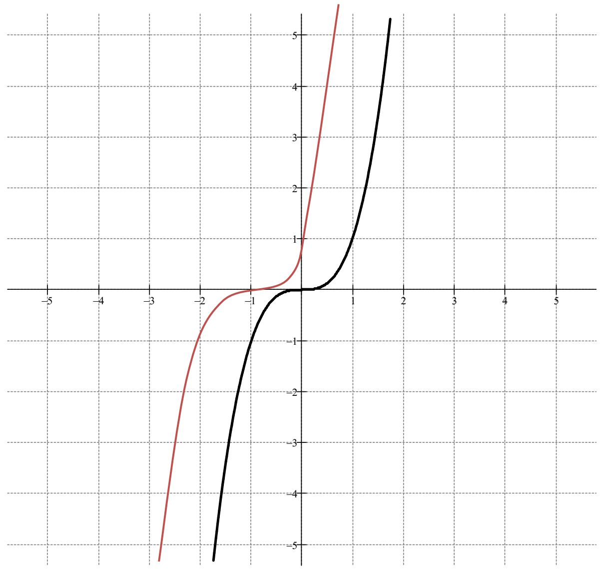
See how the green arrows point to same y-values

Look at the new x's

The y-coordinates of $y = (x - 2)^2$ are the same as $y = x^2$, but occur two *x*-units later.

Ex. On the set of axes, use a shift to graph $y = (x+1)^3$. The base curve is shown on the axes as a guide.

The +1 inside the parentheses means shifting the base function horizontally 1 unit to the LEFT

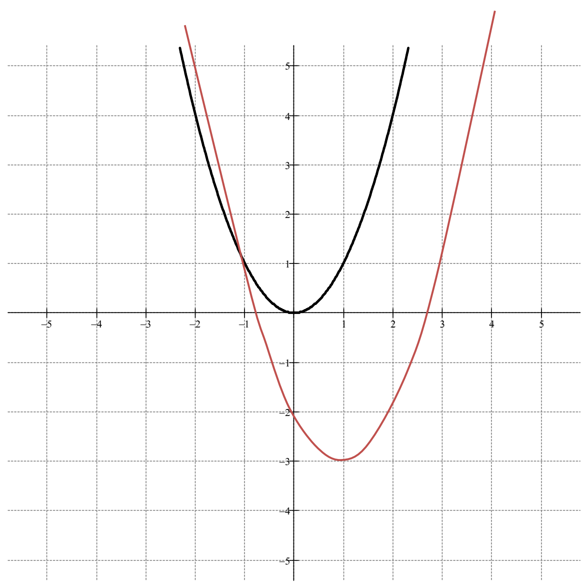


You can also combine both vertical and horizontal shifts at the same time.

Ex. Using the base graph of $y = x^2$, graph $y = (x-1)^2 - 3$

The –1 inside the parentheses means shifting the base function horizontally 1 unit to the RIGHT.

The –3 outside the parentheses indicates a vertical shift DOWN 3.



Homework: Pg. 216 #17-22, 35, 36, 46, 53-56

1.6b Transformations of Functions (Day 2)

Reflections of Graphs

Def: The graph of $y = -f(x)$ is the graph of the $y = f(x)$ reflected over the *x*-axis.

- Basically: The *y*-coordinate changes sign: (*x*, *y*) ➔ (*x*, -*y*)

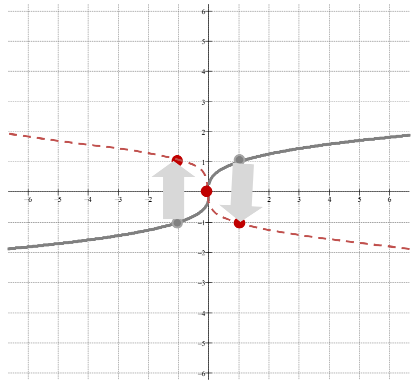
Ex. Graph the function $f(x) = -\sqrt[3]{x}$. Use the base graph of $f(x) = \sqrt[3]{x}$ as a guide.

Points of Reference:

(–1, –1) → (–1, 0)

(0, 0) → (0, 0)

(1, 1) → (1, –1)



Def: The graph of $y = f(-x)$ is the graph of the reflection of $y = f(x)$ over the *y*-axis.

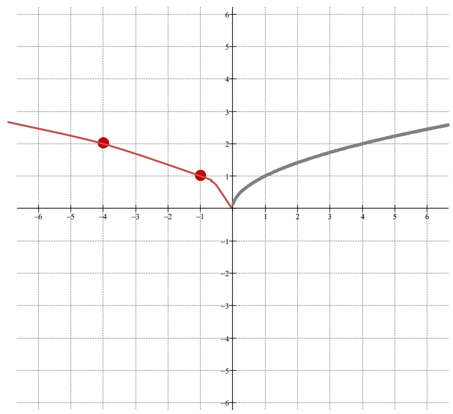
- Basically: The *x*-coordinate changes sign: (*x*, *y*) ➔ (-*x*, *y*)
- Ex. Graph the function $f(x) = \sqrt{-x}$. Use the base graph of $f(x) = \sqrt{x}$ as a guide.

Points of Reference:

(0, 0) → (0, 0)

(1, 1) → (–1, 1)

(4, 2) → (–4, 2)

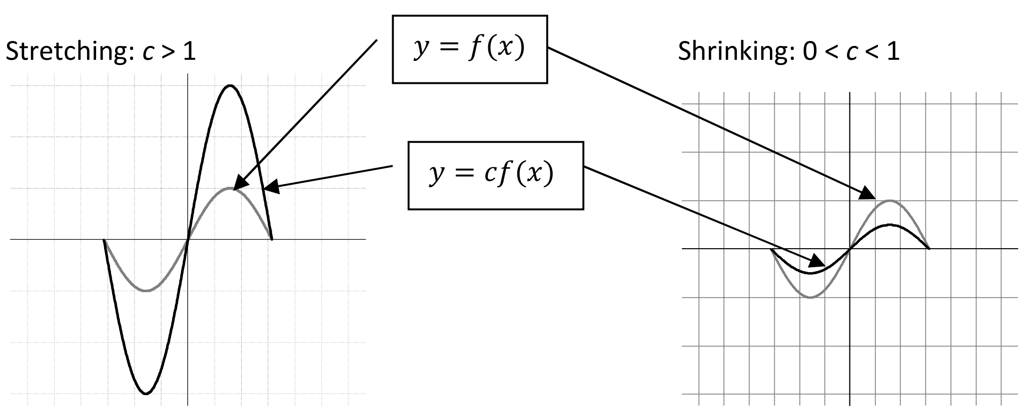


Dilations: Stretching and Shrinking

Def: Vertical Stretching and Shrinking

Let *f* be a function and *c* be a positive real number.

1. If $c > 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically stretched by multiplying each of the *y*-coordinates by *c*.
2. If $0 < c < 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically shrunk by multiplying each of the *y*-coordinates by *c*.

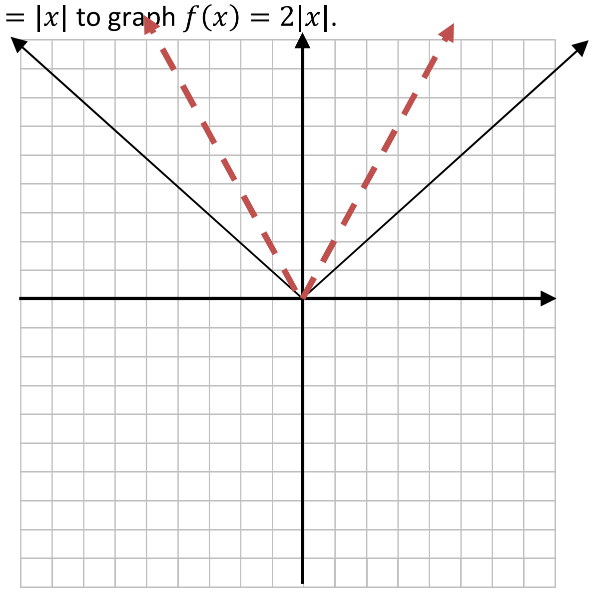


You can tell it is a vertical stretch/shrink because the x-intercepts are the same

Ex. On the set of axes to the right, use the graph of $f(x) = |x|$ to graph $f(x) = 2|x|$.

x	$\|x\|$	$2\|x\|$
–2	2	4
–1	1	2
0	0	0
1	1	2
2	2	4

The dashed line shows the graph of $f(x) = 2|x|$



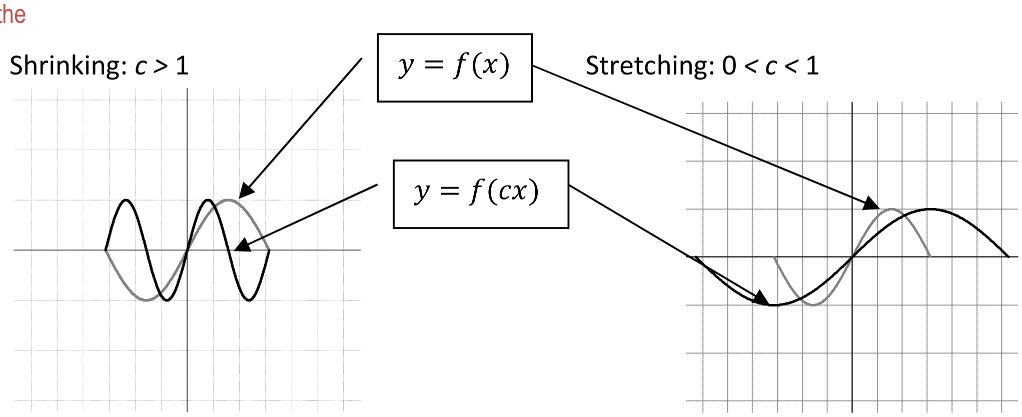
<u>Def</u>: Horizontal Stretching and Shrinking

Let f be a function and c be a positive real number.

Note the c is multiplying the x variable.

$f(2x)$ would horizontally shrink the graph

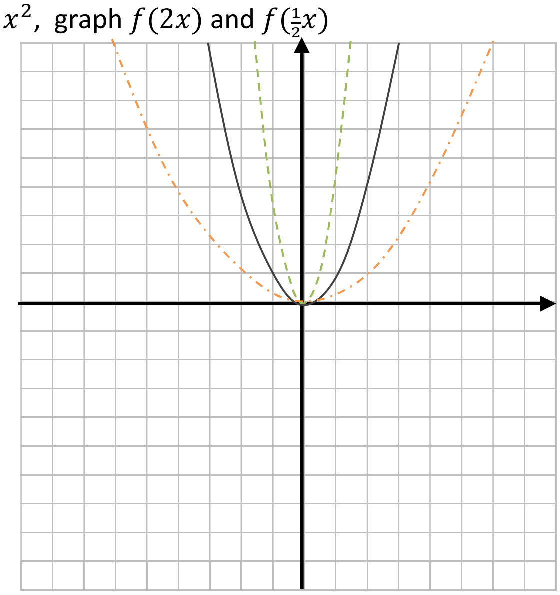
1. If $c > 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ horizontally shrunk by dividing each of the x-coordinates by c.
2. If $0 < c < 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ horizontally stretched by dividing each of the x-coordinates by c.



You can tell it is a horizontal stretch/shrink because the graphs have the same height

Ex. Use the axes to the right: Using the graph of $f(x) = x^2$, graph $f(2x)$ and $f(\frac{1}{2}x)$

x	$f(x) = x^2$	$f(2x)$	$f(\frac{1}{2}x)$
–2	4	16	1
–1	1	4	¼
0	0	0	0
1	1	4	¼
2	4	16	1
		Steeper	Flatter



Homework: Pg. 216-217 #23, 24, 27-30, 99, 101, 102

1.6c Transformations of Functions (Day 3)

Sequences of Transformations:

Recall: Transformations

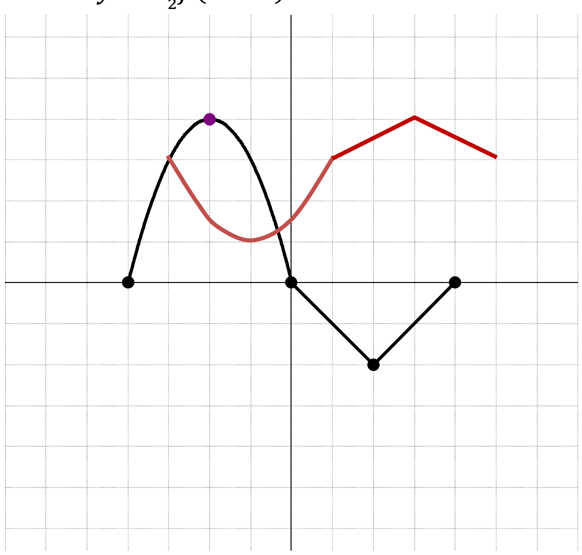
To graph	Draw the Graph of *f* and:	Changes in the Equation of $y = f(x)$
Vertical Shift: $y = f(x) \pm c$	Raise or lower the graph of *f* by *c* units (raises if *c* > 0, lowers if *c* < 0)	*c* is added to *f* or *c* is subtracted from *f*
Horizontal Shifts $y = f(x \pm c)$	Shifts the graph of *f* left or right *c* units (left if *c* > 0 and right if *c* < 0)	Replace all occurrences of *x* in *f* with *x* ± *c*
Reflections over: *x*-axis	Reflect graph of *f* over *x*-axis	Change all the signs of *f*: $y = -f(x)$
y-axis	Reflect graph of *f* over *y*-axis	Change all *x* to –*x* and simplify: $y = f(-x)$
Stretch or Shrink: Vertical: $y = cf(x)$	Multiply *y*-coordinates by *c*	$f(x)$ is multiplied by *c*: (stretch c>1, shrink 0<*c*<1)
Horiz.: $y = f(cx)$	Divide x-coordinates by *c*	All *x* are replaced by $\frac{x}{c}$: (shrink c>1, stretch 0<c<1)

A function can be transformed with more than one transformation if you perform them in the following order:

1. Horizontal Shift
2. Stretching or shrinking
3. Reflection
4. Vertical Shift

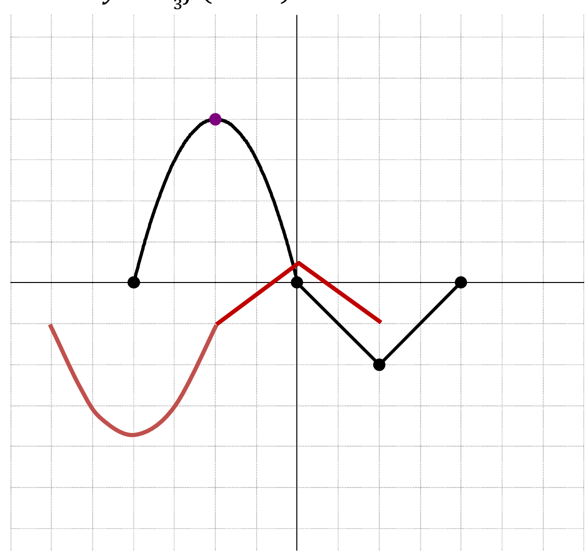
Ex. Use the graph of $y = f(x)$ to graph

$y = -\frac{1}{2}f(x-1)+3$



(x, y)	H.S 1R (x+1,y)	V Shrink (x, ½ y)	Refl over x (x, –y)	V Shift Up 3 (x, y+3)
(–4, 0)	(–3,0)	(–3,0)	(–3,0)	(–3, 3)
(–2,4)	(–1, 4)	(–1, 2)	(–1, –2)	(–1, 1)
(0, 0)	(1, 0)	(1, 0)	(1, 0)	(1, 3)
(1, –2)	(2, –2)	(2, –1)	(2, 1)	(2, 4)
(4, 0)	(5, 0)	(5, 0)	(5, 0)	(5, 3)

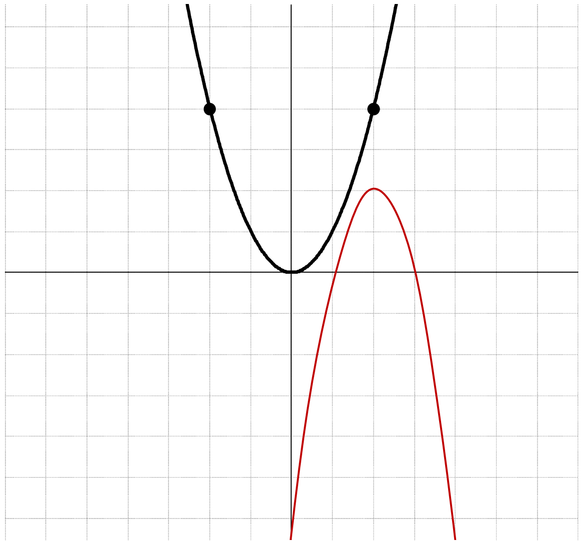
$y = -\frac{2}{3}f(x+2)-1$



(x, y)	H.S 2L (x–2,y)	V Shrink $(x, \frac{2}{3}y)$	Refl over x (x, –y)	V Shift down 1 (x, y – 1)
(–4, 0)	(–6,0)	(–6,0)	(–6,0)	(–6, –1)
(–2,4)	(–4, 4)	$(-4, \frac{8}{3})$	$(-4, -\frac{8}{3})$	$(-4, -\frac{11}{3})$
(0, 0)	(–2, 0)	(–2, 0)	(–2, 0)	(-2, –1)
(1, –2)	(–1, –2)	$(-1, -\frac{4}{3})$	$(-1, \frac{4}{3})$	$(-1, \frac{1}{3})$
(4, 0)	(2, 0)	(2, 0)	(2, 0)	(2, –1)

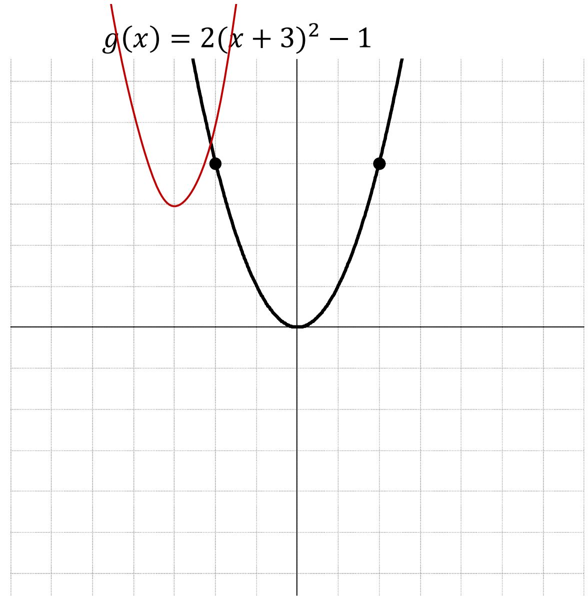
Ex. Use the graph of $f(x) = x^2$ to graph

$g(x) = -2(x-2)^2 + 2$



(x, y)	H.S 2R (x+2,y)	V Stretch (x, 2 y)	Refl over x (x, –y)	V Shift Up 2 (x, y+2)
(–2, 4)	(0, 4)	(0, 8)	(0, –8)	(0, –6)
(–1, 1)	(1, 1)	(1, 2)	(1, –2)	(1, 0)
(0, 0)	(2, 0)	(2, 0)	(2, 0)	(2, 2)
(1, 1)	(3, 1)	(3, 2)	(3, –2)	(3, 0)
(2, 4)	(4, 4)	(4, 8)	(4, –8)	(4, -6)

$g(x) = 2(x+3)^2 - 1$



(x, y)	H.S 3L (x–3,y)	V Stretch (x, 2 y)	V Shift Down 1 (x, y+3)
(–2, 4)	(–5, 4)	(–5, 8)	(–5, 11)
(–1, 1)	(–4, 1)	(–4, 2)	(–4, 5)
(0, 0)	(–3, 0)	(–3, 0)	(–3, 3)
(1, 1)	(–2, 1)	(–2, 2)	(–2, 5)
(2, 4)	(–1, 4)	(–1, 8)	(–1, 11)

Ex. Write the equation of the function given its base function, shifts, stretch/shrinks, and reflections

1. Base function: $f(x) = x^3$, vertical stretch of 2, horizontal shift 2 to the right, vertical shift 3 down, reflected over the *y*-axis.

Order:
1. Horiz. Shift
2. Stretch/Shrink
3. Reflect
4. Vertical Shift

Horiz Shift, Vertical Stretch 2, Reflect over y, Vertical Shift up 3

$x^3 \rightarrow (x-2)^3 \rightarrow 2(x-2)^3 \rightarrow 2(-(x-2))^3 \rightarrow 2(-x+2)^3 + 3$

$g(x) = 2(-x+2)^3 + 3$

2. Base function: $y = |x|$, reflected over *x*-axis, horizontal shift of 4 units left, vertical shift 2 units up, with a horizontal shrink factor of ½.

Order:
1. Horiz. Shift
2. Stretch/Shrink
3. Reflect
4. Vertical Shift

Horiz Shift, Horiz Shrink ½, Reflect over *x*, Vertical Shift up 2

$|x| \rightarrow |x+4| \rightarrow |2(x+4)| \rightarrow -|2x+8| \rightarrow -|2\,x+8| + 2$

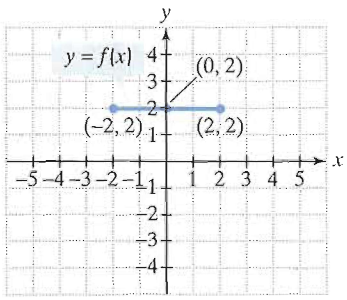
$g(x) = -|2x+8| + 2$

Homework: Pg. 216 – 219 #31, 32, 52, 93, 94, 103-106, 123, 126, 145-148

Exercise Set 1.6

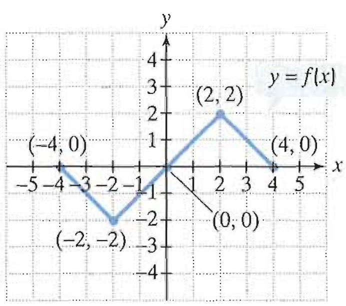
Practice Exercises

In Exercises 1–16, use the graph of $y = f(x)$ to graph each function g.



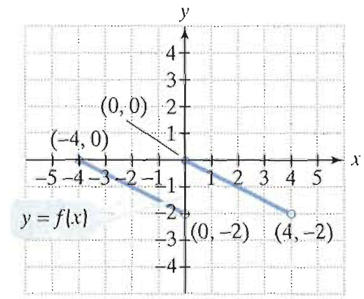
1. $g(x) = f(x) + 1$
2. $g(x) = f(x) - 1$
3. $g(x) = f(x + 1)$
4. $g(x) = f(x - 1)$
5. $g(x) = f(x - 1) - 2$
6. $g(x) = f(x + 1) + 2$
7. $g(x) = f(-x)$
8. $g(x) = -f(x)$
9. $g(x) = -f(x) + 3$
10. $g(x) = f(-x) + 3$
11. $g(x) = \frac{1}{2}f(x)$
12. $g(x) = 2f(x)$
13. $g(x) = f\left(\frac{1}{2}x\right)$
14. $g(x) = f(2x)$
15. $g(x) = -f\left(\frac{1}{2}x\right) + 1$
16. $g(x) = -f(2x) - 1$

In Exercises 17–32, use the graph of $y = f(x)$ to graph each function g.



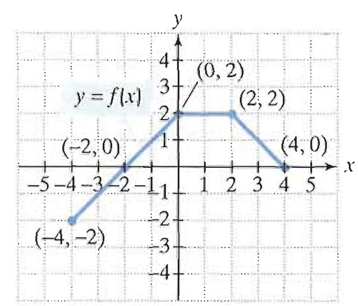
17. $g(x) = f(x) - 1$
18. $g(x) = f(x) + 1$
19. $g(x) = f(x - 1)$
20. $g(x) = f(x + 1)$
21. $g(x) = f(x - 1) + 2$
22. $g(x) = f(x + 1) - 2$
23. $g(x) = -f(x)$
24. $g(x) = f(-x)$
25. $g(x) = f(-x) + 1$
26. $g(x) = -f(x) + 1$
27. $g(x) = 2f(x)$
28. $g(x) = \frac{1}{2}f(x)$
29. $g(x) = f(2x)$
30. $g(x) = f\left(\frac{1}{2}x\right)$
31. $g(x) = 2f(x + 2) + 1$
32. $g(x) = 2f(x + 2) - 1$

In Exercises 33–44, use the graph of $y = f(x)$ to graph each function g.



33. $g(x) = f(x) + 2$
34. $g(x) = f(x) - 2$
35. $g(x) = f(x + 2)$
36. $g(x) = f(x - 2)$
37. $g(x) = -f(x + 2)$
38. $g(x) = -f(x - 2)$
39. $g(x) = -\frac{1}{2}f(x + 2)$
40. $g(x) = -\frac{1}{2}f(x - 2)$
41. $g(x) = -\frac{1}{2}f(x + 2) - 2$
42. $g(x) = -\frac{1}{2}f(x - 2) + 2$
43. $g(x) = \frac{1}{2}f(2x)$
44. $g(x) = 2f\left(\frac{1}{2}x\right)$

In Exercises 45–52, use the graph of $y = f(x)$ to graph each function g.



45. $g(x) = f(x - 1) - 1$
46. $g(x) = f(x + 1) + 1$
47. $g(x) = -f(x - 1) + 1$
48. $g(x) = -f(x + 1) - 1$
49. $g(x) = 2f\left(\frac{1}{2}x\right)$
50. $g(x) = \frac{1}{2}f(2x)$
51. $g(x) = \frac{1}{2}f(x + 1)$
52. $g(x) = 2f(x - 1)$

In Exercises 53–66, begin by graphing the standard quadratic function, $f(x) = x^2$. Then use transformations of this graph to graph the given function.

53. $g(x) = x^2 - 2$
54. $g(x) = x^2 - 1$
55. $g(x) = (x - 2)^2$
56. $g(x) = (x - 1)^2$
57. $h(x) = -(x - 2)^2$
58. $h(x) = -(x - 1)^2$
59. $h(x) = (x - 2)^2 + 1$
60. $h(x) = (x - 1)^2 + 2$
61. $g(x) = 2(x - 2)^2$
62. $g(x) = \frac{1}{2}(x - 1)^2$
63. $h(x) = 2(x - 2)^2 - 1$
64. $h(x) = \frac{1}{2}(x - 1)^2 - 1$
65. $h(x) = -2(x + 1)^2 + 1$
66. $h(x) = -2(x + 2)^2 + 1$

In Exercises 67–80, begin by graphing the square root function, $f(x) = \sqrt{x}$. Then use transformations of this graph to graph the given function.

67. $g(x) = \sqrt{x} + 2$
68. $g(x) = \sqrt{x} + 1$
69. $g(x) = \sqrt{x + 2}$
70. $g(x) = \sqrt{x + 1}$
71. $h(x) = -\sqrt{x + 2}$
72. $h(x) = -\sqrt{x + 1}$
73. $h(x) = \sqrt{-x + 2}$
74. $h(x) = \sqrt{-x + 1}$
75. $g(x) = \frac{1}{2}\sqrt{x + 2}$
76. $g(x) = 2\sqrt{x + 1}$
77. $h(x) = \sqrt{x + 2} - 2$
78. $h(x) = \sqrt{x + 1} - 1$
79. $g(x) = 2\sqrt{x + 2} - 2$
80. $g(x) = 2\sqrt{x + 1} - 1$

In Exercises 81–94, begin by graphing the absolute value function, $f(x) = |x|$. Then use transformations of this graph to graph the given function.

81. $g(x) = |x| + 4$
82. $g(x) = |x| + 3$
83. $g(x) = |x + 4|$
84. $g(x) = |x + 3|$
85. $h(x) = |x + 4| - 2$
86. $h(x) = |x + 3| - 2$
87. $h(x) = -|x + 4|$
88. $h(x) = -|x + 3|$
89. $g(x) = -|x + 4| + 1$
90. $g(x) = -|x + 4| + 2$
91. $h(x) = 2|x + 4|$
92. $h(x) = 2|x + 3|$
93. $g(x) = -2|x + 4| + 1$
94. $g(x) = -2|x + 3| + 2$

In Exercises 95–106, begin by graphing the standard cubic function, $f(x) = x^3$. Then use transformations of this graph to graph the given function.

95. $g(x) = x^3 - 3$
96. $g(x) = x^3 - 2$
97. $g(x) = (x - 3)^3$
98. $g(x) = (x - 2)^3$
99. $h(x) = -x^3$
100. $h(x) = -(x - 2)^3$
101. $h(x) = \frac{1}{2}x^3$
102. $h(x) = \frac{1}{4}x^3$
103. $r(x) = (x - 3)^3 + 2$
104. $r(x) = (x - 2)^3 + 1$
105. $h(x) = \frac{1}{2}(x - 3)^3 - 2$
106. $h(x) = \frac{1}{2}(x - 2)^3 - 1$

In Exercises 107–118, begin by graphing the cube root function, $f(x) = \sqrt[3]{x}$. Then use transformations of this graph to graph the given function.

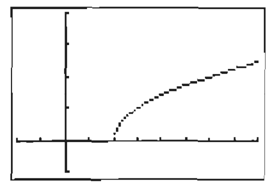
107. $g(x) = \sqrt[3]{x} + 2$
108. $g(x) = \sqrt[3]{x} - 2$
109. $g(x) = \sqrt[3]{x + 2}$
110. $g(x) = \sqrt[3]{x - 2}$
111. $h(x) = \frac{1}{2}\sqrt[3]{x + 2}$
112. $h(x) = \frac{1}{2}\sqrt[3]{x - 2}$
113. $r(x) = \frac{1}{2}\sqrt[3]{x + 2} - 2$
114. $r(x) = \frac{1}{2}\sqrt[3]{x - 2} + 2$
115. $h(x) = -\sqrt[3]{x + 2}$
116. $h(x) = -\sqrt[3]{x - 2}$
117. $g(x) = \sqrt[3]{-x - 2}$
118. $g(x) = \sqrt[3]{-x + 2}$

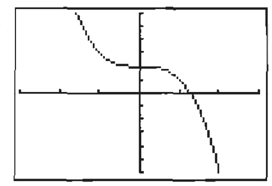
Practice Plus

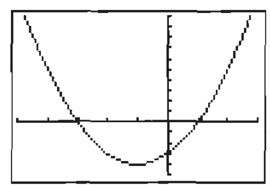
In Exercises 119–122, use transformations of the graph of the greatest integer function, $f(x) = \text{int}(x)$, to graph each function. (The graph of $f(x) = \text{int}(x)$ is shown in ***Figure 1.34*** *on page 171.)*

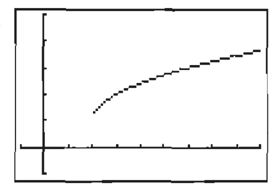
119. $g(x) = 2\ \text{int}(x + 1)$
120. $g(x) = 3\ \text{int}(x - 1)$
121. $h(x) = \text{int}(-x) + 1$
122. $h(x) = \text{int}(-x) - 1$

In Exercises 123–126, write a possible equation for the function whose graph is shown. Each graph shows a transformation of a common function.

123.

[−2, 8, 1] by [−1, 4, 1]

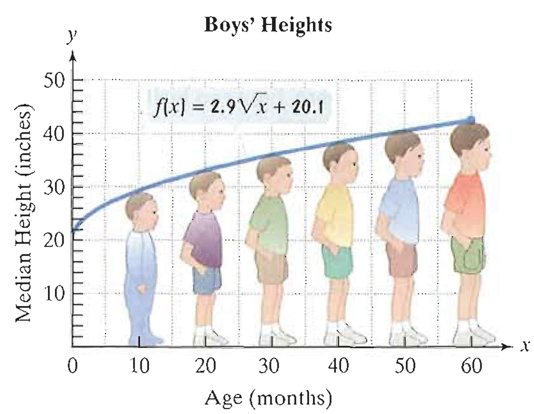
124.

[−3, 3, 1] by [−6, 6, 1]

125.

[−5, 3, 1] by [−5, 10, 1]

126.

[−1, 9, 1] by [−1, 5, 1]

Application Exercises

127. The function $f(x) = 2.9\sqrt{x} + 20.1$ models the median height, $f(x)$, in inches, of boys who are x months of age. The graph of f is shown.



Source: Laura Walther Nathanson, *The Portable Pediatrician for Parents*

a. Describe how the graph can be obtained using transformations of the square root function $f(x) = \sqrt{x}$.

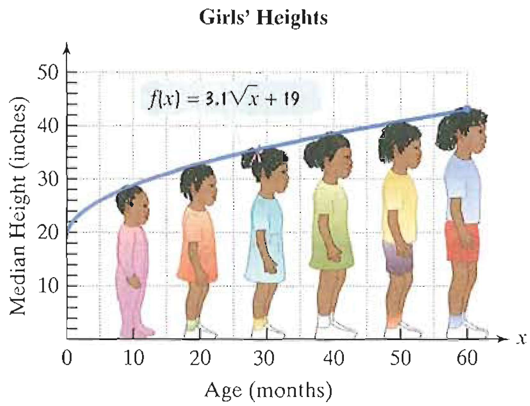
b. According to the model, what is the median height of boys who are 48 months, or four years, old? Use a calculator and round to the nearest tenth of an inch. The actual median height for boys at 48 months is 40.8 inches. How well does the model describe the actual height?

(This exercise continues on the next page.)

c. Use the model to find the average rate of change, in inches per month, between birth and 10 months. Round to the nearest tenth.

d. Use the model to find the average rate of change, in inches per month, between 50 and 60 months. Round to the nearest tenth. How does this compare with your answer in part (c)? How is this difference shown by the graph?

128. The function $f(x) = 3.1\sqrt{x} + 19$ models the median height, $f(x)$, in inches, of girls who are x months of age. The graph of f is shown.



Source: Laura Walther Nathanson, *The Portable Pediatrician for Parents*

a. Describe how the graph can be obtained using transformations of the square root function $f(x) = \sqrt{x}$.

b. According to the model, what is the median height of girls who are 48 months, or four years, old? Use a calculator and round to the nearest tenth of an inch. The actual median height for girls at 48 months is 40.2 inches. How well does the model describe the actual height?

c. Use the model to find the average rate of change, in inches per month, between birth and 10 months. Round to the nearest tenth.

d. Use the model to find the average rate of change, in inches per month, between 50 and 60 months. Round to the nearest tenth. How does this compare with your answer in part (c)? How is this difference shown by the graph?

Writing in Mathematics

129. What must be done to a function's equation so that its graph is shifted vertically upward?

130. What must be done to a function's equation so that its graph is shifted horizontally to the right?

131. What must be done to a function's equation so that its graph is reflected about the x-axis?

132. What must be done to a function's equation so that its graph is reflected about the y-axis?

133. What must be done to a function's equation so that its graph is stretched vertically?

134. What must be done to a function's equation so that its graph is shrunk horizontally?

Technology Exercises

135. a. Use a graphing utility to graph $f(x) = x^2 + 1$.

b. Graph $f(x) = x^2 + 1$, $g(x) = f(2x)$, $h(x) = f(3x)$, and $k(x) = f(4x)$ in the same viewing rectangle.

c. Describe the relationship among the graphs of f, g, h, and k, with emphasis on different values of x for points on all four graphs that give the same y-coordinate.

d. Generalize by describing the relationship between the graph of f and the graph of g, where $g(x) = f(cx)$ for $c > 1$.

e. Try out your generalization by sketching the graphs of $f(cx)$ for $c = 1$, $c = 2$, $c = 3$, and $c = 4$ for a function of your choice.

136. a. Use a graphing utility to graph $f(x) = x^2 + 1$.

b. Graph $f(x) = x^2 + 1$, $g(x) = f\left(\frac{1}{2}x\right)$, and $h(x) = f\left(\frac{1}{4}x\right)$ in the same viewing rectangle.

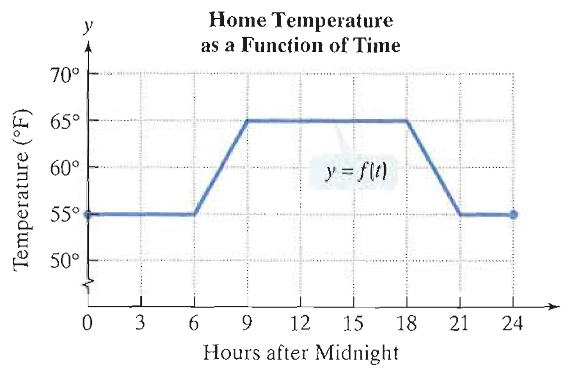
c. Describe the relationship among the graphs of f, g, and h, with emphasis on different values of x for points on all three graphs that give the same y-coordinate.

d. Generalize by describing the relationship between the graph of f and the graph of g, where $g(x) = f(cx)$ for $0 < c < 1$.

e. Try out your generalization by sketching the graphs of $f(cx)$ for $c = 1$, and $c = \frac{1}{2}$, and $c = \frac{1}{4}$ for a function of your choice.

Critical Thinking Exercises

Make Sense? *During the winter, you program your home thermostat so that at midnight, the temperature is 55°. This temperature is maintained until 6 A.M. Then the house begins to warm up so that by 9 A.M. the temperature is 65°. At 6 P.M. the house begins to cool. By 9 P.M., the temperature is again 55°. The graph illustrates home temperature, $f(t)$, as a function of hours after midnight, t.*



In Exercises 137–140, determine whether each statement makes sense or does not make sense, and explain your reasoning. If the statement makes sense, graph the new function on the domain [0, 24]. If the statement does not make sense, correct the function in the statement and graph the corrected function on the domain [0, 24].

137. I decided to keep the house 5° warmer than before, so I reprogrammed the thermostat to $y = f(t) + 5$.

138. I decided to keep the house 5° cooler than before, so I reprogrammed the thermostat to $y = f(t) - 5$.

139. I decided to change the heating schedule to start one hour earlier than before, so I reprogrammed the thermostat to $y = f(t - 1)$.

140. I decided to change the heating schedule to start one hour later than before, so I reprogrammed the thermostat to $y = f(t + 1)$.

In Exercises 141–144, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

141. If $f(x) = |x|$ and $g(x) = |x + 3| + 3$, then the graph of g is a translation of the graph of f three units to the right and three units upward.

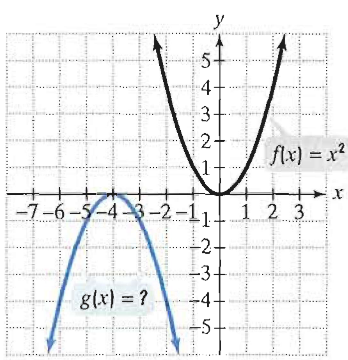
142. If $f(x) = -\sqrt{x}$ and $g(x) = \sqrt{-x}$, then f and g have identical graphs.

143. If $f(x) = x^2$ and $g(x) = 5(x^2 - 2)$, then the graph of g can be obtained from the graph of f by stretching f five units followed by a downward shift of two units.

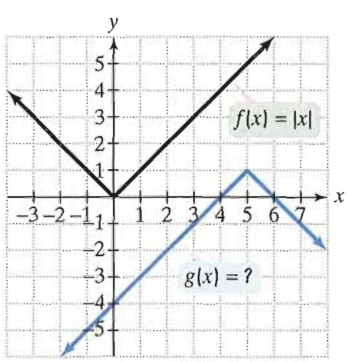
144. If $f(x) = x^3$ and $g(x) = -(x - 3)^3 - 4$, then the graph of g can be obtained from the graph of f by moving f three units to the right, reflecting about the x-axis, and then moving the resulting graph down four units.

In Exercises 145–148, functions f and g are graphed in the same rectangular coordinate system. If g is obtained from f through a sequence of transformations, find an equation for g.

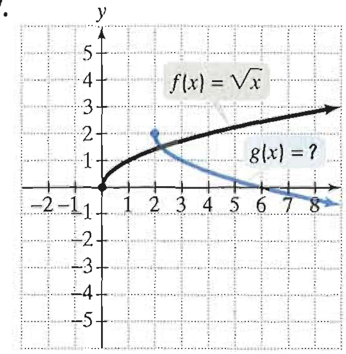
145.



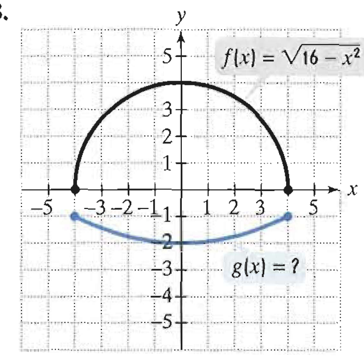
146.



147.



148.



For Exercises 149–152, assume that (a, b) is a point on the graph of f. What is the corresponding point on the graph of each of the following functions?

149. $y = f(-x)$

150. $y = 2f(x)$

151. $y = f(x - 3)$

152. $y = f(x) - 3$

Preview Exercises

Exercises 153–155 will help you prepare for the material covered in the next section.

In Exercises 153–154, perform the indicated operation or operations.

153. $(2x - 1)(x^2 + x - 2)$

154. $(f(x))^2 - 2f(x) + 6$, where $f(x) = 3x - 4$

155. Simplify: $\dfrac{2}{\frac{3}{x} - 1}$.

1.7 Composite Functions

Finding Domain: If you do not have the graph of a function $y = f(x)$

- To find the domain of a function, you are answering the following question: *What are all the possible values of x?*
  - Or better: *What values CAN'T x be?*
- How do you find domain?
  1. Assume (unless stated) the domain is All Real values
  2. If *x* is in the bottom of a fraction, then set the bottom ≠ 0 and solve for *x*
  3. If *x* is in a radical (even root), then set the inside of the radical ≥ 0 and solve for *x*
  4. If *x* is in a radical which is the bottom of a fraction, then set the inside of the radical > 0 and solve for *x*

Ex. Find the domain of each of the following:

1. $f(x) = x^2 - 7x$

   $D: \mathbb{R}$

2. $g(x) = \frac{3x+2}{x^2-2x-3}$

   $x^2 - 2x - 3 = 0 \rightarrow (x-3)(x+1) = 0 \rightarrow x = -1, x = 3$

   $D: x \neq -1, 3$

3. $h(x) = \sqrt{3x - 12}$

   $3x - 12 \geq 0$

   $3x \geq 12$

   $x \geq 4$

   $D: x \geq 4$

4. $f(x) = x^2 + 3x - 15$

   $D: \mathbb{R}$

5. $g(x) = \frac{3x}{x^2-3}$

   $x^2 - 36 = 0 \rightarrow (x-6)(x+6) = 0 \rightarrow x = -6, x = 6$

   $D: x \neq \pm 6$

6. $h(x) = \sqrt{9x + 15}$

   $9x + 15 \geq 0$

   $9x \geq -15$

   $x \geq -\frac{15}{9}$

   $D: x \geq -\frac{15}{9}$

Algebra of Functions: Let *f* and *g* be two functions. The following functions are defined:

1. Sum of 2 functions: $(f + g)(x) = f(x) + g(x)$
2. Difference of 2 functions: $(f - g)(x) = f(x) - g(x)$
3. Product of 2 functions: $(fg)(x) = f(x) \cdot g(x)$
4. Quotient of 2 functions: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$

*** In all cases, the domains will be the set of real numbers common to the domains of *f* and *g*: $D_f \cap D_g$

Ex. Let $f(x) = 2x - 1$ and $g(x) = x^2 + x - 2$. Find and state the domain of:

1. $(f + g)(x)$

   $f(x) + g(x)$

   $2x - 1 + (x^2 + x - 2)$

   $x^2 + 3x - 3$

   D: All reals ($\mathbb{R}$)

2. $(fg)(x)$

   $f(x) \cdot g(x)$

   $(2x - 1)(x^2 + x - 2)$

   $2x^3 + 2x^2 - 4x - x^2 - x + 2$

   $2x^3 + x^2 - 5x + 2$

   D: All reals ($\mathbb{R}$)

3. $\left(\frac{f}{g}\right)(x)$

   $\frac{f(x)}{g(x)}$

   $\frac{(2x-1)}{(x^2+x-2)}$

   D: $x \neq -2, 1$

Composite Functions: Let *f* and *g* be functions, then the composite of *f* with *g*, written $(f \circ g)(x)$ is denoted by:

$$(f \circ g)(x) = f[g(x)] = f(g(x))$$

** The domain of the composite function is the set of all *x* such that

1. *x* is in the domain of *g*
2. $g(x)$ is in the domain of *f*.

The idea here is that wherever there is an x in f, insert g(x) – USE PARENTHESES!!!

Ex. Given $f(x) = 3x - 4$ and $(x) = x^2 - 2x + 6$, find each of the following:

Find $g(2)$ and insert value into f

1. $(f \circ g)(x)$

$(f \circ g) = 3(x^2 - 2x + 6) - 4$

$(f \circ g)(x) = 3x^2 - 6x + 14$

Note:

$(f \circ g)(2) = 3(4) - 6(2) + 14$

$(f \circ g)(2) = 12 - 12 + 14 = 14$

3. $(f \circ g)(2)$

$g(2) = 4 - 4 + 6 = 6$

$f(6) = 3(6) - 4 = 14$

2. $(g \circ f)(x)$

$(g \circ f)(x) = (3x - 4)^2 - 2(3x - 4) + 6$

$(g \circ f)(x) = 9x^2 - 24x + 16 - 6x + 8 + 6$

$(g \circ f)(x) = 9x^2 - 30x + 30$

4. $(f \circ f)(x)$

$(f \circ f)(x) = 3(x - 4) - 4$

$(f \circ f) = 3x - 16$

Note:

$D_f: x \neq 1$

$D_g: x \neq 0$

Ex. Given $f(x) = \dfrac{2}{x-1}$ and $g(x) = \dfrac{3}{x}$, find each of the following and state its domain.

1. $(f \circ g)(x)$

$(f \circ g)(x) = \dfrac{2}{\left(\frac{3}{x}\right) - 1}$

Multiply top and bottom by *x*

$(f \circ g) = \dfrac{2x}{3 - x}$

Domain: $x \neq 0,\ 3$

You must keep the domain of the inserted function as well as the new!

2. $(g \circ f)(x)$

$(g \circ f)(x) = \dfrac{3}{\frac{2}{x-1}}$

$(f \circ g) = \dfrac{3(x-1)}{2} = \dfrac{3x-3}{2}$

Domain: $x \neq 1$

Ex. Express each of the following as a composition of 2 functions *f* and *g*:

1. $h(x) = \sqrt[3]{x^2 + 1}$

2. $k(x) = (x^2 - 4)^3$

What you want to do is make *f* the outside function and *g* the inside function

$f(x) = \sqrt[3]{x}$ $\qquad g(x) = x^2 + 1$ $\qquad f(x) = x^3$ $\qquad g(x) = x^2 - 4$

$$h(x) = \sqrt[3]{x^2 + 1}$$

Inside function

Once the inside function is removed, rewrite the function with *x* in its place and you have the outside function

$$h(x) = \sqrt[3]{x}$$

Homework: Pg. 229 – 231 #1-19odd, 33-48 (3's), 65-71odd, 81, 84, 89-92

5 Write functions as compositions.

Decomposing Functions

When you form a composite function, you "compose" two functions to form a new function. It is also possible to reverse this process. That is, you can "decompose" a given function and express it as a composition of two functions. Although there is more than one way to do this, there is often a "natural" selection that comes to mind first. For example, consider the function h defined by

$$h(x) = (3x^2 - 4x + 1)^5.$$

The function h takes $3x^2 - 4x + 1$ and raises it to the power 5. A natural way to write h as a composition of two functions is to raise the function $g(x) = 3x^2 - 4x + 1$ to the power 5. Thus, if we let

$$f(x) = x^5 \text{ and } g(x) = 3x^2 - 4x + 1, \text{ then}$$

$$(f \circ g)(x) = f(g(x)) = f(3x^2 - 4x + 1) = (3x^2 - 4x + 1)^5.$$

EXAMPLE 6 Writing a Function as a Composition

Express $h(x)$ as a composition of two functions:

$$h(x) = \sqrt[3]{x^2 + 1}.$$

Solution The function h takes $x^2 + 1$ and takes its cube root. A natural way to write h as a composition of two functions is to take the cube root of the function $g(x) = x^2 + 1$. Thus, we let

$$f(x) = \sqrt[3]{x} \text{ and } g(x) = x^2 + 1.$$

We can check this composition by finding $(f \circ g)(x)$. This should give the original function, namely $h(x) = \sqrt[3]{x^2 + 1}$.

$$(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = \sqrt[3]{x^2 + 1} = h(x)$$

Study Tip

Suppose the form of function h is $h(x) = (\text{algebraic expression})^{\text{power}}$. Function h can be expressed as a composition, $f \circ g$, using

$f(x) = x^{\text{power}}$

$g(x) = \text{algebraic expression}.$

Check Point 6 Express $h(x)$ as a composition of two functions:

$$h(x) = \sqrt{x^2 + 5}.$$

Exercise Set 1.7

Practice Exercises

In Exercises 1–30, find the domain of each function.

1. $f(x) = 3(x - 4)$
2. $f(x) = 2(x + 5)$
3. $g(x) = \dfrac{3}{x - 4}$
4. $g(x) = \dfrac{2}{x + 5}$
5. $f(x) = x^2 - 2x - 15$
6. $f(x) = x^2 + x - 12$
7. $g(x) = \dfrac{3}{x^2 - 2x - 15}$
8. $g(x) = \dfrac{2}{x^2 + x - 12}$
9. $f(x) = \dfrac{1}{x + 7} + \dfrac{3}{x - 9}$
10. $f(x) = \dfrac{1}{x + 8} + \dfrac{3}{x - 10}$
11. $g(x) = \dfrac{1}{x^2 + 1} - \dfrac{1}{x^2 - 1}$
12. $g(x) = \dfrac{1}{x^2 + 4} - \dfrac{1}{x^2 - 4}$
13. $h(x) = \dfrac{4}{\dfrac{3}{x} - 1}$
14. $h(x) = \dfrac{5}{\dfrac{4}{x} - 1}$
15. $f(x) = \dfrac{1}{\dfrac{4}{x - 1} - 2}$
16. $f(x) = \dfrac{1}{\dfrac{4}{x - 2} - 3}$
17. $f(x) = \sqrt{x - 3}$
18. $f(x) = \sqrt{x + 2}$

19. $g(x) = \dfrac{1}{\sqrt{x-3}}$
20. $g(x) = \dfrac{1}{\sqrt{x+2}}$
21. $g(x) = \sqrt{5x+35}$
22. $g(x) = \sqrt{7x-70}$
23. $f(x) = \sqrt{24-2x}$
24. $f(x) = \sqrt{84-6x}$
25. $h(x) = \sqrt{x-2} + \sqrt{x+3}$
26. $h(x) = \sqrt{x-3} + \sqrt{x+4}$
27. $g(x) = \dfrac{\sqrt{x-2}}{x-5}$
28. $g(x) = \dfrac{\sqrt{x-3}}{x-6}$
29. $f(x) = \dfrac{2x+7}{x^3-5x^2-4x+20}$
30. $f(x) = \dfrac{7x+2}{x^3-2x^2-9x+18}$

In Exercises 31–48, find $f+g, f-g, fg$, and $\frac{f}{g}$. Determine the domain for each function.

31. $f(x) = 2x+3, g(x) = x-1$
32. $f(x) = 3x-4, g(x) = x+2$
33. $f(x) = x-5, g(x) = 3x^2$
34. $f(x) = x-6, g(x) = 5x^2$
35. $f(x) = 2x^2-x-3, g(x) = x+1$
36. $f(x) = 6x^2-x-1, g(x) = x-1$
37. $f(x) = 3-x^2, g(x) = x^2+2x-15$
38. $f(x) = 5-x^2, g(x) = x^2+4x-12$
39. $f(x) = \sqrt{x}, g(x) = x-4$
40. $f(x) = \sqrt{x}, g(x) = x-5$
41. $f(x) = 2+\dfrac{1}{x}, g(x) = \dfrac{1}{x}$
42. $f(x) = 6-\dfrac{1}{x}, g(x) = \dfrac{1}{x}$
43. $f(x) = \dfrac{5x+1}{x^2-9}, g(x) = \dfrac{4x-2}{x^2-9}$
44. $f(x) = \dfrac{3x+1}{x^2-25}, g(x) = \dfrac{2x-4}{x^2-25}$
45. $f(x) = \sqrt{x+4}, g(x) = \sqrt{x-1}$
46. $f(x) = \sqrt{x+6}, g(x) = \sqrt{x-3}$
47. $f(x) = \sqrt{x-2}, g(x) = \sqrt{2-x}$
48. $f(x) = \sqrt{x-5}, g(x) = \sqrt{5-x}$

In Exercises 49–64, find

a. $(f \circ g)(x)$; **b.** $(g \circ f)(x)$; **c.** $(f \circ g)(2)$.

49. $f(x) = 2x, g(x) = x+7$
50. $f(x) = 3x, g(x) = x-5$
51. $f(x) = x+4, g(x) = 2x+1$
52. $f(x) = 5x+2, g(x) = 3x-4$
53. $f(x) = 4x-3, g(x) = 5x^2-2$
54. $f(x) = 7x+1, g(x) = 2x^2-9$
55. $f(x) = x^2+2, g(x) = x^2-2$
56. $f(x) = x^2+1, g(x) = x^2-3$
57. $f(x) = 4-x, g(x) = 2x^2+x+5$
58. $f(x) = 5x-2, g(x) = -x^2+4x-1$
59. $f(x) = \sqrt{x}, g(x) = x-1$
60. $f(x) = \sqrt{x}, g(x) = x+2$
61. $f(x) = 2x-3, g(x) = \dfrac{x+3}{2}$
62. $f(x) = 6x-3, g(x) = \dfrac{x+3}{6}$
63. $f(x) = \dfrac{1}{x}, g(x) = \dfrac{1}{x}$
64. $f(x) = \dfrac{2}{x}, g(x) = \dfrac{2}{x}$

In Exercises 65–72, find

a. $(f \circ g)(x)$; **b.** *the domain of* $f \circ g$.

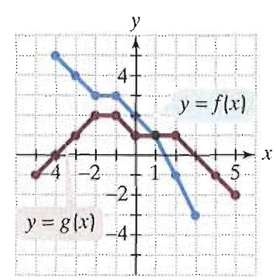
65. $f(x) = \dfrac{2}{x+3}, g(x) = \dfrac{1}{x}$
66. $f(x) = \dfrac{5}{x+4}, g(x) = \dfrac{1}{x}$
67. $f(x) = \dfrac{x}{x+1}, g(x) = \dfrac{4}{x}$
68. $f(x) = \dfrac{x}{x+5}, g(x) = \dfrac{6}{x}$
69. $f(x) = \sqrt{x}, g(x) = x-2$
70. $f(x) = \sqrt{x}, g(x) = x-3$
71. $f(x) = x^2+4, g(x) = \sqrt{1-x}$
72. $f(x) = x^2+1, g(x) = \sqrt{2-x}$

In Exercises 73–80, express the given function h as a composition of two functions f and g so that $h(x) = (f \circ g)(x)$.

73. $h(x) = (3x-1)^4$
74. $h(x) = (2x-5)^3$
75. $h(x) = \sqrt[3]{x^2-9}$
76. $h(x) = \sqrt{5x^2+3}$
77. $h(x) = |2x-5|$
78. $h(x) = |3x-4|$
79. $h(x) = \dfrac{1}{2x-3}$
80. $h(x) = \dfrac{1}{4x+5}$

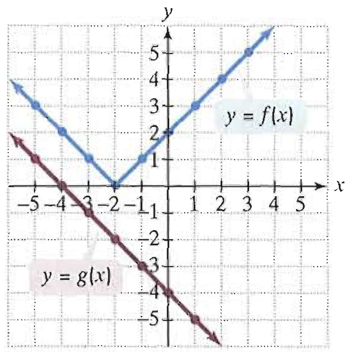
Practice Plus

Use the graphs of f and g to solve Exercises 81–88.



81. Find $(f+g)(-3)$.
82. Find $(g-f)(-2)$.
83. Find $(fg)(2)$.
84. Find $\left(\dfrac{g}{f}\right)(3)$.
85. Find the domain of $f+g$.
86. Find the domain of $\dfrac{f}{g}$.
87. Graph $f+g$.
88. Graph $f-g$.

In Exercises 89–92, use the graphs of f and g to evaluate each composite function.



89. $(f \circ g)(-1)$

90. $(f \circ g)(1)$

91. $(g \circ f)(0)$

92. $(g \circ f)(-1)$

In Exercises 93–94, find all values of x satisfying the given conditions.

93. $f(x) = 2x - 5$, $g(x) = x^2 - 3x + 8$, and $(f \circ g)(x) = 7$.

94. $f(x) = 1 - 2x$, $g(x) = 3x^2 + x - 1$, and $(f \circ g)(x) = -5$.

Application Exercises

We opened the section with functions that model the numbers of births and deaths in the United States from 2000 through 2005:

$B(x) = 7.4x^2 - 15x + 4046$ $\quad D(x) = -3.5x^2 + 20x + 2405.$

Number of births, $B(x)$, in thousands, x years after 2000

Number of deaths, $D(x)$, in thousands, x years after 2000

Use these functions to solve Exercises 95–96.

95. a. Write a function that models the change in U.S. population for each year from 2000 through 2005.

b. Use the function from part (a) to find the change in U.S. population in 2003.

c. Does the result in part (b) overestimate or underestimate the actual population change in 2003 obtained from the data in **Figure 1.61** on page 220? By how much?

96. a. Write a function that models the total number of births and deaths in the United States for each year from 2000 through 2005.

b. Use the function from part (a) to find the total number of births and deaths in the United States in 2005.

c. Does the result in part (b) overestimate or underestimate the actual number of total births and deaths in 2005 obtained from the data in **Figure 1.61** on page 220? By how much?

97. A company that sells radios has yearly fixed costs of \$600,000. It costs the company \$45 to produce each radio. Each radio will sell for \$65. The company's costs and revenue are modeled by the following functions, where x represents the number of radios produced and sold:

$C(x) = 600{,}000 + 45x$ This function models the company's costs.

$R(x) = 65x.$ This function models the company's revenue.

Find and interpret $(R - C)(20{,}000)$, $(R - C)(30{,}000)$, and $(R - C)(40{,}000)$.

98. A department store has two locations in a city. From 2004 through 2008, the profits for each of the store's two branches are modeled by the functions $f(x) = -0.44x + 13.62$ and $g(x) = 0.51x + 11.14$. In each model, x represents the number of years after 2004, and f and g represent the profit, in millions of dollars.

a. What is the slope of f? Describe what this means.

b. What is the slope of g? Describe what this means.

c. Find $f + g$. What is the slope of this function? What does this mean?

99. The regular price of a computer is x dollars. Let $f(x) = x - 400$ and $g(x) = 0.75x$.

a. Describe what the functions f and g model in terms of the price of the computer.

b. Find $(f \circ g)(x)$ and describe what this models in terms of the price of the computer.

c. Repeat part (b) for $(g \circ f)(x)$.

d. Which composite function models the greater discount on the computer, $f \circ g$ or $g \circ f$? Explain.

100. The regular price of a pair of jeans is x dollars. Let $f(x) = x - 5$ and $g(x) = 0.6x$.

a. Describe what functions f and g model in terms of the price of the jeans.

b. Find $(f \circ g)(x)$ and describe what this models in terms of the price of the jeans.

c. Repeat part (b) for $(g \circ f)(x)$.

d. Which composite function models the greater discount on the jeans, $f \circ g$ or $g \circ f$? Explain.

Writing in Mathematics

101. If a function is defined by an equation, explain how to find its domain.

102. If equations for f and g are given, explain how to find $f - g$.

103. If equations for two functions are given, explain how to obtain the quotient function and its domain.

104. Describe a procedure for finding $(f \circ g)(x)$. What is the name of this function?

105. Describe the values of x that must be excluded from the domain of $(f \circ g)(x)$.

Technology Exercises

106. Graph $y_1 = x^2 - 2x$, $y_2 = x$, and $y_3 = y_1 \div y_2$ in the same $[-10, 10, 1]$ by $[-10, 10, 1]$ viewing rectangle. Then use the TRACE feature to trace along y_3. What happens at $x = 0$? Explain why this occurs.

107. Graph $y_1 = \sqrt{2 - x}$, $y_2 = \sqrt{x}$, and $y_3 = \sqrt{2 - y_2}$ in the same $[-4, 4, 1]$ by $[0, 2, 1]$ viewing rectangle. If y_1 represents f and y_2 represents g, use the graph of y_3 to find the domain of $f \circ g$. Then verify your observation algebraically.

Critical Thinking Exercises

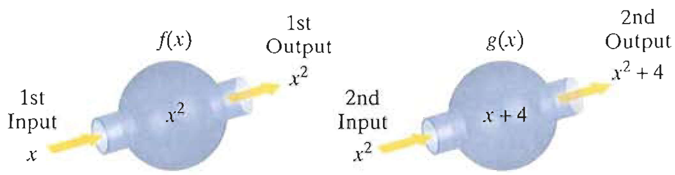
Make Sense? *In Exercises 108–111, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

108. I used a function to model data from 1980 through 2005. The independent variable in my model represented the number of years after 1980, so the function's domain was $\{x \mid x = 0, 1, 2, 3, \ldots, 25\}$.

109. I have two functions. Function f models total world population x years after 2000 and function g models population of the world's more-developed regions x years after 2000. I can use $f - g$ to determine the population of the world's less-developed regions for the years in both function's domains.

110. I must have made a mistake in finding the composite functions $f \circ g$ and $g \circ f$, because I notice that $f \circ g$ is not the same function as $g \circ f$.

111. This diagram illustrates that $f(g(x)) = x^2 + 4$.



In Exercises 112–115, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

112. If $f(x) = x^2 - 4$ and $g(x) = \sqrt{x^2 - 4}$, then $(f \circ g)(x) = -x^2$ and $(f \circ g)(5) = -25$.

113. There can never be two functions f and g, where $f \neq g$, for which $(f \circ g)(x) = (g \circ f)(x)$.

114. If $f(7) = 5$ and $g(4) = 7$, then $(f \circ g)(4) = 35$.

115. If $f(x) = \sqrt{x}$ and $g(x) = 2x - 1$, then $(f \circ g)(5) = g(2)$.

116. Prove that if f and g are even functions, then fg is also an even function.

117. Define two functions f and g so that $f \circ g = g \circ f$.

Preview Exercises

Exercises 118–120 will help you prepare for the material covered in the next section.

118. Consider the function defined by

$$\{(-2, 4), (-1, 1), (1, 1), (2, 4)\}.$$

Reverse the components of each ordered pair and write the resulting relation. Is this relation a function?

119. Solve for y: $x = \frac{5}{y} + 4$.

120. Solve for y: $x = y^2 - 1, y \geq 0$.

Section 1.8 Inverse Functions

Objectives

1. Verify inverse functions.
2. Find the inverse of a function.
3. Use the horizontal line test to determine if a function has an inverse function.
4. Use the graph of a one-to-one function to graph its inverse function.
5. Find the inverse of a function and graph both functions on the same axes.

In most societies, women say they prefer to marry men who are older than themselves, whereas men say they prefer women who are younger. Evolutionary psychologists attribute these preferences to female concern with a partner's material resources and male concern with a partner's fertility (*Source:* David M. Buss, *Psychological Inquiry*, 6, 1–30). When the man is considerably older than the woman, people rarely comment. However, when the woman is older, as in the relationship between actors Ashton Kutcher and Demi Moore, people take notice.

Figure 1.65 on the next page shows the preferred age difference in a mate in five selected countries.

1.8 Inverse Functions

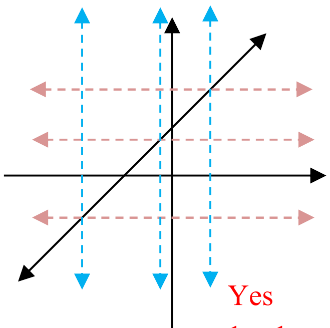
One to One Functions

Def: A function *f(x)* is a **one-to-one function** on the domain D if $f(a) \neq f(b)$ whenever $a \neq b$

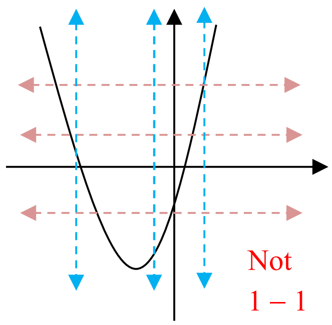
For every *x* there exists one and only one *y* and for every *y* there exists one and only one *x*.

Passes both the vertical and horizontal line test → **Any vertical line or horizontal line, when drawn, can intersect *f*(*x*) in at most one point!**

Ex. Are the following functions one-to-one

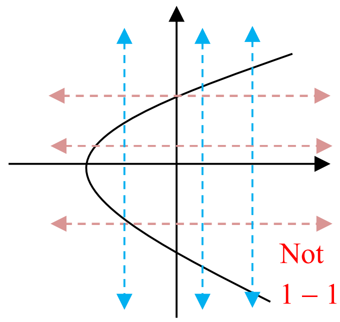


Yes 1 – 1



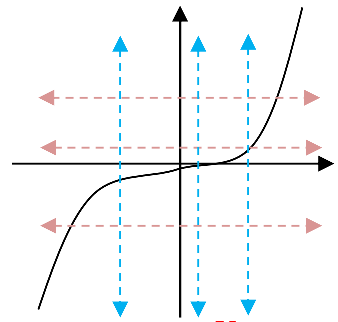
Not 1 – 1

Fails Horizontal



Not 1 – 1

Fails Vertical



Yes 1 – 1

Inverses:

Def: A function defined by reversing a one-to-one function *f* is the **inverse of *f***.

Notation $f^{-1}(x)$ "$y = f(x) \rightarrow x = f(y)$"

- Graphically: The inverse of a function $y = f(x)$ is the reflection over the line *y* = *x*
- Algebraically: The inverse satisfies $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$
  If you composite the function and its inverse, you just get *x*

To find an Inverse: Change $f(x)$ to *y*

1. Interchange x and y in the original equation
2. Solve for y which is $f^{-1}(x)$

Property:

$D_f = R_{f^{-1}}$

$R_f = D_{f^{-1}}$

The domain of the function is the same as the range of the inverse.

***** The range of the function is the domain of the inverse *******

Ex. Find the inverse of *y* = –2*x* + 4. Algebraically prove it is the inverse.

$x = -2y + 4 \rightarrow y = -2x + 4$ (*wow quick!!*)

$f^{-1}(x) = -2x + 4$

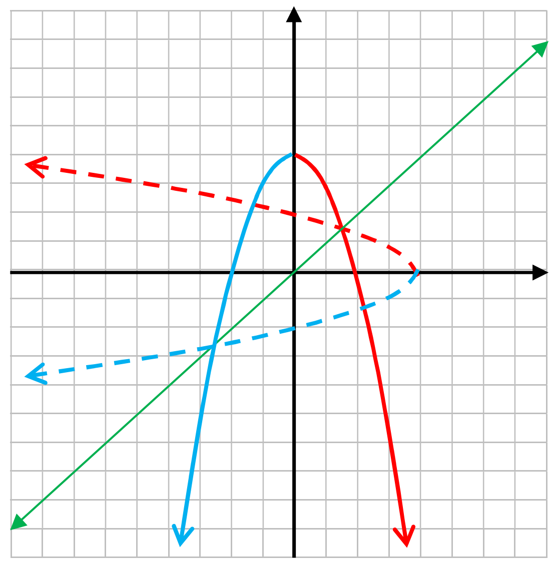
Ex. Find the inverse of $y = x^2$ with domain $x \geq 0$. Algebraically prove it is the inverse. Show graphically as well.

$x = y^2 \rightarrow y = \pm\sqrt{x}$

Since the domain of *f* is $x \geq 0$, then the range of the inverse would be the same with *y*: $y \geq 0$, making it positive. So, the inverse is $f^{-1}(x) = \sqrt{x}$

Ex. Consider the function $f(x) = 4 - x^2$ with domain $x \geq 0$.

(a) On the set of axes, graph $f(x)$ on the indicated domain.
(b) According to the graph, what is the range of $f(x)$?
(c) Write the equation for $f^{-1}(x)$ and state its domain and range
(d) On the same set of axes, graph and label $f^{-1}(x)$
(e) On the same set of axes, graph and label the line $y = x$
(f) According to your graph, is $f^{-1}(x)$ the image of $f(x)$ after a reflection over $y = x$?
(g) Show, using $(f \circ f^{-1})(x)$, that your $f^{-1}(x)$ is correct.



(b) Since the highest point of the curve is 4, the range of the function is $y \leq 4$

(c) $x = 4 - y^2$

$y^2 = 4 - x \quad \rightarrow \quad y = \pm\sqrt{4 - x}$

Since $D_f: x \geq 0$, *then* $R_{f^{-1}}: y \geq 0$

Therefore, $f^{-1}(x) = \sqrt{4 - x}$

If the function was the other half of the parabola (where x is negative), then $f^{-1}(x) = -\sqrt{4 - x}$

$D_{f^{-1}}: \quad 4 - x \geq 0 \rightarrow D_{f^{-1}}: x \leq 4$

$R_{f^{-1}} = D_f: \quad y \geq 0$

(d) Quick way: Switch (x, y) for main points

$(0,4) \rightarrow (4,0) \quad (1,3) \rightarrow (3,1) \quad (2,0) \rightarrow (0,2)$

(e) Green line

(f) Yes

(g) $(f \circ f^{-1})(x) = 4 - (\sqrt{4 - x})^2 = 4 - (4 - x) = 4 - 4 + x = x \leftarrow$ *winner*

Homework: Pg. 240-242 #1-27o, 29-39, 43, 47

5 Find the inverse of a function and graph both functions on the same axes.

In our final example, we will first find f^{-1}. Then we will graph f and f^{-1} in the same rectangular coordinate system.

EXAMPLE 7 **Finding the Inverse of a Domain-Restricted Function**

Find the inverse of $f(x) = x^2 - 1$ if $x \geq 0$. Graph f and f^{-1} in the same rectangular coordinate system.

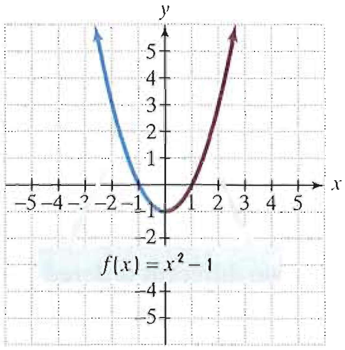


Figure 1.71

Solution The graph of $f(x) = x^2 - 1$ is the graph of the standard quadratic function shifted vertically down 1 unit. **Figure 1.71** shows the function's graph. This graph fails the horizontal line test, so the function $f(x) = x^2 - 1$ does not have an inverse function. By restricting the domain to $x \geq 0$, as given, we obtain a new function whose graph is shown in red in **Figure 1.71**. This red portion of the graph is increasing on the interval $(0, \infty)$ and passes the horizontal line test. This tells us that $f(x) = x^2 - 1$ has an inverse function if we restrict its domain to $x \geq 0$. We use our four-step procedure to find this inverse function. Begin with $f(x) = x^2 - 1, x \geq 0$.

Step 1 **Replace $f(x)$ with y:** $y = x^2 - 1, x \geq 0$.

Step 2 **Interchange x and y:** $x = y^2 - 1, y \geq 0$.

Step 3 **Solve for y:**

$$x = y^2 - 1, y \geq 0 \quad \text{This is the equation from step 2.}$$

$$x + 1 = y^2 \quad \text{Add 1 to both sides.}$$

$$\sqrt{x+1} = y \quad \text{Apply the square root property.}$$

Because $y \geq 0$, take only the principal square root and not the negative square root.

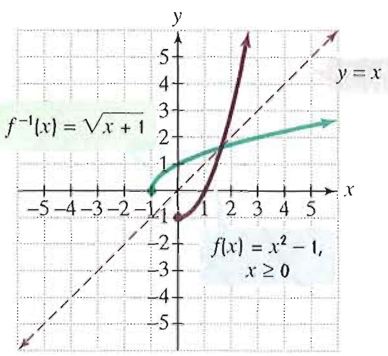


Figure 1.72

Step 4 **Replace y with $f^{-1}(x)$:** $f^{-1}(x) = \sqrt{x+1}$.

Thus, the inverse of $f(x) = x^2 - 1, x \geq 0$, is $f^{-1}(x) = \sqrt{x+1}$. The graphs of f and f^{-1} are shown in **Figure 1.72**. We obtained the graph of $f^{-1}(x) = \sqrt{x+1}$ by shifting the graph of the square root function, $y = \sqrt{x}$, horizontally to the left 1 unit. Note that the green graph of f^{-1} is the reflection of the red graph of f about the line $y = x$.

Check Point 7 Find the inverse of $f(x) = x^2 + 1$ if $x \geq 0$. Graph f and f^{-1} in the same rectangular coordinate system.

Exercise Set 1.8

Practice Exercises

In Exercises 1–10, find $f(g(x))$ and $g(f(x))$ and determine whether each pair of functions f and g are inverses of each other.

1. $f(x) = 4x$ and $g(x) = \frac{x}{4}$
2. $f(x) = 6x$ and $g(x) = \frac{x}{6}$
3. $f(x) = 3x + 8$ and $g(x) = \frac{x-8}{3}$
4. $f(x) = 4x + 9$ and $g(x) = \frac{x-9}{4}$
5. $f(x) = 5x - 9$ and $g(x) = \frac{x+5}{9}$
6. $f(x) = 3x - 7$ and $g(x) = \frac{x+3}{7}$
7. $f(x) = \frac{3}{x-4}$ and $g(x) = \frac{3}{x} + 4$
8. $f(x) = \frac{2}{x-5}$ and $g(x) = \frac{2}{x} + 5$
9. $f(x) = -x$ and $g(x) = -x$
10. $f(x) = \sqrt[3]{x-4}$ and $g(x) = x^3 + 4$

The functions in Exercises 11–28 are all one-to-one. For each function,

a. *Find an equation for $f^{-1}(x)$, the inverse function.*

b. *Verify that your equation is correct by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.*

11. $f(x) = x + 3$

12. $f(x) = x + 5$

13. $f(x) = 2x$

14. $f(x) = 4x$

15. $f(x) = 2x + 3$

16. $f(x) = 3x - 1$

17. $f(x) = x^3 + 2$

18. $f(x) = x^3 - 1$

19. $f(x) = (x + 2)^3$

20. $f(x) = (x - 1)^3$

21. $f(x) = \frac{1}{x}$

22. $f(x) = \frac{2}{x}$

23. $f(x) = \sqrt{x}$

24. $f(x) = \sqrt[3]{x}$

25. $f(x) = \frac{7}{x} - 3$

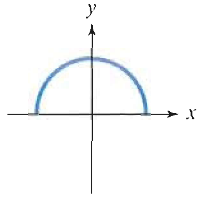
26. $f(x) = \frac{4}{x} + 9$

27. $f(x) = \frac{2x + 1}{x - 3}$

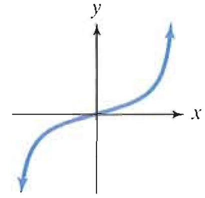
28. $f(x) = \frac{2x - 3}{x + 1}$

Which graphs in Exercises 29–34 represent functions that have inverse functions?

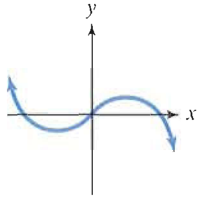
29.



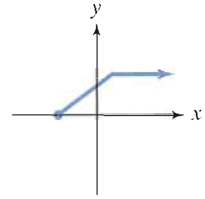
30.



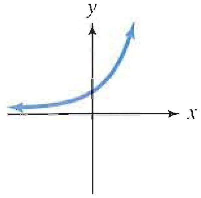
31.



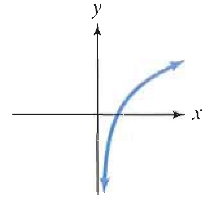
32.



33.

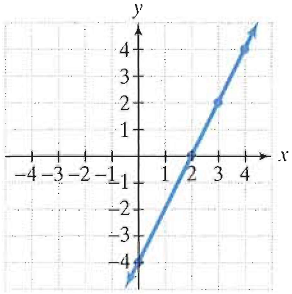


34.

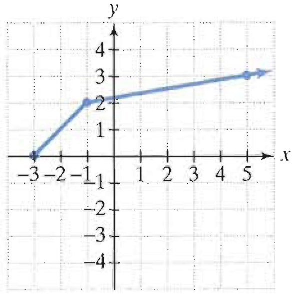


In Exercises 35–38, use the graph of f to draw the graph of its inverse function.

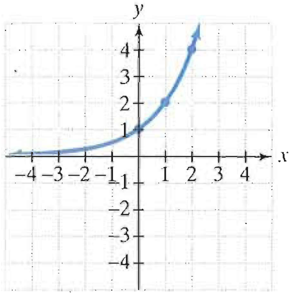
35.



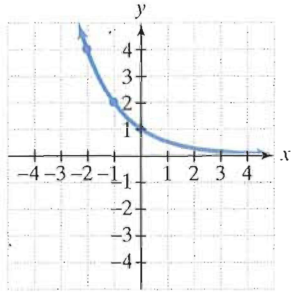
36.



37.



38.



In Exercises 39–52,

a. *Find an equation for $f^{-1}(x)$.*

b. *Graph f and f^{-1} in the same rectangular coordinate system.*

c. *Use interval notation to give the domain and the range of f and f^{-1}.*

39. $f(x) = 2x - 1$

40. $f(x) = 2x - 3$

41. $f(x) = x^2 - 4, x \geq 0$

42. $f(x) = x^2 - 1, x \leq 0$

43. $f(x) = (x - 1)^2, x \leq 1$

44. $f(x) = (x - 1)^2, x \geq 1$

45. $f(x) = x^3 - 1$

46. $f(x) = x^3 + 1$

47. $f(x) = (x + 2)^3$

48. $f(x) = (x - 2)^3$

(Hint for Exercises 49–52: To solve for a variable involving an nth root, raise both sides of the equation to the nth power: $(\sqrt[n]{y})^n = y$.)

49. $f(x) = \sqrt{x - 1}$

50. $f(x) = \sqrt{x} + 2$

51. $f(x) = \sqrt[3]{x} + 1$

52. $f(x) = \sqrt[3]{x - 1}$

Practice Plus

In Exercises 53–58, f and g are defined by the following tables. Use the tables to evaluate each composite function.

x	$f(x)$
−1	1
0	4
1	5
2	−1

x	$g(x)$
−1	0
1	1
4	2
10	−1

53. $f(g(1))$

54. $f(g(4))$

55. $(g \circ f)(-1)$

56. $(g \circ f)(0)$

57. $f^{-1}(g(10))$

58. $f^{-1}(g(1))$

In Exercises 59–64, let

$$f(x) = 2x - 5$$
$$g(x) = 4x - 1$$
$$h(x) = x^2 + x + 2.$$

Evaluate the indicated function without finding an equation for the function.

59. $(f \circ g)(0)$

60. $(g \circ f)(0)$

61. $f^{-1}(1)$

62. $g^{-1}(7)$

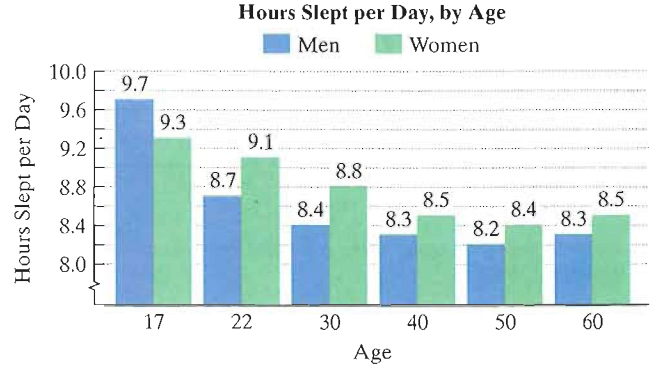
63. $g(f[h(1)])$

64. $f(g[h(1)])$

Application Exercises

The bar graph shows the average number of hours that Americans sleep per day, by age group. Use this information to solve Exercises 65–66.

Hours Slept per Day, by Age



Source: ATUS, Bureau of Labor Statistics

(In Exercises 65–66, refer to the graph at the bottom of the previous page.)

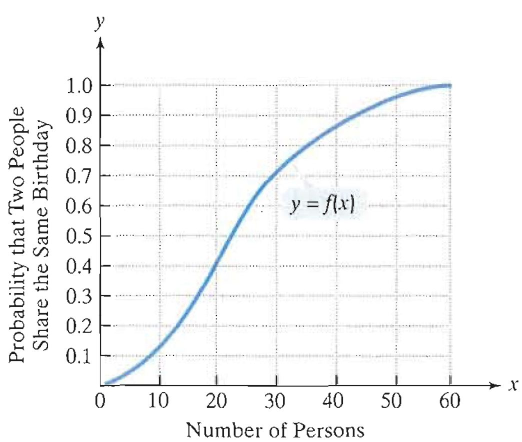
65. a. Consider a function, f, whose domain is the set of six ages shown. Let the range be the average number of hours that men sleep per day. Write the function f as a set of ordered pairs.

b. Write the relation that is the inverse of f as a set of ordered pairs. Based on these ordered pairs, is f a one-to-one function? Explain your answer.

66. a. Consider a function, g, whose domain is the set of six ages shown. Let the range be the average number of hours that women sleep per day. Write the function g as a set of ordered pairs.

b. Write the relation that is the inverse of g as a set of ordered pairs. Based on these ordered pairs, is g a one-to-one function? Explain your answer.

67. The graph represents the probability of two people in the same room sharing a birthday as a function of the number of people in the room. Call the function f.

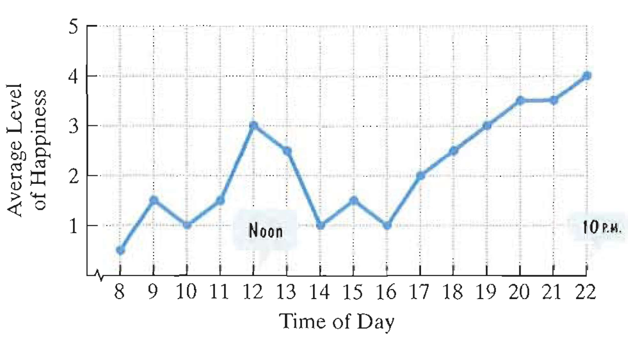


a. Explain why f has an inverse that is a function.

b. Describe in practical terms the meaning of $f^{-1}(0.25)$, $f^{-1}(0.5)$, and $f^{-1}(0.7)$.

68. A study of 900 working women in Texas showed that their feelings changed throughout the day. As the graph indicates, the women felt better as time passed, except for a blip (that's slang for relative maximum) at lunchtime.

Average Level of Happiness at Different Times of Day



Source: D. Kahneman et al., "A Survey Method for Characterizing Daily Life Experience," *Science*

a. Does the graph have an inverse that is a function? Explain your answer.

b. Identify two or more times of day when the average happiness level is 3. Express your answers as ordered pairs.

c. Do the ordered pairs in part (b) indicate that the graph represents a one-to-one function? Explain your answer.

69. The formula

$$y = f(x) = \frac{9}{5}x + 32$$

is used to convert from x degrees Celsius to y degrees Fahrenheit. The formula

$$y = g(x) = \frac{5}{9}(x - 32)$$

is used to convert from x degrees Fahrenheit to y degrees Celsius. Show that f and g are inverse functions.

Writing in Mathematics

70. Explain how to determine if two functions are inverses of each other.

71. Describe how to find the inverse of a one-to-one function.

72. What is the horizontal line test and what does it indicate?

73. Describe how to use the graph of a one-to-one function to draw the graph of its inverse function.

74. How can a graphing utility be used to visually determine if two functions are inverses of each other?

75. What explanations can you offer for the trends shown by the graph in Exercise 68?

Technology Exercises

In Exercises 76–83, use a graphing utility to graph the function. Use the graph to determine whether the function has an inverse that is a function (that is, whether the function is one-to-one).

76. $f(x) = x^2 - 1$

77. $f(x) = \sqrt[3]{2 - x}$

78. $f(x) = \frac{x^3}{2}$

79. $f(x) = \frac{x^4}{4}$

80. $f(x) = \text{int}(x - 2)$

81. $f(x) = |x - 2|$

82. $f(x) = (x - 1)^3$

83. $f(x) = -\sqrt{16 - x^2}$

In Exercises 84–86, use a graphing utility to graph f and g in the same viewing rectangle. In addition, graph the line y = x and visually determine if f and g are inverses.

84. $f(x) = 4x + 4,\ g(x) = 0.25x - 1$

85. $f(x) = \frac{1}{x} + 2,\ g(x) = \frac{1}{x - 2}$

86. $f(x) = \sqrt[3]{x} - 2,\ g(x) = (x + 2)^3$

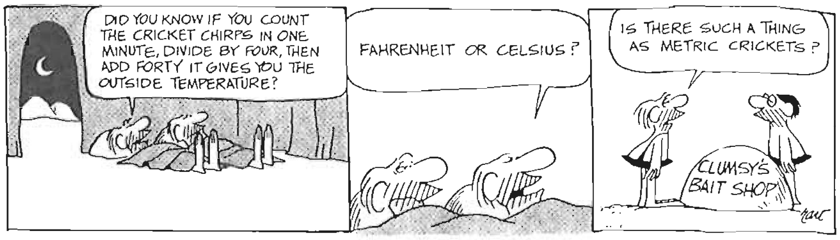
Critical Thinking Exercises

Make Sense? *In Exercises 87–90, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

87. I found the inverse of $f(x) = 5x - 4$ in my head: The reverse of multiplying by 5 and subtracting 4 is adding 4 and dividing by 5, so $f^{-1}(x) = \frac{x + 4}{5}$.

88. I'm working with the linear function $f(x) = 3x + 5$ and I do not need to find f^{-1} in order to determine the value of $(f \circ f^{-1})(17)$.

Exercises 89–90 are based on the following cartoon.



B.C. by permission of Johnny Hart and Creators Syndicate, Inc.

89. Assuming that there is no such thing as metric crickets, I modeled the information in the first frame of the cartoon with the function

$$T(n) = \frac{n}{4} + 40,$$

where $T(n)$ is the temperature, in degrees Fahrenheit, and n is the number of cricket chirps per minute.

90. I used the function in Exercise 89 and found an equation for $T^{-1}(n)$, which expresses the number of cricket chirps per minute as a function of Fahrenheit temperature.

In Exercises 91–94, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

91. The inverse of $\{(1,4),(2,7)\}$ is $\{(2,7),(1,4)\}$.

92. The function $f(x) = 5$ is one-to-one.

93. If $f(x) = 3x$, then $f^{-1}(x) = \frac{1}{3x}$.

94. The domain of f is the same as the range of f^{-1}.

95. If $f(x) = 3x$ and $g(x) = x + 5$, find $(f \circ g)^{-1}(x)$ and $(g^{-1} \circ f^{-1})(x)$.

96. Show that

$$f(x) = \frac{3x-2}{5x-3}$$

is its own inverse.

97. *Freedom 7* was the spacecraft that carried the first American into space in 1961. Total flight time was 15 minutes and the spacecraft reached a maximum height of 116 miles. Consider a function, s, that expresses *Freedom 7*'s height, $s(t)$, in miles, after t minutes. Is s a one-to-one function? Explain your answer.

98. If $f(2) = 6$, and f is one-to-one, find x satisfying $8 + f^{-1}(x-1) = 10$.

Group Exercise

99. In Tom Stoppard's play *Arcadia*, the characters dream and talk about mathematics, including ideas involving graphing, composite functions, symmetry, and lack of symmetry in things that are tangled, mysterious, and unpredictable. Group members should read the play. Present a report on the ideas discussed by the characters that are related to concepts that we studied in this chapter. Bring in a copy of the play and read appropriate excerpts.

Preview Exercises

Exercises 100–102 will help you prepare for the material covered in the next section.

100. Let $(x_1, y_1) = (7, 2)$ and $(x_2, y_2) = (1, -1)$. Find $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Express the answer in simplified radical form.

101. Use a rectangular coordinate system to graph the circle with center $(1, -1)$ and radius 1.

102. Solve by completing the square: $y^2 - 6y - 4 = 0$.

Section 1.9 Distance and Midpoint Formulas; Circles

Objectives

1. Find the distance between two points.
2. Find the midpoint of a line segment.
3. Write the standard form of a circle's equation.
4. Give the center and radius of a circle whose equation is in standard form.
5. Convert the general form of a circle's equation to standard form.

It's a good idea to know your way around a circle. Clocks, angles, maps, and compasses are based on circles. Circles occur everywhere in nature: in ripples on water, patterns on a moth's wings, and cross sections of trees. Some consider the circle to be the most pleasing of all shapes.

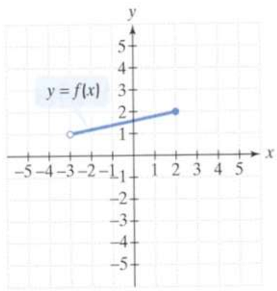
The rectangular coordinate system gives us a unique way of knowing a circle. It enables us to translate a circle's geometric definition into an algebraic equation. To do this, we must first develop a formula for the distance between any two points in rectangular coordinates.

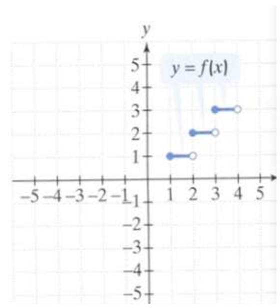
Name____ Worksheet: 1.1 thru 1.8

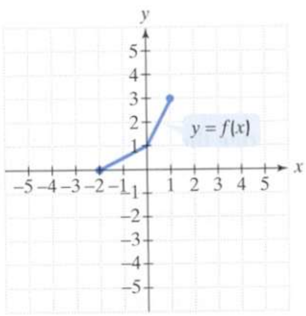
Precalculus – Period ________

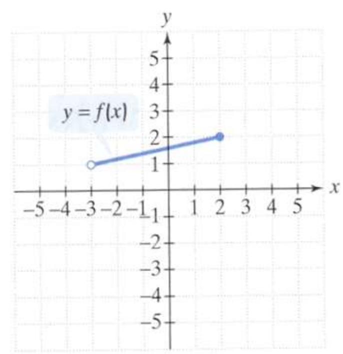
Answer each of the following questions. USE YOUR OWN PAPER FOR THIS ASSIGNMENT

For exercises 1-6, determine whether the relation is a function. Give the domain and range.

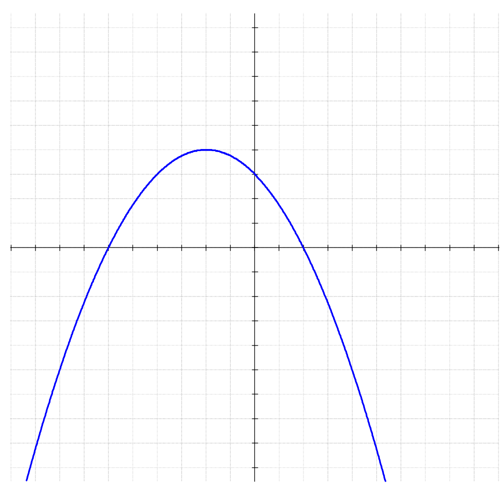
1. {(2,6), (1,4), (2,–6)}
2. {(0,1), (2,1), (3,4)}
3. 



5. 

6. 

Use the graph of *f* to solve exercises 7 – 22



7. Explain why *f* represents the graph of a function.
8. Find the domain of *f*
9. Find the range of *f*
10. What are the *x*-intercept(s)
11. What are the *y*-intercepts(s)
12. For what interval(s) is *f* increasing?
13. For what interval(s) is *f* decreasing?
14. At what number does *f* have a relative maximum?
15. What are the coordinates of the relative maximum?
16. What is the value of *f*(–4)?
17. For what value(s) of *x* is *f*(*x*) = –2?
18. For what value(s) of *x* is *f*(*x*) = 0?
19. For what values of *x* is *f*(*x*) > 0?
20. For what values of *x* is *f*(*x*) < 0?
21. If *f*(*x*) even, odd, or neither? Why or why not?
22. What is the average rate of change for *f* from *x* = –4 to *x* = 4?

On graph paper, graph exercises 23 – 27

23. $y = -2x$
24. $4x - 2y = 8$
25. $f(x) = |x| - 4$
26. $f(x) = x^2 - 4$

27. $f(x)=\begin{cases}-1, & x\le 0\\ 2x+1, & x>0\end{cases}$

28. Let $f(x) = -2x^2 + x - 4$

    a. Find $f(-x)$. Is *f* even, odd, or neither?

    b. Find $\frac{f(x+h)-f(x)}{h}, h \neq 0$

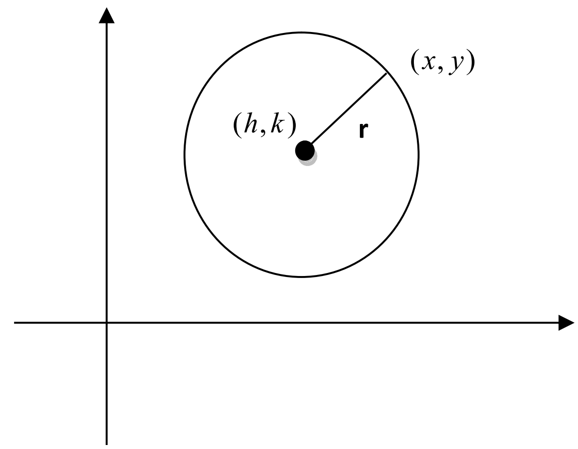
29. If $C(x)=\begin{cases}30, & 0\le t\le 200\\ 30+0.25(t-200), & t>200\end{cases}$. (a) Find *C*(150) (b) *C*(250)

For exercises 30-33, write the equation of the line, in slope-intercept form, that satisfies the following condition

30. Slope = 5, passes through (2,–1).
31. Passes through (4,2) and (–1,12)
32. Passes through (5,1) and is parallel to $4x - 3y - 1 = 0$
33. Passes through (–4, –3) and is perpendicular to $2x - y + 3 = 0$

34. Is the line that passes through (2,–4) and (3,2) parallel or perpendicular to the line passes through (–4,2) and (–3,8)?
35. What is the domain of $g(x) = \sqrt{5x - 10}$?
36. What is the domain of $f(x) = \frac{x-4}{x^2-4}$?
37. Find the domain of $h(x) = \frac{4}{\frac{3}{x}-1}$
38. Express $h(x) = \sqrt[3]{2x^2 + 3x - 6}$ as composite of two functions f and g.
39. Use the graph the graph of $f(x) = x^2$ to graph $g(x) = -f(x+1) - 3$
40. What is the domain of $f(x) = \frac{x^2-4}{x^2-3x+2}$?

Def: A circle is the set of all points (called a *locus*) that are equidistant from a fixed point, called the center. The fixed distance from the circle's center to any point on the circle is called the radius.



On a set of axes, the distance of the radius would be:

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

If you square both sides, you get:

$$(x-h)^2 + (y-k)^2 = r^2$$

This is the ***Standard Form for the Equation of a Circle***.

Ex. Write the standard form equation of a circle whose

1. center is (3,4) and radius is 4

$$(x-3)^2 + (y-4)^2 = 4^2$$
$$(x-3)^2 + (y-4)^2 = 16$$

2. center is (5,4) and passes thru (-1, -4)

$$r^2 = (5-(-1))^2 + (4-(-4))^2 = 6^2 + 8^2 = 100$$

$$(x-5)^2 + (y-4)^2 = 100$$

3. has a diameter whose endpoints are (-4, 3) and (4, 9)

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(4-(-4))^2 + (9-3)^2} = \sqrt{64+36} = 10 \rightarrow \quad r = 5 \quad \rightarrow r^2 = 25$$

Center is the midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-4+4}{2}, \frac{3+9}{2}\right) = (0,6)$

Equation is: $(x-0)^2 + (y-6)^2 = 25 \rightarrow x^2 + (y-6)^2 = 25$

Def: The ***General Form of the Equation of a Circle*** is

$x^2 + y^2 + Dx + Ey + F = 0$, where *D, E,* and *F* are real number.

Ex. Write the General Form equation of a circle whose

1. standard form is $(x-3)^2 + (y-2)^2 = 16$

$$x^2 - 6x + 9 + y^2 - 4x + 4 = 16$$
$$x^2 + y^2 - 6x - 4y - 3 = 0$$

2. Center is (5,6) and radius $\sqrt{8}$

$$(x-5)^2 + (y-6)^2 = (\sqrt{8})^2$$
$$x^2 - 10x + 25 + y^2 - 12x + 36 = 8$$
$$x^2 + y^2 - 10x - 12y + 53 = 0$$

Ex. What is the center and radius of a circle whose equation is $x^2 + y^2 + 4x - 6y - 23 = 0$?

Convert to standard form: Using Completing the Square

$x^2 + 4x + \_\_\_ + y^2 - 6y + \_\_\_ = 23 + \_\_\_ + \_\_\_$

$\frac{1}{2}(-4) = -2 \rightarrow +4$

$x^2 - 4x + 4 + y^2 - 6y + 9 = 23 + 4 + 9$

$\frac{1}{2}(-6) = -3 \rightarrow +9$

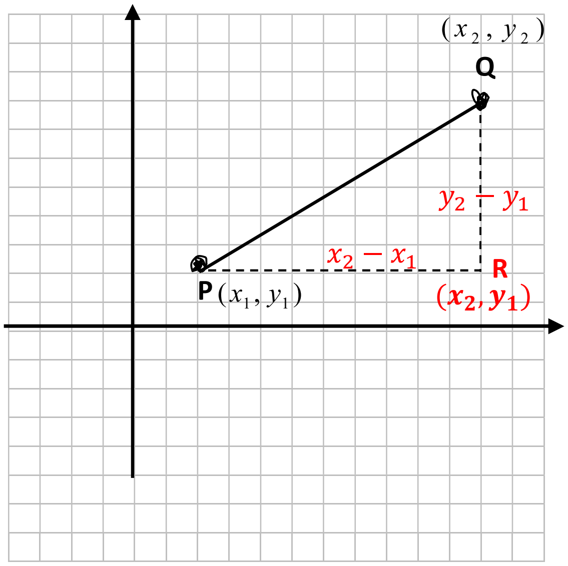
$(x-2)^2 + (y-3)^2 = 36$

Completing the square:
1. Take ½ of the linear term
2. Square it and add to both sides

Center is (2,3)
radius $= \sqrt{36} = 6$

Homework: Pg. 250 – 251 #3-18(3's), 19-27o, 31-39o, 43, 47, 51, 55, 57

1.9 Distance and Midpoint Formulas: Circles



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Suppose two points P and Q have coordinates (x_1, y_1) and (x_2, y_2) respectively.

Th: The distance, d, between points P and Q, is defined:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Proof:

Using the Pythagorean Theorem on Right ΔPRQ:

$$PQ^2 = PR^2 + QR^2$$
$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$PQ = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ex. Find the distance between the following points:

1. (4,2) and (8,5)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(8 - 4)^2 + (5 - 2)^2}$
$d = \sqrt{16 + 9} = \sqrt{25}$
$d = 5$

2. (-5,3) and (7,-2)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(7 + 5)^2 + (-2 - 3)^2}$
$d = \sqrt{144 + 25} = \sqrt{169}$
$d = 13$

3. (-2,4) and (2, -4)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(2 + 2)^2 + (-4 - 4)^2}$
$d = \sqrt{16 + 64} = \sqrt{80}$
$d = 4\sqrt{5}$

Th: Consider a line segment whose endpoints are (x_1, y_1) and (x_2, y_2) The coordinates of the segment's midpoint are:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Ex. Find the midpoint of the segment whose endpoints are

1. (4,2) and (10,8)

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$\left(\frac{4 + 10}{2}, \frac{2 + 8}{2}\right)$
$(7, 5)$

2. (5,1) and (10, 5)

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$\left(\frac{5 + 10}{2}, \frac{1 + 5}{2}\right)$
$\left(\frac{15}{2}, 3\right)$

3. (3, -2) and (-9, 7)

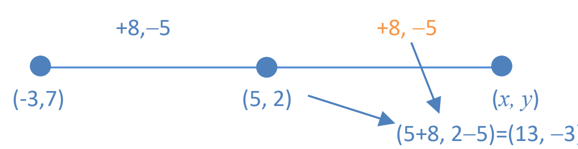
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$\left(\frac{3 - 9}{2}, \frac{-2 + 7}{2}\right)$
$\left(-3, \frac{5}{2}\right)$

Ex. If the midpoint of a segment is (5, 2) and one endpoint is (-3,7), find the other endpoint.

$x_m = \frac{x_1 + x_2}{2} \rightarrow 2x_m = x_1 + x_2 \rightarrow x_2 = 2x_m - x_1$ $\quad y_2 = 2y_m - y_1$

$x_2 = 2(5) - (-3) = 10 + 3 = 13$
$y_2 = 2(2) - 7 = 4 - 7 = -3$

Endpoint is $(13, -3)$



Exercise Set 1.9

Practice Exercises

In Exercises 1–18, find the distance between each pair of points. If necessary, round answers to two decimals places.

1. $(2, 3)$ and $(14, 8)$ **2.** $(5, 1)$ and $(8, 5)$

3. $(4, -1)$ and $(-6, 3)$ **4.** $(2, -3)$ and $(-1, 5)$

5. $(0, 0)$ and $(-3, 4)$ **6.** $(0, 0)$ and $(3, -4)$

7. $(-2, -6)$ and $(3, -4)$ **8.** $(-4, -1)$ and $(2, -3)$

9. $(0, -3)$ and $(4, 1)$ **10.** $(0, -2)$ and $(4, 3)$

11. $(3.5, 8.2)$ and $(-0.5, 6.2)$ **12.** $(2.6, 1.3)$ and $(1.6, -5.7)$

13. $(0, -\sqrt{3})$ and $(\sqrt{5}, 0)$

14. $(0, -\sqrt{2})$ and $(\sqrt{7}, 0)$

15. $(3\sqrt{3}, \sqrt{5})$ and $(-\sqrt{3}, 4\sqrt{5})$

16. $(2\sqrt{3}, \sqrt{6})$ and $(-\sqrt{3}, 5\sqrt{6})$

17. $\left(\frac{7}{3}, \frac{1}{5}\right)$ and $\left(\frac{1}{3}, \frac{6}{5}\right)$ **18.** $\left(-\frac{1}{4}, -\frac{1}{7}\right)$ and $\left(\frac{3}{4}, \frac{6}{7}\right)$

In Exercises 19–30, find the midpoint of each line segment with the given endpoints.

19. $(6, 8)$ and $(2, 4)$ **20.** $(10, 4)$ and $(2, 6)$

21. $(-2, -8)$ and $(-6, -2)$ **22.** $(-4, -7)$ and $(-1, -3)$

23. $(-3, -4)$ and $(6, -8)$ **24.** $(-2, -1)$ and $(-8, 6)$

25. $\left(-\frac{7}{2}, \frac{3}{2}\right)$ and $\left(-\frac{5}{2}, -\frac{11}{2}\right)$

26. $\left(-\frac{2}{5}, \frac{7}{15}\right)$ and $\left(-\frac{2}{5}, -\frac{4}{15}\right)$

27. $(8, 3\sqrt{5})$ and $(-6, 7\sqrt{5})$

28. $(7\sqrt{3}, -6)$ and $(3\sqrt{3}, -2)$

29. $(\sqrt{18}, -4)$ and $(\sqrt{2}, 4)$ **30.** $(\sqrt{50}, -6)$ and $(\sqrt{2}, 6)$

In Exercises 31–40, write the standard form of the equation of the circle with the given center and radius.

31. Center $(0, 0), r = 7$ **32.** Center $(0, 0), r = 8$

33. Center $(3, 2), r = 5$ **34.** Center $(2, -1), r = 4$

35. Center $(-1, 4), r = 2$ **36.** Center $(-3, 5), r = 3$

37. Center $(-3, -1), r = \sqrt{3}$ **38.** Center $(-5, -3), r = \sqrt{5}$

39. Center $(-4, 0), r = 10$ **40.** Center $(-2, 0), r = 6$

In Exercises 41–52, give the center and radius of the circle described by the equation and graph each equation. Use the graph to identify the relation's domain and range.

41. $x^2 + y^2 = 16$ **42.** $x^2 + y^2 = 49$

43. $(x - 3)^2 + (y - 1)^2 = 36$

44. $(x - 2)^2 + (y - 3)^2 = 16$

45. $(x + 3)^2 + (y - 2)^2 = 4$

46. $(x + 1)^2 + (y - 4)^2 = 25$

47. $(x + 2)^2 + (y + 2)^2 = 4$

48. $(x + 4)^2 + (y + 5)^2 = 36$

49. $x^2 + (y - 1)^2 = 1$ **50.** $x^2 + (y - 2)^2 = 4$

51. $(x + 1)^2 + y^2 = 25$ **52.** $(x + 2)^2 + y^2 = 16$

In Exercises 53–64, complete the square and write the equation in standard form. Then give the center and radius of each circle and graph the equation.

53. $x^2 + y^2 + 6x + 2y + 6 = 0$

54. $x^2 + y^2 + 8x + 4y + 16 = 0$

55. $x^2 + y^2 - 10x - 6y - 30 = 0$

56. $x^2 + y^2 - 4x - 12y - 9 = 0$

57. $x^2 + y^2 + 8x - 2y - 8 = 0$

58. $x^2 + y^2 + 12x - 6y - 4 = 0$

59. $x^2 - 2x + y^2 - 15 = 0$

60. $x^2 + y^2 - 6y - 7 = 0$

61. $x^2 + y^2 - x + 2y + 1 = 0$

62. $x^2 + y^2 + x + y - \frac{1}{2} = 0$

63. $x^2 + y^2 + 3x - 2y - 1 = 0$

64. $x^2 + y^2 + 3x + 5y + \frac{9}{4} = 0$

Practice Plus

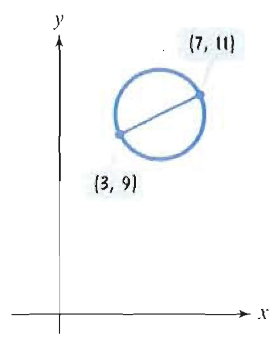
In Exercises 65–66, a line segment through the center of each circle intersects the circle at the points shown.

a. *Find the coordinates of the circle's center.*

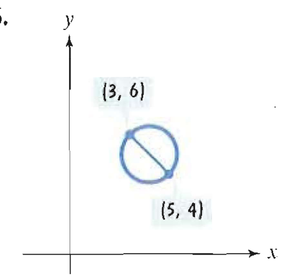
b. *Find the radius of the circle.*

c. *Use your answers from parts (a) and (b) to write the standard form of the circle's equation.*

65.



66.



In Exercises 67–70, graph both equations in the same rectangular coordinate system and find all points of intersection. Then show that these ordered pairs satisfy the equations.

67. $x^2 + y^2 = 16$
$x - y = 4$

68. $x^2 + y^2 = 9$
$x - y = 3$

69. $(x - 2)^2 + (y + 3)^2 = 4$
$y = x - 3$

70. $(x - 3)^2 + (y + 1)^2 = 9$
$y = x - 1$

Application Exercises

The cell phone screen shows coordinates of six cities from a rectangular coordinate system placed on North America by long-distance telephone companies. Each unit in this system represents $\sqrt{0.1}$ mile.

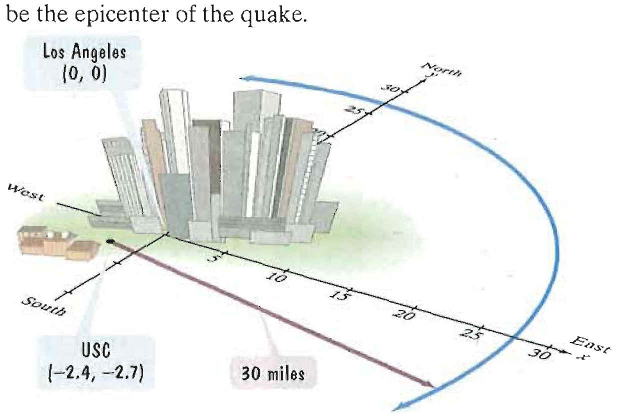
Source: Peter H. Dana

In Exercises 71–72, use this information to find the distance, to the nearest mile, between each pair of cities.

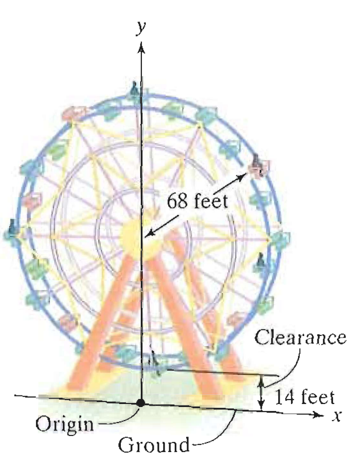
71. Boston and San Francisco

72. New Orleans and Houston

73. A rectangular coordinate system with coordinates in miles is placed with the origin at the center of Los Angeles. The figure indicates that the University of Southern California is located 2.4 miles west and 2.7 miles south of central Los Angeles. A seismograph on the campus shows that a small earthquake occurred. The quake's epicenter is estimated to be approximately 30 miles from the university. Write the standard form of the equation for the set of points that could be the epicenter of the quake.



74. The Ferris wheel in the figure has a radius of 68 feet. The clearance between the wheel and the ground is 14 feet. The rectangular coordinate system shown has its origin on the ground directly below the center of the wheel. Use the coordinate system to write the equation of the circular wheel.



Writing in Mathematics

75. In your own words, describe how to find the distance between two points in the rectangular coordinate system.

76. In your own words, describe how to find the midpoint of a line segment if its endpoints are known.

77. What is a circle? Without using variables, describe how the definition of a circle can be used to obtain a form of its equation.

78. Give an example of a circle's equation in standard form. Describe how to find the center and radius for this circle.

79. How is the standard form of a circle's equation obtained from its general form?

80. Does $(x - 3)^2 + (y - 5)^2 = 0$ represent the equation of a circle? If not, describe the graph of this equation.

81. Does $(x - 3)^2 + (y - 5)^2 = -25$ represent the equation of a circle? What sort of set is the graph of this equation?

82. Write and solve a problem about the flying time between a pair of cities shown on the cell phone screen for Exercises 71–72. Do not use the pairs in Exercise 71 or Exercise 72. Begin by determining a reasonable average speed, in miles per hour, for a jet flying between the cities.

Technology Exercises

In Exercises 83–85, use a graphing utility to graph each circle whose equation is given.

83. $x^2 + y^2 = 25$

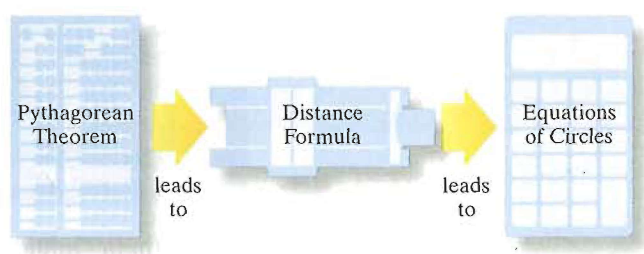
84. $(y + 1)^2 = 36 - (x - 3)^2$

85. $x^2 + 10x + y^2 - 4y - 20 = 0$

Critical Thinking Exercises

Make Sense? *In Exercises 86–89, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

86. I've noticed that in mathematics, one topic often leads logically to a new topic:



87. To avoid sign errors when finding h and k, I place parentheses around the numbers that follow the subtraction signs in a circle's equation.

88. I used the equation $(x + 1)^2 + (y - 5)^2 = -4$ to identify the circle's center and radius.

89. My graph of $(x - 2)^2 + (y + 1)^2 = 16$ is my graph of $x^2 + y^2 = 16$ translated two units right and one unit down.

In Exercises 90–93, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

90. The equation of the circle whose center is at the origin with radius 16 is $x^2 + y^2 = 16$.

91. The graph of $(x - 3)^2 + (y + 5)^2 = 36$ is a circle with radius 6 centered at $(-3, 5)$.

92. The graph of $(x - 4) + (y + 6) = 25$ is a circle with radius 5 centered at $(4, -6)$.

93. The graph of $(x - 3)^2 + (y + 5)^2 = -36$ is a circle with radius 6 centered at $(3, -5)$.

94. Show that the points $A(1, 1 + d)$, $B(3, 3 + d)$, and $C(6, 6 + d)$ are collinear (lie along a straight line) by showing that the distance from A to B plus the distance from B to C equals the distance from A to C.

95. Prove the midpoint formula by using the following procedure.

a. Show that the distance between (x_1, y_1) and $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is equal to the distance between (x_2, y_2) and $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

b. Use the procedure from Exercise 94 and the distances from part (a) to show that the points (x_1, y_1), $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, and (x_2, y_2) are collinear.

96. Find the area of the donut-shaped region bounded by the graphs of $(x - 2)^2 + (y + 3)^2 = 25$ and $(x - 2)^2 + (y + 3)^2 = 36$.

97. A **tangent line** to a circle is a line that intersects the circle at exactly one point. The tangent line is perpendicular to the radius of the circle at this point of contact. Write an equation in point-slope form for the line tangent to the circle whose equation is $x^2 + y^2 = 25$ at the point $(3, -4)$.

Preview Exercises

Exercises 98–100 will help you prepare for the material covered in the next section.

98. Write an algebraic expression for the fare increase if a \$200 plane ticket is increased to x dollars.

99. Find the perimeter and the area of each rectangle with the given dimensions:

a. 40 yards by 30 yards **b.** 50 yards by 20 yards.

100. Solve for h: $\pi r^2 h = 22$. Then rewrite $2\pi r^2 + 2\pi r h$ in terms of r.

Section 1.10 Modeling with Functions

Objectives

1. Construct functions from verbal descriptions.
2. Construct functions from formulas.

A can of Coca-Cola is sold every six seconds throughout the world.

In 2005, to curb consumption of sugared soda, the Center for Science in the Public Interest (CSPI) urged the FDA to slap cigarette-style warning labels on these drinks, calling them "liquid candy." Despite the variety of its reputations throughout the world, the soft drink industry has spent far more time reducing the amount of aluminum in its cylindrical cans than addressing the problems of the nutritional disaster floating within its packaging. In the 1960s, one pound of aluminum made fewer than 20 cans; today, almost 30 cans come out of the same amount. The thickness of the can wall is less than five-thousandths of an inch, about the same as a magazine cover.

Many real-world problems involve constructing mathematical models that are functions. The problem of minimizing the amount of aluminum needed to manufacture a 12-ounce soft-drink can first requires that we express the surface area of all such cans as a function of their radius. In constructing such a function, we must be able to translate a verbal description into a mathematical representation—that is, a mathematical model.

Study Tip

In calculus, you will solve problems involving maximum or minimum values of functions. Such problems often require creating the function that is to be maximized or minimized. Quite often, the calculus is fairly routine. It is the algebraic setting up of the function that causes difficulty. That is why the material in this section is so important.

Chapter 1: Functions and Graphs — Review Sheet

Things to Know

Def: A <u>function</u> is a relation such that each element in the domain corresponds to exactly ONE element in the range.

$f(x)$ is pronounced "f of x" and means "the value of function f at x".

<u>Evaluating a function</u>:
Given function $y = f(x)$, then $f(2)$ means the value of function when x = 2.

A graph is a function if it passes the ***Vertical Line Test (VLT)***
- The vertical line test means that if you draw vertical lines through the graph, that it would only cross through the graph 1 time.
- If the graph fails the VLT, then the relation is NOT a function.

<u>Identifying Domain and Range Using a Function's Graph</u>:
Suppose you have the graph on the right:

To find the domain – Look at the x-coordinates of the left most and right most points.

To find the range - Look at the y-coordinates of the lowest and highest points.

Def: A function is
1. <u>increasing</u> on an open interval *I*, if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.
2. <u>decreasing</u> on an open interval *I*, if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.
3. <u>constant</u> on an open interval *I*, if $f(x_1) = f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.

<u>Relative Extrema</u>: (Relative Maximums and Minimums)
1. A function has a <u>relative maximum</u> at *x=c* if the function changes from an increasing function to a decreasing function at *x = c*.
   - For all x around c, $f(x) < f(c)$
2. A function has a <u>relative minimum</u> at *x=c* if the function changes from a decreasing function to an increasing function at *x = c*.
   - For all x around c, $f(x) > f(c)$

<u>Intercepts</u>:
<u>Def</u>: An intercept is where a graph crosses an axis.
(a) The <u>*x*-intercept</u> is where the graph crosses the *x*-axis.
- Has coordinates (*a,* 0)

(b) The y<u>-intercept</u> is where the graph crosses the *y*-axis
- Has coordinates (0, *b*)

<u>To find the x-intercept</u>: Set the y value equal to 0 and solve for x.

<u>To find the y-intercept</u>: Set the x value equal to 0 and solve for y.

<u>Def</u>: A function $y = f(x)$ is
1. an <u>even function</u> if $f(-x) = f(x)$ for all *x* in the domain of *f.*
   - This means if you plug in $-x$ in for x and simplify, you get the original function $f(x)$
   - Even functions are symmetric about the *y*-axis (y-axis divides the function into mirror images)
2. an <u>odd function</u> if $f(-x) = -f(x)$ for all *x* in the domain of *f.*
   - This mean if you plug in $-x$ in for x and simplify, you get the original function $f(x)$ with all the signs changed.
   - Odd functions are symmetric about the origin (it looks the same upside down as right side up)

<u>Def</u>: A function that is defined by two or more equations over a specified domain is called a <u>piecewise function</u>.

Ex.

$$y = \begin{cases} x^2 - 1, & x < 1 \\ 2x - 2, & x \geq 1 \end{cases}$$

For *x* < 1, you use the first expression, for all other values of *x*, use the 2nd expression (2*x*-2)

<u>Def</u>: The <u>slope</u> of a line through distinct points (x_1, y_1) and (x_2, y_2) is

$$\frac{\Delta y}{\Delta x} = \frac{change\, in\, y}{change\, in\, x} = \frac{y_2 - y_1}{x_2 - x_1}$$

this is also referred to as $\frac{"rise"}{"run"}$

provided that $x_1 \neq x_2$

<u>There are 4 Kinds of Sloped Lines</u>:
1. Positive Slope – as you move from left to right, the line goes up
2. Negative Slope – as you move from left to right, the line goes down
3. Zero Slope – horizontal line
4. No Slope – vertical line

<u>Property</u>: 2 Lines are <u>parallel</u> if they have the <u>same slope</u>.

$$m_1 = m_2$$

<u>Property</u>: 2 lines are <u>perpendicular</u> if their slopes are <u>negative reciprocals</u>.

$$m_1 = -\frac{1}{m_2}$$

<u>Equations of Lines</u>: There are several different forms of equations for lines.
1. <u>Point-Slope Form</u>: For a non-vertical line whose slope is *m* and passes through a point (x_1, y_1)

   $$y - y_1 = m(x - x_1)$$
2. <u>Slope-Intercept Form</u>: For a non-vertical line with slope *m* and *y*-intercept *b*

   $$y = mx + b$$
3. <u>Equation of a Horizontal Line</u>: All horizontal line are of the form $y = k$, where *k* is any real number

   Ex. *y* = 4   *y* = -2   *y* = 0
4. <u>Equation of a Vertical Line</u>: All vertical line are of the form $x = k$, where *k* is any real number

   Ex. *x* = 8   *x*= -5   *x* = 0
5. <u>General Form of a Line</u>: Every line has an equation that can be written in the general form: $Ax + By + C = 0$

Def: **The Average Rate of Change of a Function**

Let $(x_1, f(x_1))$ and $(x_2, f(x_2))$ be two distinct points on the graph of $y = f(x)$.

The **average rate of change** of *f* from x_1 to x_2, denoted by

$\frac{\Delta y}{\Delta x}$ ("delta y over delta x" = "change in y over the change in x") is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Def: Suppose an object's position is expressed by the function $s(t)$, where t is time. The **average velocity** of the object from time t_1 to t_2 is

$$\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

TRANSFORMATIONS OF FUNCTIONS

Vertical Shift

- A vertical shift occurs to a standard graph, when the graph "shifts" up or down.

Def: A function $f(x)$ will have a vertical shift into a new function $g(x)$ if for any real number *k*: $g(x) = f(x) + k$

- The curve will have the exact same shape, it will just be higher or lower than its original form.
- The shift will be upward if $k > 0$
- The shift will be downward if $k < 0$

Horizontal Shift

- A horizontal shift to a standard graph when the graph "shifts" left or right.

Def: A function $f(x)$ will have a horizontal shift into a new function $g(x)$ if for any real number *k*: $g(x) = f(x + k)$

- The curve will have the exact same shape, it will just be left or right than its original form.
- The shift will be to the right *k* units if $k < 0$
- The shift will be to the left *k* units if $k > 0$

Reflections of Graphs

Def: The graph of $y = -f(x)$ is the graph of the $y = f(x)$ reflected over the *x*-axis.

- Basically: The *y*-coordinate changes sign: $(x, y) \rightarrow (x, -y)$

Def: The graph of $y = f(-x)$ is the graph of the reflection of $y = f(x)$ over the *y*-axis.

- Basically: The *x*-coordinate changes sign: $(x, y) \rightarrow (-x, y)$

Dilations: Stretching and Shrinking

Def: Vertical Stretching and Shrinking

Let *f* be a function and *c* be a positive real number.

1. If $c > 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically stretched by multiplying each of the *y*-coordinates by *c*.
2. If $0 < c < 1$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ vertically shrunk by multiplying each of the *y*-coordinates by *c*.

Def: Horizontal Stretching and Shrinking

Let *f* be a function and *c* be a positive real number.

1. If $c > 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ horizontally shrunk by dividing each of the *x*-coordinates by *c*.
2. If $0 < c < 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ horizontally stretched by dividing each of the *x*-coordinates by *c*.

A function can be transformed with more than one transformation if you perform them in the following order:

1. Horizontal Shift
2. Stretching or shrinking
3. Reflection
4. Vertical Shift

Algebra of Functions: Let *f* and *g* be two functions. The following functions are defined:

1. Sum of 2 functions: $(f + g)(x) = f(x) + g(x)$
2. Difference of 2 functions: $(f - g)(x) = f(x) - g(x)$
3. Product of 2 functions: $(fg)(x) = f(x) \cdot g(x)$
4. Quotient of 2 functions: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$

*** In all cases, the domains will be the set of real numbers common to the domains of *f* and *g*: $D_f \cap D_g$

Finding Domain: If you do not have the graph of a function $y = f(x)$

- To find the domain of a function, you are answering the following question: *What are all the possible values of x?*
  - Or better: *What values CAN'T x be?*
- How do you find domain?
  1. Assume (unless stated) the domain is All Real values
  2. If *x* is in the bottom of a fraction, then set the bottom ≠ 0 and solve for *x*
  3. If *x* is in a radical (even root), then set the inside of the radical ≥ 0 and solve for *x*
  4. If *x* is in a radical which is the bottom of a fraction, then set the inside of the radical > 0 and solve for *x*

Composite Functions:

Let *f* and *g* be functions, then the composite of *f* with *g*, written $(f \circ g)(x)$ is denoted by: $(f \circ g)(x) = f[g(x)] = f(g(x))$

** The domain of the composite function is the set of all *x* such that

1. *x* is in the domain of *g*
2. $g(x)$ is in the domain of *f*.

Circles:

Standard Form for the Equation of a Circle: $(x-h)^2 + (y-k)^2 = r^2$

Center: (h, k) radius = r

General Form for a Circle: $x^2 + y^2 + Dx + Ey + F = 0$

Convert from General to Standard:

1. Group x's and y's and move constant to the other side (set up Completing the square)
2. Complete the square for both x and y (add the constants to the other side as well)
3. Write the quadratics in factored form: i.e. $(x-h)^2$

Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Def: A function defined by reversing a one-to-one function *f* is the **inverse of *f***.

Notation $f^{-1}(x)$

- Graphically: The inverse of a function $y = f(x)$ is the reflection over the line $y = x$
- Algebraically: The inverse satisfies

$$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$$

To find an Inverse: Change $f(x)$ to y

1. Interchange x and y in the original equation
2. Solve for y which is $f^{-1}(x)$

Property:

$D_f = R_{f^{-1}}$

$R_f = D_{f^{-1}}$

Name____ Chapter 1 Sample Test

Precalculus – Period_______ Form A

A. Answer each of the following questions. Place your answers in the space provided on the right. [2 points each]

1. Given $f(x) = x^2 - x - 4$, find $f(-2)$. 1.____________

2. Is the function $f(x) = x^2 - 8$ an even function, odd function, or neither? 2.____________

3. What is the distance between the points whose coordinates are (5, 3) and (-3, 18)? 3.____________

4. What is the equation of the line perpendicular to $3x - 4y + 6 = 0$ and passes through (1,1)? 4.____________

5. What is the domain of $f(x) = \frac{x^2-4}{x^2-3x+2}$? 5.____________

6. If $f(x) = x^2 - 3x$ and $g(x) = 2x + 1$, then find $(f \circ g)(2)$ 6.____________

7. If $f(x) = 5 + x^2$ and $g(x) = x^2 - 1$, then what is the value of $(f + g)(2)$? 7.____________

8. What is the slope of the line that passes through (2,1) and (-1,7)? 8.____________

9. What is the domain of $g(x) = \sqrt{5x - 10}$? 9.____________

10. What is the inverse of $f(x) = 3x - 2$? 10.____________

11. If $f(x) = 4x^2 - 3x$, the what is $f(x + 1)$ in simplest form? 11.____________

12. Express $h(x) = \sqrt[3]{2x^2 + 3x - 6}$ as composite of two functions f and g. 12. f(x)=_________

g(x)=_________

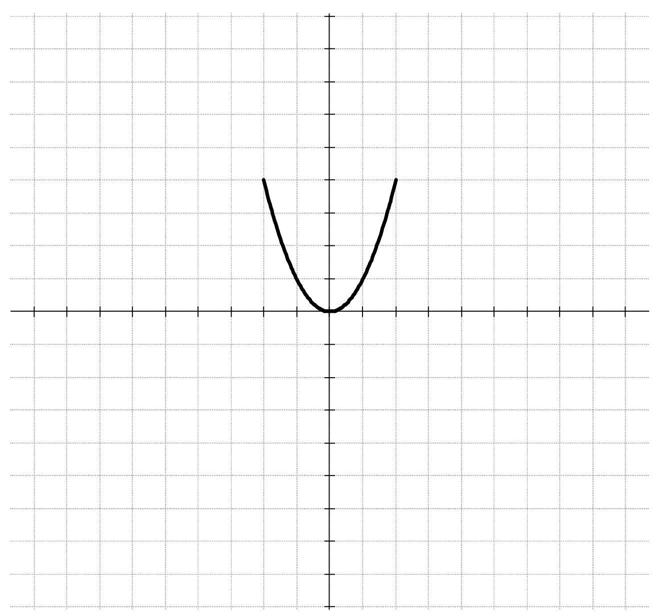
B. Answer each of the following questions. Show all work for partial credit.

13. What is the *y*-intercept of the line perpendicular to $4x - 5y + 8 = 0$ that passes through (1,1)?

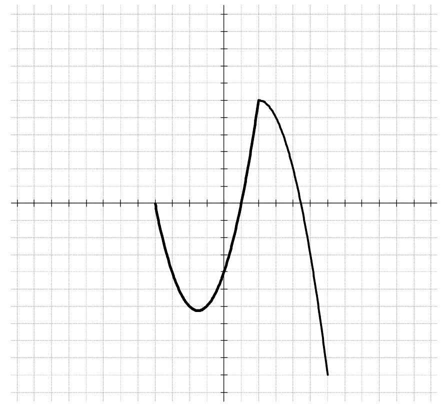
14. On the set of axes, the graph of $f(x) = x^2$ is shown.

(a) Use the graph to graph $g(x) = -f(x + 1) - 3$

(b) What is the equation of $g(x)$?



15. Consider the graph of $y = f(x)$. Answer each of the following questions based on the graph.



(a) What is the range of *f* ?
(b) On what interval is *f* increasing?
(c) On what interval is *f* decreasing?
(d) What are the coordinates of the relative maximum?
(e) What are the coordinates of the relative minimum?
(f) What are the *x* intercept(s)?
(g) What are the *y* intercept(s)?

16. Consider the function $f(x) = \frac{x-3}{x+2}$.

(a) What is the domain?
(b) Find $f^{-1}(x)$
(c) What is the domain of $f^{-1}(x)$?
(d) What is the range of *f* ?

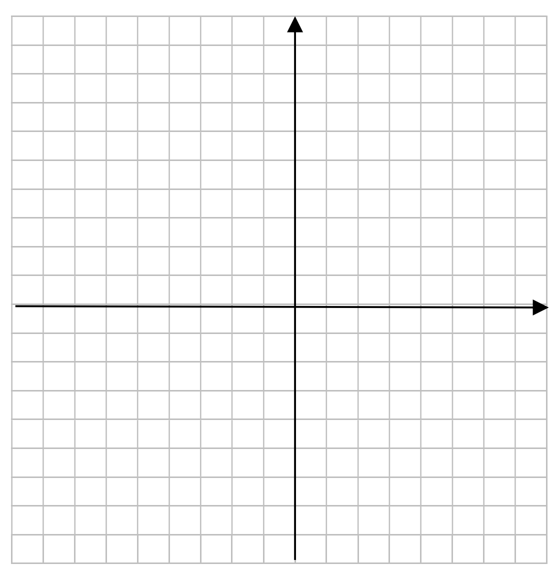
17. Consider the circle whose general form equation is $x^2 + y^2 + 4x - 6y - 3 = 0$

(a) Write the equation of the circle in Standard Form.
(b) What is the center of the circle?
(c) What is the radius of the circle?
(d) On the set of axes, draw, to the best of your ability, the circle.

(a)____________________

(b)____________________

(c)____________________



Use the graph of g to solve Exercises 71–76.

71. Find $g(-4)$.

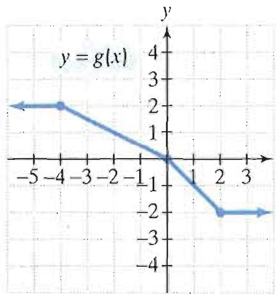
72. Find $g(2)$.

73. Find $g(-10)$.

74. Find $g(10)$.

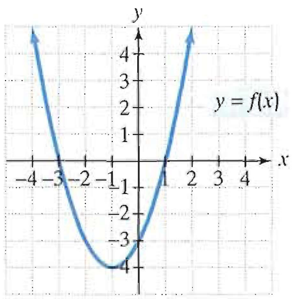
75. For what value of x is $g(x) = 1$?

76. For what value of x is $g(x) = -1$?



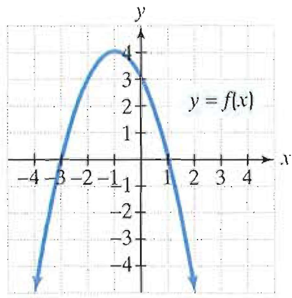
In Exercises 77–92, use the graph to determine **a.** *the function's domain;* **b.** *the function's range;* **c.** *the x-intercepts, if any;* **d.** *the y-intercept, if any; and* **e.** *the missing function values, indicated by question marks, below each graph.*

77.



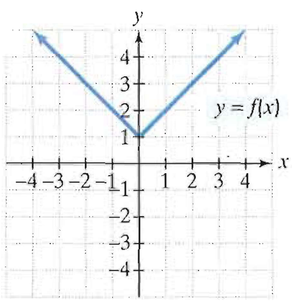
$f(-2) = ?$ $f(2) = ?$

78.



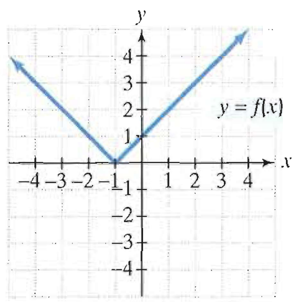
$f(-2) = ?$ $f(2) = ?$

79.



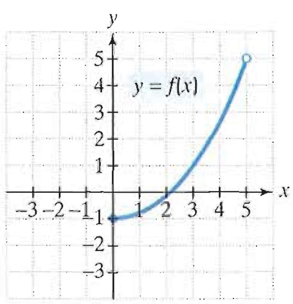
$f(-1) = ?$ $f(3) = ?$

80.



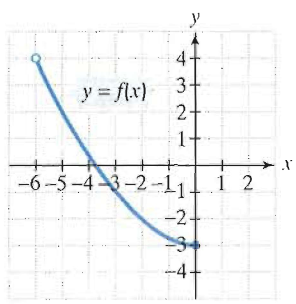
$f(-4) = ?$ $f(3) = ?$

81.



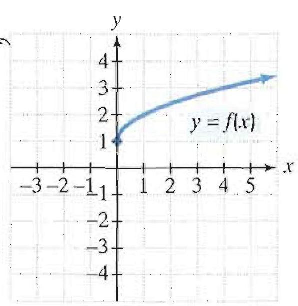
$f(3) = ?$

82.



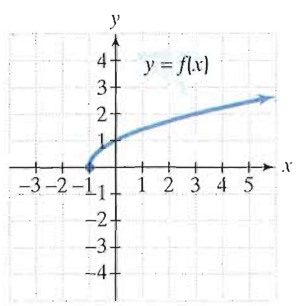
$f(-5) = ?$

83.



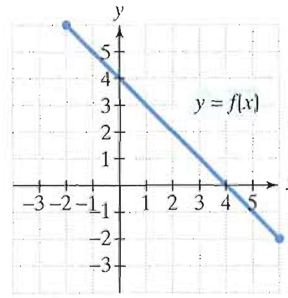
$f(4) = ?$

84.



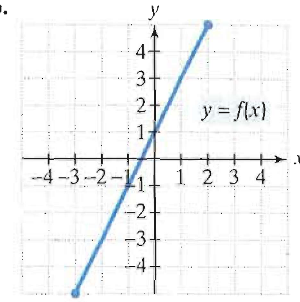
$f(3) = ?$

85.



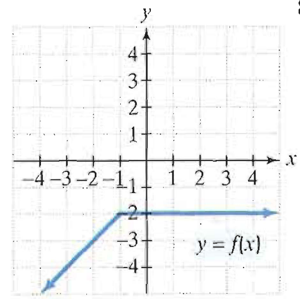
$f(-1) = ?$

86.



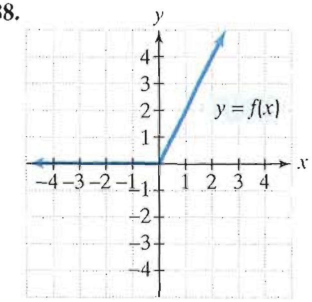
$f(-2) = ?$

87.



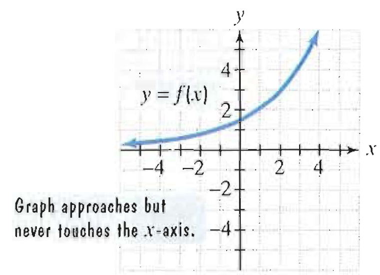
$f(-4) = ?$ $f(4) = ?$

88.



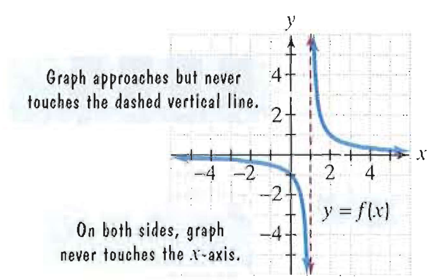
$f(-2) = ?$ $f(2) = ?$

89.



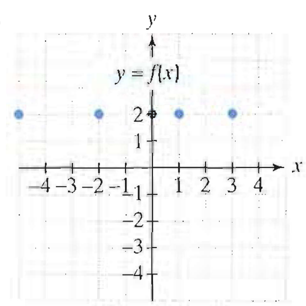
$f(4) = ?$

90.



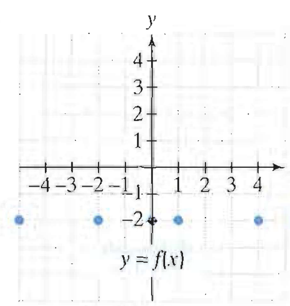
$f(2) = ?$

91.



$f(-5) + f(3) = ?$

92.



$f(-5) + f(4) = ?$

Practice Plus

In Exercises 93–94, let $f(x) = x^2 - x + 4$ *and* $g(x) = 3x - 5$.

93. Find $g(1)$ and $f(g(1))$. **94.** Find $g(-1)$ and $f(g(-1))$.

In Exercises 95–96, let f and g be defined by the following table:

x	$f(x)$	$g(x)$
−2	6	0
−1	3	4
0	−1	1
1	−4	−3
2	0	−6

95. Find $\sqrt{f(-1) - f(0)} - [g(2)]^2 + f(-2) \div g(2) \cdot g(-1)$.

96. Find $|f(1) - f(0)| - [g(1)]^2 + g(1) \div f(-1) \cdot g(2)$.

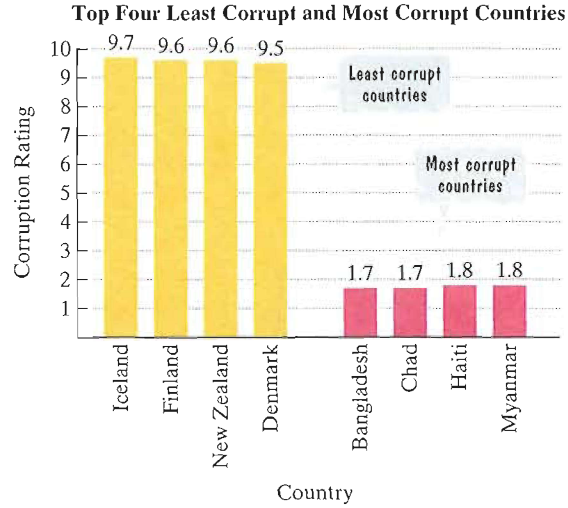
In Exercises 97–98, find $f(-x) - f(x)$ *for the given function* f. *Then simplify the expression.*

97. $f(x) = x^3 + x - 5$ **98.** $f(x) = x^2 - 3x + 7$

Application Exercises

The Corruption Perceptions Index uses perceptions of the general public, business people, and risk analysts to rate countries by how likely they are to accept bribes. The ratings are on a scale from 0 to 10, where higher scores represent less corruption. The graph shows the corruption ratings for the world's least corrupt and most corrupt countries. (The rating for the United States is 7.6.) Use the graph to solve Exercises 99–100.

Top Four Least Corrupt and Most Corrupt Countries



Source: Transparency International, *Corruption Perceptions Index*

99. Use the four least corrupt countries to solve this exercise.

a. Write a set of four ordered pairs in which countries correspond to corruption ratings. Each ordered pair should be in the form

(country, corruption rating).

b. Is the relation in part (a) a function? Explain your answer.

c. Write a set of four ordered pairs in which corruption ratings correspond to countries. Each ordered pair should be in the form

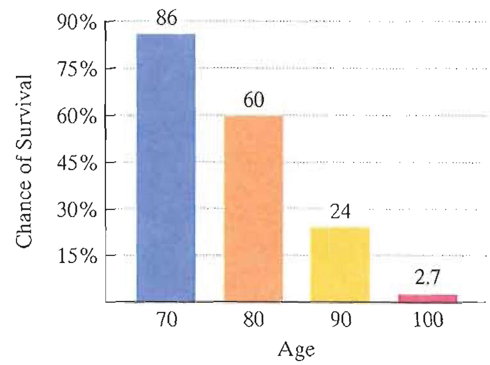
(corruption rating, country).

d. Is the relation in part (c) a function? Explain your answer.

100. Repeat parts (a) through (d) in Exercise 99 using the four most corrupt countries.

The bar graph shows your chances of surviving to various ages once you reach 60.

Chances of 60-Year-Olds Surviving to Various Ages



Source: National Center for Health Statistics

The functions

$$f(x) = -2.9x + 286$$
$$\text{and} \quad g(x) = 0.01x^2 - 4.9x + 370$$

model the chance, as a percent, that a 60-year-old will survive to age x. Use this information to solve Exercises 101–102.

101. a. Find and interpret $f(70)$. **b.** Find and interpret $g(70)$.

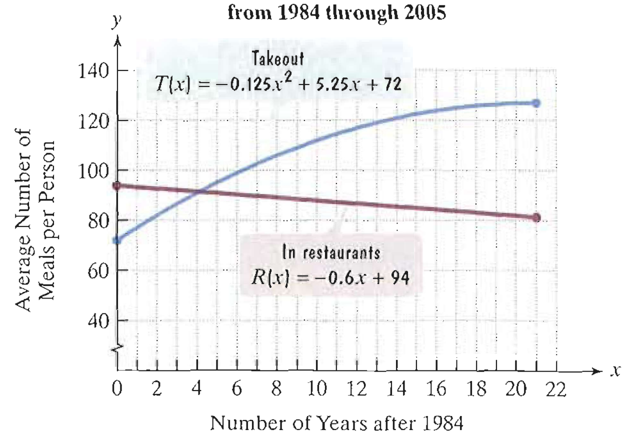
c. Which function serves as a better model for the chance of surviving to age 70?

102. a. Find and interpret $f(90)$. **b.** Find and interpret $g(90)$.

c. Which function serves as a better model for the chance of surviving to age 90?

To go, please. The graphs show that more Americans are ordering their food to go, instead of dining inside restaurants. A quadratic function models the average number of meals per person that Americans ordered for takeout and a linear function models the average number of meals ordered for eating in restaurants. In each model, x represents the number of years after 1984. Use the graphs and the displayed equations to solve Exercises 103–104.

Models for Average Number of Meals per Person Americans Ordered from 1984 through 2005



Source: NPD Group

(In Exercises 103–104, refer to the graphs and their equations at the bottom of the previous page.)

103. **a.** Use the equation for function T to find and interpret $T(20)$. How is this shown on the graph of T?

b. Use the equation for function R to find and interpret $R(0)$. How is this shown on the graph of R?

c. According to the graphs, in which year did the average number of takeout orders approximately equal the average number of in-restaurant orders? Use the equations for T and R to find the average number of meals per person for each kind of order in that year.

104. **a.** Use the equation for function T to find and interpret $T(18)$. How is this shown on the graph of T?

b. Use the equation for function R to find and interpret $R(20)$. How is this shown on the graph of R?

In Exercises 105–108, you will be developing functions that model given conditions.

105. A company that manufactures bicycles has a fixed cost of \$100,000. It costs \$100 to produce each bicycle. The total cost for the company is the sum of its fixed cost and variable costs. Write the total cost, C, as a function of the number of bicycles produced, x. Then find and interpret $C(90)$.

106. A car was purchased for \$22,500. The value of the car decreased by \$3200 per year for the first six years. Write a function that describes the value of the car, V, after x years, where $0 \le x \le 6$. Then find and interpret $V(3)$.

107. You commute to work a distance of 40 miles and return on the same route at the end of the day. Your average rate on the return trip is 30 miles per hour faster than your average rate on the outgoing trip. Write the total time, T, in hours, devoted to your outgoing and return trips as a function of your rate on the outgoing trip, x. Then find and interpret $T(30)$. Hint:

$$\text{Time traveled} = \frac{\text{Distance traveled}}{\text{Rate of travel}}.$$

108. A chemist working on a flu vaccine needs to mix a 10% sodium-iodine solution with a 60% sodium-iodine solution to obtain a 50-milliliter mixture. Write the amount of sodium iodine in the mixture, S, in milliliters, as a function of the number of milliliters of the 10% solution used, x. Then find and interpret $S(30)$.

Writing in Mathematics

109. What is a relation? Describe what is meant by its domain and its range.

110. Explain how to determine whether a relation is a function. What is a function?

111. How do you determine if an equation in x and y defines y as a function of x?

112. Does $f(x)$ mean f times x when referring to a function f? If not, what does $f(x)$ mean? Provide an example with your explanation.

113. What is the graph of a function?

114. Explain how the vertical line test is used to determine whether a graph represents a function.

115. Explain how to identify the domain and range of a function from its graph.

116. For people filing a single return, federal income tax is a function of adjusted gross income because for each value of adjusted gross income there is a specific tax to be paid. By contrast, the price of a house is not a function of the lot size on which the house sits because houses on same-sized lots can sell for many different prices.

a. Describe an everyday situation between variables that is a function.

b. Describe an everyday situation between variables that is not a function.

Technology Exercise

117. Use a graphing utility to verify any five pairs of graphs that you drew by hand in Exercises 39–54.

Critical Thinking Exercises

Make Sense? *In Exercises 118–121, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

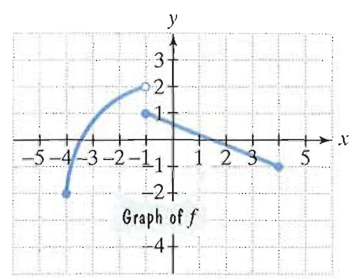
118. My body temperature is a function of the time of day.

119. Using $f(x) = 3x + 2$, I found $f(50)$ by applying the distributive property to $(3x + 2)50$.

120. I graphed a function showing how paid vacation days depend on the number of years a person works for a company. The domain was the number of paid vacation days.

121. I graphed a function showing how the average number of annual physician visits depends on a person's age. The domain was the average number of annual physician visits.

Use the graph of f to determine whether each statement in Exercises 122–125 is true or false.



122. The domain of f is $[-4, -1) \cup (-1, 4]$.

123. The range of f is $[-2, 2]$.

124. $f(-1) - f(4) = 2$

125. $f(0) = 2.1$

126. If $f(x) = 3x + 7$, find $\dfrac{f(a + h) - f(a)}{h}$.

127. Give an example of a relation with the following characteristics: The relation is a function containing two ordered pairs. Reversing the components in each ordered pair results in a relation that is not a function.

128. If $f(x + y) = f(x) + f(y)$ and $f(1) = 3$, find $f(2), f(3)$, and $f(4)$. Is $f(x + y) = f(x) + f(y)$ for all functions?

Preview Exercises

Exercises 129–131 will help you prepare for the material covered in the next section.

129. The function $C(t) = 20 + 0.40(t - 60)$ describes the monthly cost, $C(t)$, in dollars, for a cellular phone plan for t calling minutes, where $t > 60$. Find and interpret $C(100)$.

130. Use point plotting to graph $f(x) = x + 2$ if $x \le 1$.

131. Simplify: $2(x + h)^2 + 3(x + h) + 5 - (2x^2 + 3x + 5)$.

Solution

a. We find $f(x+h)$ by replacing x with $x+h$ each time that x appears in the equation.

$$f(x) = 2x^2 \quad - \quad x \quad + \quad 3$$

Replace x with $x+h$. | Replace x with $x+h$. | Replace x with $x+h$. | Copy the 3. There is no x in this term.

$$\begin{aligned} f(x+h) &= 2(x+h)^2 - (x+h) + 3 \\ &= 2(x^2+2xh+h^2) - x - h + 3 \\ &= 2x^2 + 4xh + 2h^2 - x - h + 3 \end{aligned}$$

b. Using our result from part (a), we obtain the following:

This is $f(x+h)$ from part (a). | This is $f(x)$ from the given equation.

$$\frac{f(x+h)-f(x)}{h} = \frac{\boxed{2x^2+4xh+2h^2-x-h+3} - (2x^2-x+3)}{h}$$

$$= \frac{2x^2+4xh+2h^2-x-h+3-2x^2+x-3}{h}$$ Remove parentheses and change the sign of each term in the parentheses.

$$= \frac{(2x^2-2x^2)+(-x+x)+(3-3)+4xh+2h^2-h}{h}$$ Group like terms.

$$= \frac{4xh+2h^2-1h}{h}$$ Simplify.

We wrote $-h$ as $-1h$ to avoid possible errors in the next factoring step.

$$= \frac{h(4x+2h-1)}{h}$$ Factor h from the numerator.

$$= 4x+2h-1$$ Divide out identical factors of h in the numerator and denominator.

Check Point 5 If $f(x) = -2x^2 + x + 5$, find and simplify each expression:

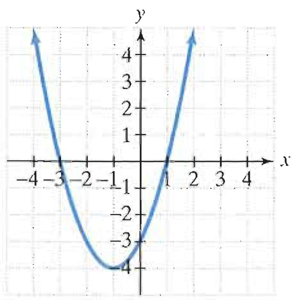
a. $f(x+h)$ **b.** $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$.

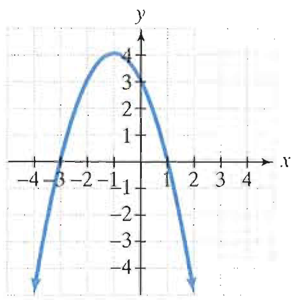
Exercise Set 1.3

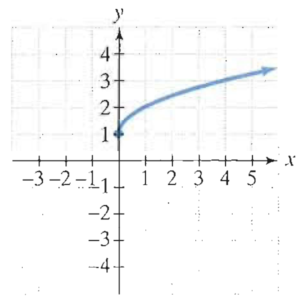
Practice Exercises

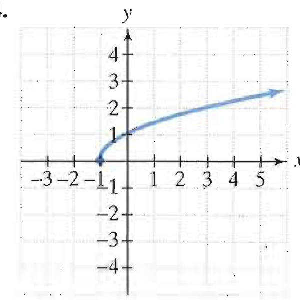
In Exercises 1–12, use the graph to determine

a. *intervals on which the function is increasing, if any.*
b. *intervals on which the function is decreasing, if any.*
c. *intervals on which the function is constant, if any.*

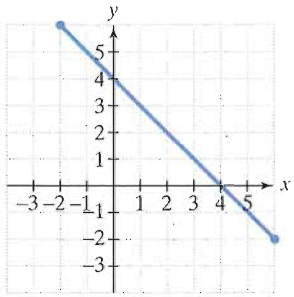
1.


2.


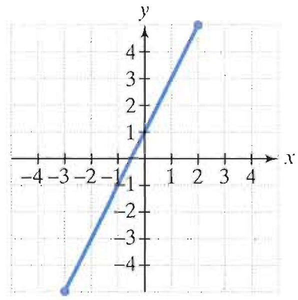
3.


4.


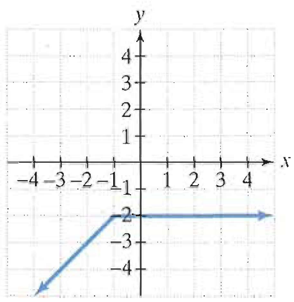
5.



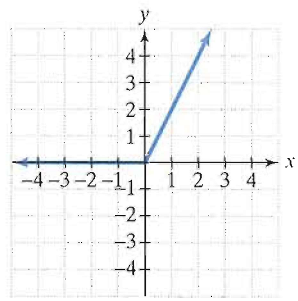
6.



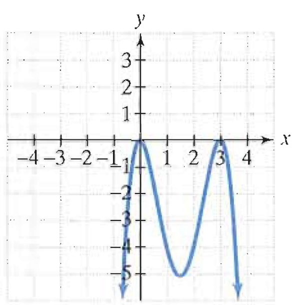
7.



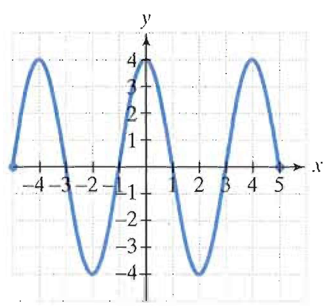
8.



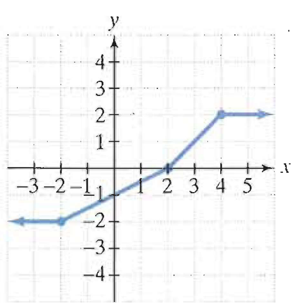
9.



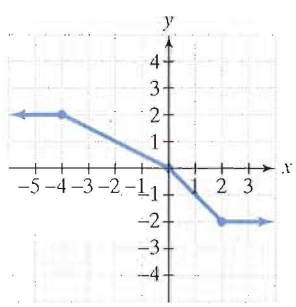
10.



11.



12.

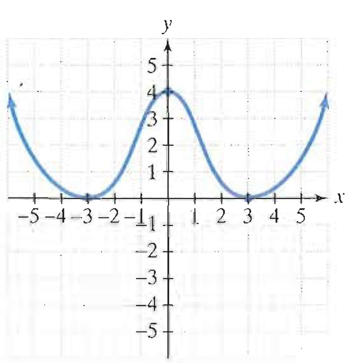


In Exercises 13–16, the graph of a function f is given. Use the graph to find each of the following:

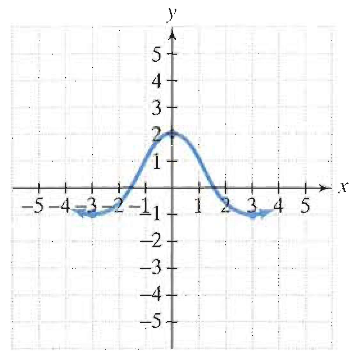
a. *The numbers, if any, at which f has a relative maximum. What are these relative maxima?*

b. *The numbers, if any, at which f has a relative minimum. What are these relative minima?*

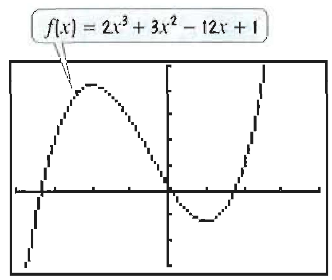
13.



14.

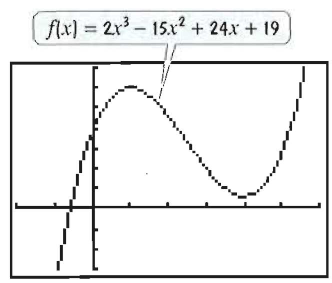


15.



[−4, 4, 1] by [−15, 25, 5]

16.



[−2, 6, 1] by [−15, 35, 5]

In Exercises 17–28, determine whether each function is even, odd, or neither.

17. $f(x) = x^3 + x$

18. $f(x) = x^3 - x$

19. $g(x) = x^2 + x$

20. $g(x) = x^2 - x$

21. $h(x) = x^2 - x^4$

22. $h(x) = 2x^2 + x^4$

23. $f(x) = x^2 - x^4 + 1$

24. $f(x) = 2x^2 + x^4 + 1$

25. $f(x) = \frac{1}{5}x^6 - 3x^2$

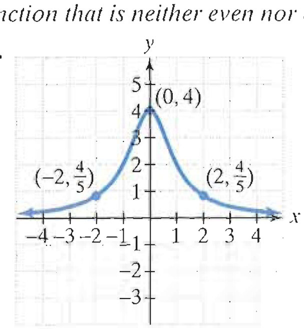
26. $f(x) = 2x^3 - 6x^5$

27. $f(x) = x\sqrt{1 - x^2}$

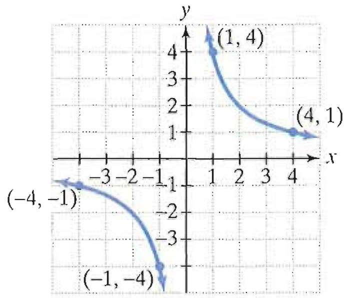
28. $f(x) = x^2\sqrt{1 - x^2}$

In Exercises 29–32, use possible symmetry to determine whether each graph is the graph of an even function, an odd function, or a function that is neither even nor odd.

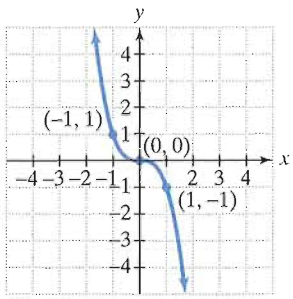
29.



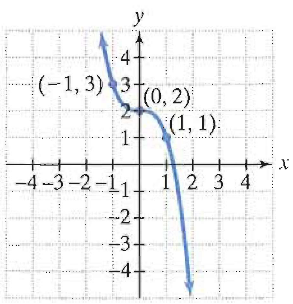
30.



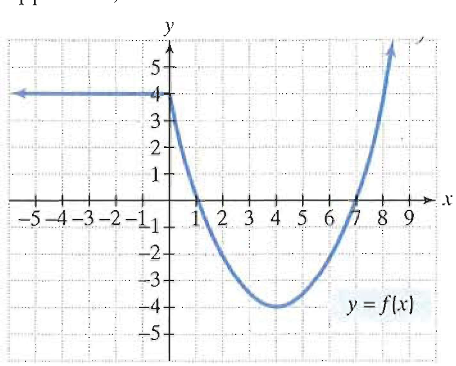
31.



32.



33. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the x-intercepts

d. the y-intercept

e. intervals on which f is increasing

f. intervals on which f is decreasing

g. intervals on which f is constant

h. the number at which f has a relative minimum

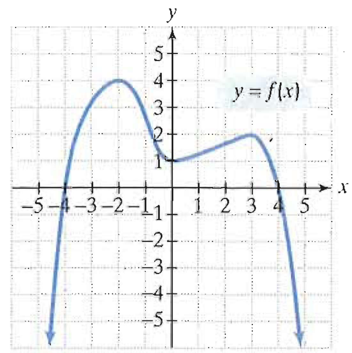
i. the relative minimum of f

j. $f(-3)$

k. the values of x for which $f(x) = -2$

l. Is f even, odd, or neither?

34. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the x-intercepts

d. the y-intercept

e. intervals on which f is increasing

f. intervals on which f is decreasing

g. values of x for which $f(x) \leq 0$

h. the numbers at which f has a relative maximum

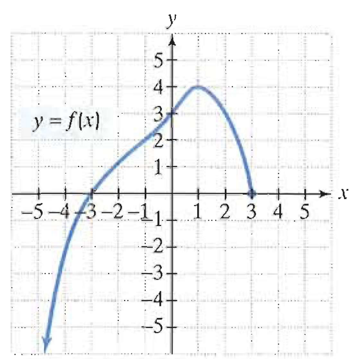
i. the relative maxima of f

j. $f(-2)$

k. the values of x for which $f(x) = 0$

l. Is f even, odd, or neither?

35. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the zeros of f

d. $f(0)$

e. intervals on which f is increasing

f. intervals on which f is decreasing

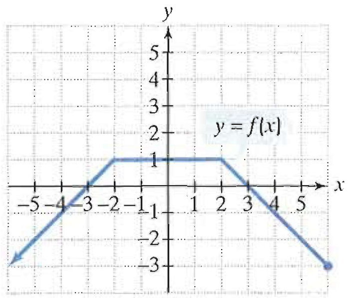
g. values of x for which $f(x) \leq 0$

h. any relative maxima and the numbers at which they occur

i. the value of x for which $f(x) = 4$

j. Is $f(-1)$ positive or negative?

36. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f
b. the range of f
c. the zeros of f
d. $f(0)$
e. intervals on which f is increasing
f. intervals on which f is decreasing
g. intervals on which f is constant
h. values of x for which $f(x) > 0$
i. values of x for which $f(x) = -2$
j. Is $f(4)$ positive or negative?
k. Is f even, odd, or neither?
l. Is $f(2)$ a relative maximum?

In Exercises 37–42, evaluate each piecewise function at the given values of the independent variable.

37. $f(x) = \begin{cases} 3x + 5 & \text{if } x < 0 \\ 4x + 7 & \text{if } x \geq 0 \end{cases}$

a. $f(-2)$ **b.** $f(0)$ **c.** $f(3)$

38. $f(x) = \begin{cases} 6x - 1 & \text{if } x < 0 \\ 7x + 3 & \text{if } x \geq 0 \end{cases}$

a. $f(-3)$ **b.** $f(0)$ **c.** $f(4)$

39. $g(x) = \begin{cases} x + 3 & \text{if } x \geq -3 \\ -(x + 3) & \text{if } x < -3 \end{cases}$

a. $g(0)$ **b.** $g(-6)$ **c.** $g(-3)$

40. $g(x) = \begin{cases} x + 5 & \text{if } x \geq -5 \\ -(x + 5) & \text{if } x < -5 \end{cases}$

a. $g(0)$ **b.** $g(-6)$ **c.** $g(-5)$

41. $h(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

a. $h(5)$ **b.** $h(0)$ **c.** $h(3)$

42. $h(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5} & \text{if } x \neq 5 \\ 10 & \text{if } x = 5 \end{cases}$

a. $h(7)$ **b.** $h(0)$ **c.** $h(5)$

In Exercises 43–54, the domain of each piecewise function is $(-\infty, \infty)$.

a. *Graph each function.*

b. *Use your graph to determine the function's range.*

43. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

44. $f(x) = \begin{cases} x & \text{if } x < 0 \\ -x & \text{if } x \geq 0 \end{cases}$

45. $f(x) = \begin{cases} 2x & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$

46. $f(x) = \begin{cases} \frac{1}{2}x & \text{if } x \leq 0 \\ 3 & \text{if } x > 0 \end{cases}$

47. $f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ x - 3 & \text{if } x \geq -2 \end{cases}$

48. $f(x) = \begin{cases} x + 2 & \text{if } x < -3 \\ x - 2 & \text{if } x \geq -3 \end{cases}$

49. $f(x) = \begin{cases} 3 & \text{if } x \leq -1 \\ -3 & \text{if } x > -1 \end{cases}$

50. $f(x) = \begin{cases} 4 & \text{if } x \leq -1 \\ -4 & \text{if } x > -1 \end{cases}$

51. $f(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x < 1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$

52. $f(x) = \begin{cases} -\frac{1}{2}x^2 & \text{if } x < 1 \\ 2x + 1 & \text{if } x \geq 1 \end{cases}$

53. $f(x) = \begin{cases} 0 & \text{if } x < -4 \\ -x & \text{if } -4 \leq x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

54. $f(x) = \begin{cases} 0 & \text{if } x < -3 \\ -x & \text{if } -3 \leq x < 0 \\ x^2 - 1 & \text{if } x \geq 0 \end{cases}$

In Exercises 55–76, find and simplify the difference quotient

$$\frac{f(x + h) - f(x)}{h}, h \neq 0$$

for the given function.

55. $f(x) = 4x$

56. $f(x) = 7x$

57. $f(x) = 3x + 7$

58. $f(x) = 6x + 1$

59. $f(x) = x^2$

60. $f(x) = 2x^2$

61. $f(x) = x^2 - 4x + 3$

62. $f(x) = x^2 - 5x + 8$

63. $f(x) = 2x^2 + x - 1$

64. $f(x) = 3x^2 + x + 5$

65. $f(x) = -x^2 + 2x + 4$

66. $f(x) = -x^2 - 3x + 1$

67. $f(x) = -2x^2 + 5x + 7$

68. $f(x) = -3x^2 + 2x - 1$

69. $f(x) = -2x^2 - x + 3$

70. $f(x) = -3x^2 + x - 1$

71. $f(x) = 6$

72. $f(x) = 7$

73. $f(x) = \frac{1}{x}$

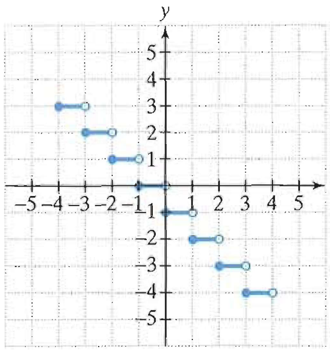
74. $f(x) = \frac{1}{2x}$

75. $f(x) = \sqrt{x}$

76. $f(x) = \sqrt{x - 1}$

Practice Plus

In Exercises 77–78, let f be defined by the following graph:



77. Find

$$\sqrt{f(-1.5) + f(-0.9)} - [f(\pi)]^2 + f(-3) \div f(1) \cdot f(-\pi).$$

78. Find

$$\sqrt{f(-2.5) - f(1.9)} - [f(-\pi)]^2 + f(-3) \div f(1) \cdot f(\pi).$$

A cellular phone company offers the following plans. Also given are the piecewise functions that model these plans. Use this information to solve Exercises 79–80.

Plan A

- *\$30 per month buys 120 minutes.*
- *Additional time costs \$0.30 per minute.*

$$C(t) = \begin{cases} 30 & \text{if } 0 \le t \le 120 \\ 30 + 0.30(t - 120) & \text{if } t > 120 \end{cases}$$

Plan B

- *\$40 per month buys 200 minutes.*
- *Additional time costs \$0.30 per minute.*

$$C(t) = \begin{cases} 40 & \text{if } 0 \le t \le 200 \\ 40 + 0.30(t - 200) & \text{if } t > 200 \end{cases}$$

79. Simplify the algebraic expression in the second line of the piecewise function for plan A. Then use point-plotting to graph the function.

80. Simplify the algebraic expression in the second line of the piecewise function for plan B. Then use point-plotting to graph the function.

In Exercises 81–82, write a piecewise function that models each cellular phone billing plan. Then graph the function.

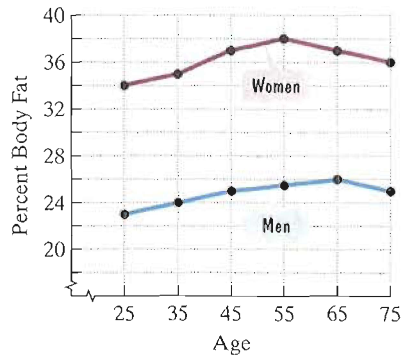
81. \$50 per month buys 400 minutes. Additional time costs \$0.30 per minute.

82. \$60 per month buys 450 minutes. Additional time costs \$0.35 per minute.

Application Exercises

With aging, body fat increases and muscle mass declines. The line graphs show the percent body fat in adult women and men as they age from 25 to 75 years. Use the graphs to solve Exercises 83–90.

Percent Body Fat in Adults



Source: Thompson et al., *The Science of Nutrition*, Benjamin Cummings, 2008

83. State the intervals on which the graph giving the percent body fat in women is increasing and decreasing.

84. State the intervals on which the graph giving the percent body fat in men is increasing and decreasing.

85. For what age does the percent body fat in women reach a maximum? What is the percent body fat for that age?

86. At what age does the percent body fat in men reach a maximum? What is the percent body fat for that age?

87. Use interval notation to give the domain and the range for the graph of the function for women.

88. Use interval notation to give the domain and the range for the graph of the function for men.

89. The function $p(x) = -0.002x^2 + 0.15x + 22.86$ models percent body fat, $p(x)$, where x is the number of years a person's age exceeds 25. Use the graphs to determine whether this model describes percent body fat in women or in men.

90. The function $p(x) = -0.004x^2 + 0.25x + 33.64$ models percent body fat, $p(x)$, where x is the number of years a person's age exceeds 25. Use the graphs to determine whether this model describes percent body fat in women or in men.

Here is the 2007 Federal Tax Rate Schedule X that specifies the tax owed by a single taxpayer.

If Your Taxable Income Is Over	But Not Over	The Tax You Owe Is	Of the Amount Over
\$ 0	\$ 7825	10%	\$ 0
\$ 7825	\$ 31,850	\$ 782.50 + 15%	\$ 7825
\$ 31,850	\$ 77,100	\$ 4386.25 + 25%	\$ 31,850
\$ 77,100	\$160,850	\$ 15,698.75 + 28%	\$ 77,100
\$160,850	\$349,700	\$ 39,148.75 + 33%	\$160,850
\$349,700	—	\$101,469.25 + 35%	\$349,700

The tax table on the previous page can be modeled by a piecewise function, where x represents the taxable income of a single taxpayer and $T(x)$ is the tax owed:

$$T(x) = \begin{cases} 0.10x & \text{if} \quad 0 < x \le 7825 \\ 782.50 + 0.15(x - 7825) & \text{if} \quad 7825 < x \le 31{,}850 \\ 4386.25 + 0.25(x - 31{,}850) & \text{if} \quad 31{,}850 < x \le 77{,}100 \\ 15{,}698.75 + 0.28(x - 77{,}100) & \text{if} \quad 77{,}100 < x \le 160{,}850 \\ \underline{\qquad ? \qquad} & \text{if} \quad 160{,}850 < x \le 349{,}700 \\ \underline{\qquad ? \qquad} & \text{if} \quad x > 349{,}700. \end{cases}$$

Use this information to solve Exercises 91–94.

91. Find and interpret $T(20{,}000)$.

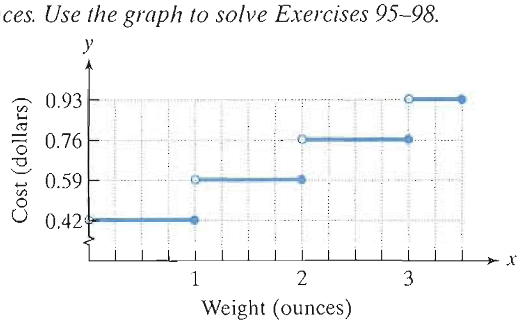
92. Find and interpret $T(50{,}000)$.

In Exercises 93–94, refer to the tax table on the previous page.

93. Find the algebraic expression for the missing piece of $T(x)$ that models tax owed for the domain $(160{,}850, 349{,}700]$.

94. Find the algebraic expression for the missing piece of $T(x)$ that models tax owed for the domain $(349{,}700, \infty)$.

The figure shows the cost of mailing a first-class letter, $f(x)$, as a function of its weight, x, in ounces, for weights not exceeding 3.5 ounces. Use the graph to solve Exercises 95–98.



Source: Lynn E. Baring, Postmaster, Inverness, CA

95. Find $f(3)$. What does this mean in terms of the variables in this situation?

96. Find $f(3.5)$. What does this mean in terms of the variables in this situation?

97. What is the cost of mailing a letter that weighs 1.5 ounces?

98. What is the cost of mailing a letter that weighs 1.8 ounces?

99. If $3.5 < x \le 4$, the cost of mailing a first-class letter jumps to \$1.34. The cost then increases by \$0.17 per ounce for weights not exceeding 13 ounces:

Weight Not Exceeding	Cost
5 ounces	\$1.51
6 ounces	\$1.68

etc.

Use this information to extend the graph shown above so that the function's domain is $(0, 13]$.

Writing in Mathematics

100. What does it mean if a function f is increasing on an interval?

101. Suppose that a function f whose graph contains no breaks or gaps on (a, c) is increasing on (a, b), decreasing on (b, c), and defined at b. Describe what occurs at $x = b$. What does the function value $f(b)$ represent?

102. If you are given a function's equation, how do you determine if the function is even, odd, or neither?

103. If you are given a function's graph, how do you determine if the function is even, odd, or neither?

104. What is a piecewise function?

105. Explain how to find the difference quotient of a function f, $\dfrac{f(x + h) - f(x)}{h}$, if an equation for f is given.

Technology Exercises

106. The function

$$f(x) = -0.00002x^3 + 0.008x^2 - 0.3x + 6.95$$

models the number of annual physician visits, $f(x)$, by a person of age x. Graph the function in a $[0, 100, 5]$ by $[0, 40, 2]$ viewing rectangle. What does the shape of the graph indicate about the relationship between one's age and the number of annual physician visits? Use the TABLE or minimum function capability to find the coordinates of the minimum point on the graph of the function. What does this mean?

In Exercises 107–112, use a graphing utility to graph each function. Use a $[-5, 5, 1]$ by $[-5, 5, 1]$ viewing rectangle. Then find the intervals on which the function is increasing, decreasing, or constant.

108. $f(x) = x^3 - 6x^2 + 9x + 1$

108. $g(x) = |4 - x^2|$

109. $h(x) = |x - 2| + |x + 2|$

110. $f(x) = x^{\frac{1}{3}}(x - 4)$

111. $g(x) = x^{\frac{2}{3}}$

112. $h(x) = 2 - x^{\frac{2}{5}}$

113. a. Graph the functions $f(x) = x^n$ for $n = 2, 4$, and 6 in a $[-2, 2, 1]$ by $[-1, 3, 1]$ viewing rectangle.

b. Graph the functions $f(x) = x^n$ for $n = 1, 3$, and 5 in a $[-2, 2, 1]$ by $[-2, 2, 1]$ viewing rectangle.

c. If n is positive and even, where is the graph of $f(x) = x^n$ increasing and where is it decreasing?

d. If n is positive and odd, what can you conclude about the graph of $f(x) = x^n$ in terms of increasing or decreasing behavior?

e. Graph all six functions in a $[-1, 3, 1]$ by $[-1, 3, 1]$ viewing rectangle. What do you observe about the graphs in terms of how flat or how steep they are?

Critical Thinking Exercises

Make Sense? *In Exercises 114–117, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

114. My graph is decreasing on $(-\infty, a)$ and increasing on (a, ∞), so $f(a)$ must be a relative maximum.

115. This work by artist Scott Kim (1955–) has the same kind of symmetry as an even function.

"DYSLEXIA," 1981

116. I graphed

$$f(x) = \begin{cases} 2 & \text{if } x \neq 4 \\ 3 & \text{if } x = 4 \end{cases}$$

and one piece of my graph is a single point.

117. I noticed that the difference quotient is always zero if $f(x) = c$, where c is any constant.

118. Sketch the graph of f using the following properties. (More than one correct graph is possible.) f is a piecewise function that is decreasing on $(-\infty, 2)$, $f(2) = 0$, f is increasing on $(2, \infty)$, and the range of f is $[0, \infty)$.

119. Define a piecewise function on the intervals $(-\infty, 2]$, $(2, 5)$, and $[5, \infty)$ that does not "jump" at 2 or 5 such that one piece is a constant function, another piece is an increasing function, and the third piece is a decreasing function.

120. Suppose that $h(x) = \frac{f(x)}{g(x)}$. The function f can be even, odd, or neither. The same is true for the function g.

a. Under what conditions is h definitely an even function?

b. Under what conditions is h definitely an odd function?

Group Exercise

121. (For assistance with this exercise, refer to the discussion of piecewise functions beginning on page 169, as well as to Exercises 79–80.) Group members who have cellular phone plans should describe the total monthly cost of the plan as follows:

$_____ per month buys _____ minutes. Additional time costs $_____ per minute.

(For simplicity, ignore other charges.) The group should select any three plans, from "basic" to "premier." For each plan selected, write a piecewise function that describes the plan and graph the function. Graph the three functions in the same rectangular coordinate system. Now examine the graphs. For any given number of calling minutes, the best plan is the one whose graph is lowest at that point. Compare the three calling plans. Is one plan always a better deal than the other two? If not, determine the interval of calling minutes for which each plan is the best deal. (You can check out cellular phone plans by visiting www.point.com.)

Preview Exercises

Exercises 122–124 will help you prepare for the material covered in the next section.

122. If $(x_1, y_1) = (-3, 1)$ and $(x_2, y_2) = (-2, 4)$, find $\frac{y_2 - y_1}{x_2 - x_1}$.

123. Find the ordered pairs (_____, 0) and (0, _____) satisfying $4x - 3y - 6 = 0$.

124. Solve for y: $3x + 2y - 4 = 0$.

Section 1.4 Linear Functions and Slope

Objectives

1. Calculate a line's slope.
2. Write the point-slope form of the equation of a line.
3. Write and graph the slope-intercept form of the equation of a line.
4. Graph horizontal or vertical lines.
5. Recognize and use the general form of a line's equation.
6. Use intercepts to graph the general form of a line's equation.
7. Model data with linear functions and make predictions.

Is there a relationship between literacy and child mortality? As the percentage of adult females who are literate increases, does the mortality of children under five decrease? **Figure 1.35** on the next page indicates that this is, indeed, the case. Each point in the figure represents one country.

The slope, approximately 0.02, indicates that for each increase of one part per million in carbon dioxide concentration, the average global temperature is increasing by approximately 0.02°F.

Now we write the line's equation in slope-intercept form.

$y - y_1 = m(x - x_1)$	Begin with the point-slope form.
$y - 57.06 = 0.02(x - 326)$	Either ordered pair, (326, 57.06) or (377, 58.11), can be (x_1, y_1). Let $(x_1, y_1) = (326, 57.06)$. From above, $m \approx 0.02$.
$y - 57.06 = 0.02x - 6.52$	Apply the distributive property: 0.02 (326) = 6.52.
$y = 0.02x + 50.54$	Add 57.06 to both sides and solve for y.

A linear function that models average global temperature, $f(x)$, for an atmospheric carbon dioxide concentration of x parts per million is

$$f(x) = 0.02x + 50.54.$$

b. If carbon dioxide concentration doubles from its preindustrial level of 280 parts per million, which many experts deem very likely, the concentration will reach 280×2, or 560 parts per million. We use the linear function to predict average global temperature at this concentration.

$f(x) = 0.02x + 50.54$	Use the function from part (a).
$f(560) = 0.02(560) + 50.54$	Substitute 560 for x.
$= 11.2 + 50.54 = 61.74$	

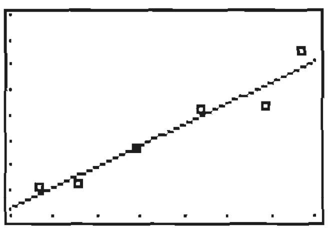
Our model projects an average global temperature of 61.74°F for a carbon dioxide concentration of 560 parts per million. Compared to the average global temperature of 58.11° for 2005 shown in **Figure 1.45(a)** on the previous page, this is an increase of

$$61.74°F - 58.11°F = 3.63°F.$$

This is consistent with a rise between 2°F and 5°F as predicted by the Intergovernmental Panel on Climate Change.

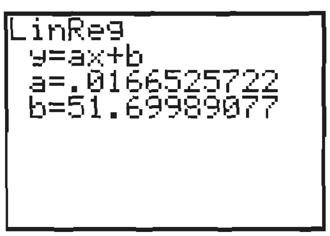
Technology

You can use a graphing utility to obtain a model for a scatter plot in which the data points fall on or near a straight line. After entering the data in **Figure 1.45(a)** on the previous page, a graphing utility displays a scatter plot of the data and the regression line, that is, the line that best fits the data.



[310, 380, 10] by [56.8, 58.4, 0.2]

Also displayed is the regression line's equation.



Check Point 9 Use the data points (317, 57.04) and (354, 57.64), shown, but not labeled, in **Figure 1.45(b)** on the previous page to obtain a linear function that models average global temperature, $f(x)$, for an atmospheric carbon dioxide concentration of x parts per million. Round m to three decimal places and b to one decimal place. Then use the function to project average global temperature at a concentration of 600 parts per million.

Exercise Set 1.4

Practice Exercises

In Exercises 1–10, find the slope of the line passing through each pair of points or state that the slope is undefined. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.

1. (4, 7) and (8, 10)
2. (2, 1) and (3, 4)
3. (−2, 1) and (2, 2)
4. (−1, 3) and (2, 4)
5. (4, −2) and (3, −2)
6. (4, −1) and (3, −1)
7. (−2, 4) and (−1, −1)
8. (6, −4) and (4, −2)
9. (5, 3) and (5, −2)
10. (3, −4) and (3, 5)

In Exercises 11–38, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

11. Slope = 2, passing through (3, 5)
12. Slope = 4, passing through (1, 3)
13. Slope = 6, passing through (−2, 5)
14. Slope = 8, passing through (4, −1)
15. Slope = −3, passing through (−2, −3)
16. Slope = −5, passing through (−4, −2)
17. Slope = −4, passing through (−4, 0)
18. Slope = −2, passing through (0, −3)

19. Slope $= -1$, passing through $\left(-\frac{1}{2}, -2\right)$

20. Slope $= -1$, passing through $\left(-4, -\frac{1}{4}\right)$

21. Slope $= \frac{1}{2}$, passing through the origin

22. Slope $= \frac{1}{3}$, passing through the origin

23. Slope $= -\frac{2}{3}$, passing through $(6, -2)$

24. Slope $= -\frac{3}{5}$, passing through $(10, -4)$

25. Passing through $(1, 2)$ and $(5, 10)$

26. Passing through $(3, 5)$ and $(8, 15)$

27. Passing through $(-3, 0)$ and $(0, 3)$

28. Passing through $(-2, 0)$ and $(0, 2)$

29. Passing through $(-3, -1)$ and $(2, 4)$

30. Passing through $(-2, -4)$ and $(1, -1)$

31. Passing through $(-3, -2)$ and $(3, 6)$

32. Passing through $(-3, 6)$ and $(3, -2)$

33. Passing through $(-3, -1)$ and $(4, -1)$

34. Passing through $(-2, -5)$ and $(6, -5)$

35. Passing through $(2, 4)$ with x-intercept $= -2$

36. Passing through $(1, -3)$ with x-intercept $= -1$

37. x-intercept $= -\frac{1}{2}$ and y-intercept $= 4$

38. x-intercept $= 4$ and y-intercept $= -2$

In Exercises 39–48, give the slope and y-intercept of each line whose equation is given. Then graph the linear function.

39. $y = 2x + 1$

40. $y = 3x + 2$

41. $f(x) = -2x + 1$

42. $f(x) = -3x + 2$

43. $f(x) = \frac{3}{4}x - 2$

44. $f(x) = \frac{3}{4}x - 3$

45. $y = -\frac{3}{5}x + 7$

46. $y = -\frac{2}{5}x + 6$

47. $g(x) = -\frac{1}{2}x$

48. $g(x) = -\frac{1}{3}x$

In Exercises 49–58, graph each equation in a rectangular coordinate system.

49. $y = -2$

50. $y = 4$

51. $x = -3$

52. $x = 5$

53. $y = 0$

54. $x = 0$

55. $f(x) = 1$

56. $f(x) = 3$

57. $3x - 18 = 0$

58. $3x + 12 = 0$

In Exercises 59–66,

a. *Rewrite the given equation in slope-intercept form.*

b. *Give the slope and y-intercept.*

c. *Use the slope and y-intercept to graph the linear function.*

59. $3x + y - 5 = 0$

60. $4x + y - 6 = 0$

61. $2x + 3y - 18 = 0$

62. $4x + 6y + 12 = 0$

63. $8x - 4y - 12 = 0$

64. $6x - 5y - 20 = 0$

65. $3y - 9 = 0$

66. $4y + 28 = 0$

In Exercises 67–72, use intercepts to graph each equation.

67. $6x - 2y - 12 = 0$

68. $6x - 9y - 18 = 0$

69. $2x + 3y + 6 = 0$

70. $3x + 5y + 15 = 0$

71. $8x - 2y + 12 = 0$

72. $6x - 3y + 15 = 0$

Practice Plus

In Exercises 73–76, find the slope of the line passing through each pair of points or state that the slope is undefined. Assume that all variables represent positive real numbers. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.

73. $(0, a)$ and $(b, 0)$

74. $(-a, 0)$ and $(0, -b)$

75. (a, b) and $(a, b + c)$

76. $(a - b, c)$ and $(a, a + c)$

In Exercises 77–78, give the slope and y-intercept of each line whose equation is given. Assume that $B \neq 0$.

77. $Ax + By = C$

78. $Ax = By - C$

In Exercises 79–80, find the value of y if the line through the two given points is to have the indicated slope.

79. $(3, y)$ and $(1, 4)$, $m = -3$

80. $(-2, y)$ and $(4, -4)$, $m = \frac{1}{3}$

In Exercises 81–82, graph each linear function.

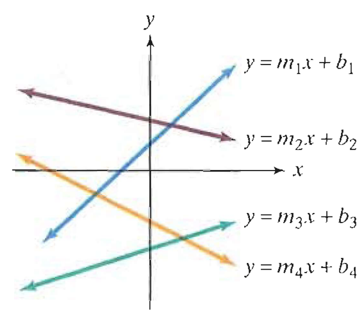
81. $3x - 4f(x) - 6 = 0$

82. $6x - 5f(x) - 20 = 0$

83. If one point on a line is $(3, -1)$ and the line's slope is -2, find the y-intercept.

84. If one point on a line is $(2, -6)$ and the line's slope is $-\frac{3}{2}$, find the y-intercept.

Use the figure to make the lists in Exercises 85–86.



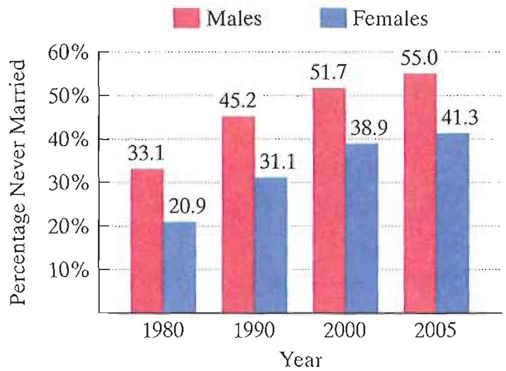
85. List the slopes m_1, m_2, m_3, and m_4 in order of decreasing size.

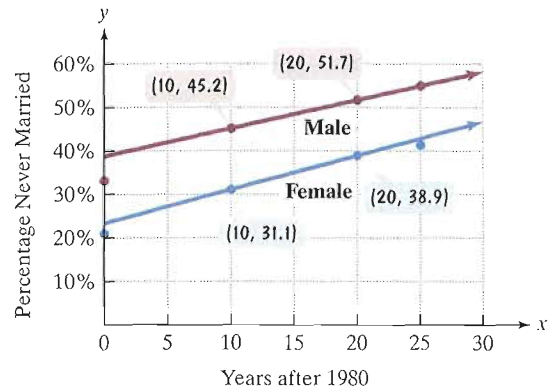
86. List the y-intercepts b_1, b_2, b_3, and b_4 in order of decreasing size.

Application Exercises

Americans are getting married later in life, or not getting married at all. In 2006, nearly half of Americans ages 25 through 29 were unmarried. The bar graph at the top of the next page shows the percentage of never-married men and women in this age group. The data are displayed on the next page as two sets of four points each, one scatter plot for the percentage of never-married American men and one for the percentage of never-married American women. Also shown for each scatter plot is a line that passes through or near the four points. Use these lines to solve Exercises 87–88.

Percentage of United States Population Never Married, Ages 25–29





Source: U.S. Census Bureau

87. In this exercise, you will use the blue line for the women shown on the scatter plot to develop a model for the percentage of never-married American females ages 25–29.

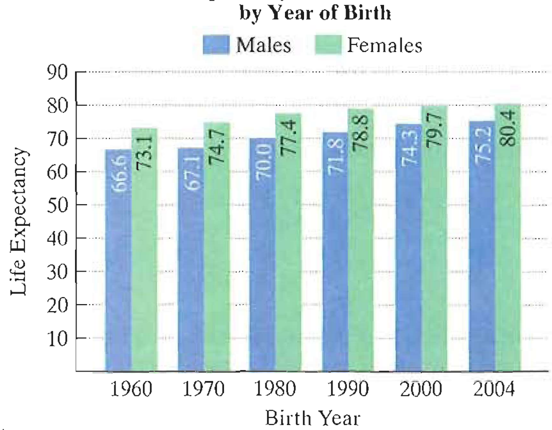
- **a.** Use the two points whose coordinates are shown by the voice balloons to find the point-slope form of the equation of the line that models the percentage of never-married American females ages 25–29, y, x years after 1980.
- **b.** Write the equation from part (a) in slope-intercept form. Use function notation.
- **c.** Use the linear function to predict the percentage of never-married American females, ages 25–29, in 2020.

88. In this exercise, you will use the red line for the men shown on the scatter plot to develop a model for the percentage of never-married American males ages 25–29.

- **a.** Use the two points whose coordinates are shown by the voice balloons to find the point-slope form of the equation of the line that models the percentage of never-married American males ages 25–29, y, x years after 1980.
- **b.** Write the equation from part (a) in slope-intercept form. Use function notation.
- **c.** Use the linear function to predict the percentage of never-married American males, ages 25–29, in 2015.

The bar graph gives the life expectancy for American men and women born in six selected years. In Exercises 89–90, you will use the data to obtain models for life expectancy and make predictions about how long American men and women will live in the future.

Life Expectancy in the United States, by Year of Birth



Source: National Center for Health Statistics

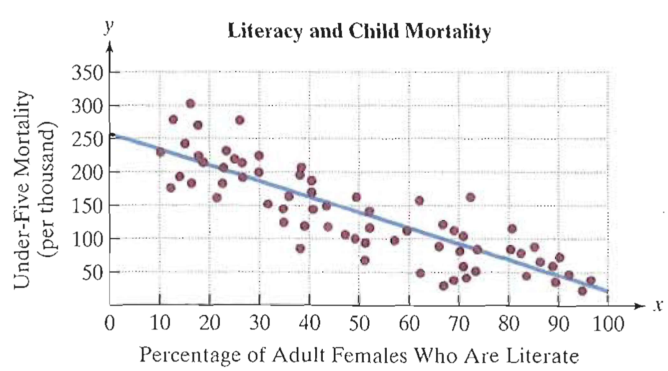
89. Use the data for males shown in the bar graph at the bottom of the previous column to solve this exercise.

- **a.** Let x represent the number of birth years after 1960 and let y represent male life expectancy. Create a scatter plot that displays the data as a set of six points in a rectangular coordinate system.
- **b.** Draw a line through the two points that show male life expectancies for 1980 and 2000. Use the coordinates of these points to write a linear function that models life expectancy, $E(x)$, for American men born x years after 1960.
- **c.** Use the function from part (b) to project the life expectancy of American men born in 2020.

90. Use the data for females shown in the bar graph at the bottom of the previous column to solve this exercise.

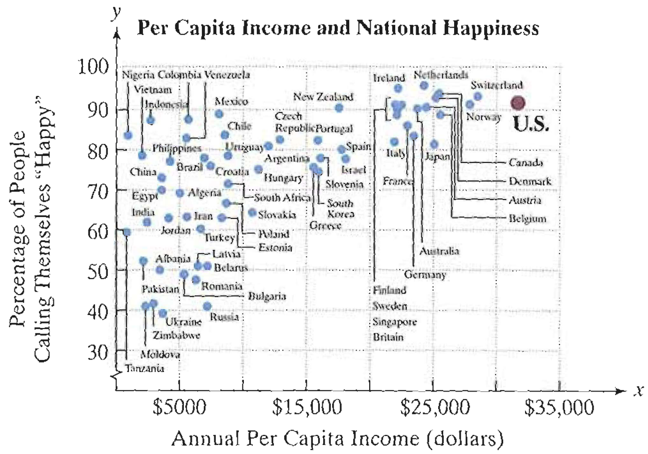
- **a.** Let x represent the number of birth years after 1960 and let y represent female life expectancy. Create a scatter plot that displays the data as a set of six points in a rectangular coordinate system.
- **b.** Draw a line through the two points that show female life expectancies for 1970 and 2000. Use the coordinates of these points to write a linear function that models life expectancy, $E(x)$, for American women born x years after 1960. Round the slope to two decimal places.
- **c.** Use the function from part (b) to project the life expectancy of American women born in 2020.

91. Shown, again, is the scatter plot that indicates a relationship between the percentage of adult females in a country who are literate and the mortality of children under five. Also shown is a line that passes through or near the points. Find a linear function that models the data by finding the slope-intercept form of the line's equation. Use the function to make a prediction about child mortality based on the percentage of adult females in a country who are literate.



Source: United Nations

92. Just as money doesn't buy happiness for individuals, the two don't necessarily go together for countries either. However, the scatter plot does show a relationship between a country's annual per capita income and the percentage of people in that country who call themselves "happy."



Source: Richard Layard, *Happiness: Lessons from a New Science,* Penguin, 2005

Draw a line that fits the data so that the spread of the data points around the line is as small as possible. Use the coordinates of two points along your line to write the slope-intercept form of its equation. Express the equation in function notation and use the linear function to make a prediction about national happiness based on per capita income.

Writing in Mathematics

93. What is the slope of a line and how is it found?

94. Describe how to write the equation of a line if the coordinates of two points along the line are known.

95. Explain how to derive the slope-intercept form of a line's equation, $y = mx + b$, from the point-slope form

$$y - y_1 = m(x - x_1).$$

96. Explain how to graph the equation $x = 2$. Can this equation be expressed in slope-intercept form? Explain.

97. Explain how to use the general form of a line's equation to find the line's slope and y-intercept.

98. Explain how to use intercepts to graph the general form of a line's equation.

99. Take another look at the scatter plot in Exercise 91. Although there is a relationship between literacy and child mortality, we cannot conclude that increased literacy causes child mortality to decrease. Offer two or more possible explanations for the data in the scatter plot.

Technology Exercises

Use a graphing utility to graph each equation in Exercises 100–103. Then use the TRACE *feature to trace along the line and find the coordinates of two points. Use these points to compute the line's slope. Check your result by using the coefficient of x in the line's equation.*

100. $y = 2x + 4$

101. $y = -3x + 6$

102. $y = -\frac{1}{2}x - 5$

103. $y = \frac{3}{4}x - 2$

104. Is there a relationship between wine consumption and deaths from heart disease? The table gives data from 19 developed countries.

Country	A	B	C	D	E	F	G (France)
Liters of alcohol from drinking wine, per person per year (x)	2.5	3.9	2.9	2.4	2.9	0.8	9.1
Deaths from heart disease, per 100,000 people per year (y)	211	167	131	191	220	297	71

Country	H	I	J	K	L	M	N	O	P	Q	R (U.S.)	S
(x)	0.8	0.7	7.9	1.8	1.9	0.8	6.5	1.6	5.8	1.3	1.2	2.7
(y)	211	300	107	167	266	227	86	207	115	285	199	172

Source: New York Times

a. Use the statistical menu of your graphing utility to enter the 19 ordered pairs of data items shown in the table.

b. Use the scatter plot capability to draw a scatter plot of the data.

c. Select the linear regression option. Use your utility to obtain values for a and b for the equation of the regression line, $y = ax + b$. You may also be given a **correlation coefficient**, r. Values of r close to 1 indicate that the points can be described by a linear relationship and the regression line has a positive slope. Values of r close to -1 indicate that the points can be described by a linear relationship and the regression line has a negative slope. Values of r close to 0 indicate no linear relationship between the variables. In this case, a linear model does not accurately describe the data.

d. Use the appropriate sequence (consult your manual) to graph the regression equation on top of the points in the scatter plot.

Critical Thinking Exercises

Make Sense? *In Exercises 105–108, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

105. The graph of my linear function at first increased, reached a maximum point, and then decreased.

106. A linear function that models tuition and fees at public four-year colleges from 2000 through 2006 has negative slope.

107. Because the variable m does not appear in $Ax + By + C = 0$, equations in this form make it impossible to determine the line's slope.

108. The federal minimum wage was \$5.15 per hour from 1997 through 2006, so $f(x) = 5.15$ models the minimum wage, $f(x)$, in dollars, for the domain $\{1997, 1998, 1999, \ldots, 2006\}$.

In Exercises 109–112, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

109. The equation $y = mx + b$ shows that no line can have a y-intercept that is numerically equal to its slope.

110. Every line in the rectangular coordinate system has an equation that can be expressed in slope-intercept form.

111. The graph of the linear function $5x + 6y - 30 = 0$ is a line passing through the point $(6, 0)$ with slope $-\frac{5}{6}$.

112. The graph of $x = 7$ in the rectangular coordinate system is the single point $(7, 0)$.

In Exercises 113–114, find the coefficients that must be placed in each shaded area so that the function's graph will be a line satisfying the specified conditions.

113. $\square x + \square y - 12 = 0$; x-intercept $= -2$; y-intercept $= 4$

114. $\square x + \square y - 12 = 0$; y-intercept $= -6$; slope $= \frac{1}{2}$

115. Prove that the equation of a line passing through $(a, 0)$ and $(0, b)(a \neq 0, b \neq 0)$ can be written in the form $\frac{x}{a} + \frac{y}{b} = 1$. Why is this called the *intercept form* of a line?

116. Excited about the success of celebrity stamps, post office officials were rumored to have put forth a plan to institute two new types of thermometers. On these new scales, $°E$ represents degrees Elvis and $°M$ represents degrees Madonna. If it is known that $40°E = 25°M$, $280°E = 125°M$, and degrees Elvis is linearly related to degrees Madonna, write an equation expressing E in terms of M.

Group Exercise

117. In Exercises 87–88, we used the data in a bar graph to develop linear functions that modeled the percentage of never-married American females and males, ages 25–29. For this group exercise, you might find it helpful to pattern your work after Exercises 87 and 88. Group members should begin by consulting an almanac, newspaper, magazine, or the Internet to find data that appear to lie approximately on or near a line. Working by hand or using a graphing utility, group members should construct scatter plots for the data that were assembled. If working by hand, draw a line that approximately fits the data in each scatter plot and then write its equation as a function in slope-intercept form. If using a graphing utility, obtain the equation of each regression line. Then use each linear function's equation to make predictions about what might occur in the future. Are there circumstances that might affect the accuracy of the prediction? List some of these circumstances.

Preview Exercises

Exercises 118–120 will help you prepare for the material covered in the next section.

118. Write the slope-intercept form of the equation of the line passing through $(-3, 1)$ whose slope is the same as the line whose equation is $y = 2x + 1$.

119. Write an equation in general form of the line passing through $(3, -5)$ whose slope is the negative reciprocal (the reciprocal with the opposite sign) of $-\frac{1}{4}$.

120. If $f(x) = x^2$, find

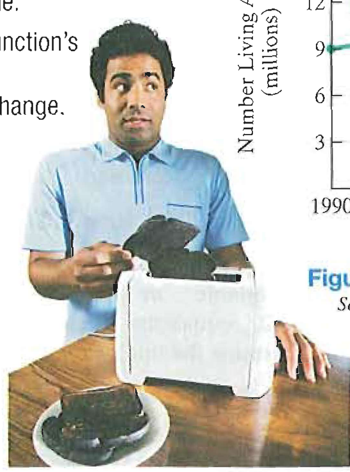
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1},$$

where $x_1 = 1$ and $x_2 = 4$.

Section 1.5 More on Slope

Objectives

1. Find slopes and equations of parallel and perpendicular lines.
2. Interpret slope as rate of change.
3. Find a function's average rate of change.



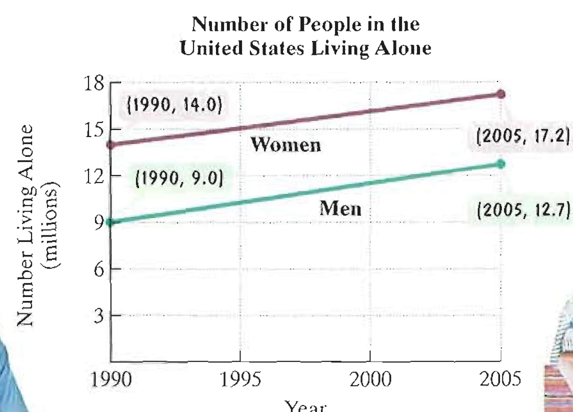


Figure 1.46
Source: U.S. Census Bureau

A best guess at the future of our nation indicates that the numbers of men and women living alone will increase each year. **Figure 1.46** shows that in 2005, 12.7 million men and 17.2 million women lived alone, an increase over the numbers displayed in the graph for 1990.

b. The ball's average velocity between 2 and 2.5 seconds is

$$\frac{\Delta s}{\Delta t} = \frac{s(2.5) - s(2)}{2.5 \text{ sec} - 2 \text{ sec}} = \frac{5(2.5)^2 - 5 \cdot 2^2}{0.5 \text{ sec}} = \frac{31.25 \text{ ft} - 20 \text{ ft}}{0.5 \text{ sec}} = 22.5 \text{ ft/sec}.$$

c. The ball's average velocity between 2 and 2.01 seconds is

$$\frac{\Delta s}{\Delta t} = \frac{s(2.01) - s(2)}{2.01 \text{ sec} - 2 \text{ sec}} = \frac{5(2.01)^2 - 5 \cdot 2^2}{0.01 \text{ sec}} = \frac{20.2005 \text{ ft} - 20 \text{ ft}}{0.01 \text{ sec}} = 20.05 \text{ ft/sec}.$$

In Example 6, observe that each calculation begins at 2 seconds and involves shorter and shorter time intervals. In calculus, this procedure leads to the concept of *instantaneous*, as opposed to *average*, velocity. Instantaneous velocity is discussed in the introduction to calculus in Chapter 11.

Check Point 6 The distance, $s(t)$, in feet, traveled by a ball rolling down a ramp is given by the function

$$s(t) = 4t^2,$$

where t is the time, in seconds, after the ball is released. Find the ball's average velocity from

a. $t_1 = 1$ second to $t_2 = 2$ seconds.

b. $t_1 = 1$ second to $t_2 = 1.5$ seconds.

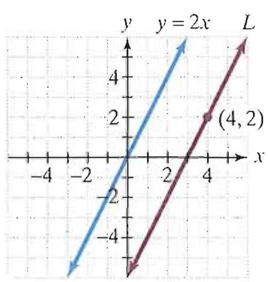
a. $t_1 = 1$ second to $t_2 = 1.01$ seconds.

Exercise Set 1.5

Practice Exercises

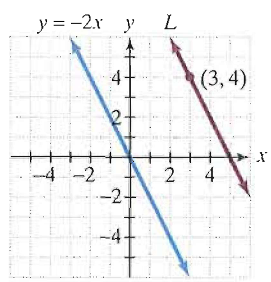
In Exercises 1–4, write an equation for line L in point-slope form and slope-intercept form.

1.



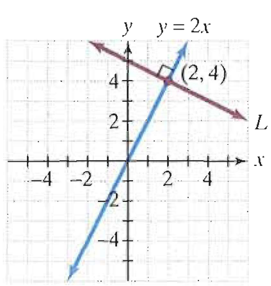
L is parallel to $y = 2x$.

2.



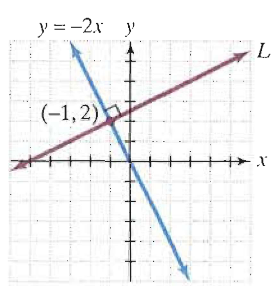
L is parallel to $y = -2x$.

3.



L is perpendicular to $y = 2x$.

4.



L is perpendicular to $y = -2x$.

In Exercises 5–8, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

5. Passing through $(-8, -10)$ and parallel to the line whose equation is $y = -4x + 3$

6. Passing through $(-2, -7)$ and parallel to the line whose equation is $y = -5x + 4$

7. Passing through $(2, -3)$ and perpendicular to the line whose equation is $y = \frac{1}{5}x + 6$

8. Passing through $(-4, 2)$ and perpendicular to the line whose equation is $y = \frac{1}{3}x + 7$

In Exercises 9–12, use the given conditions to write an equation for each line in point-slope form and general form.

9. Passing through $(-2, 2)$ and parallel to the line whose equation is $2x - 3y - 7 = 0$

10. Passing through $(-1, 3)$ and parallel to the line whose equation is $3x - 2y - 5 = 0$

11. Passing through $(4, -7)$ and perpendicular to the line whose equation is $x - 2y - 3 = 0$

12. Passing through $(5, -9)$ and perpendicular to the line whose equation is $x + 7y - 12 = 0$

In Exercises 13–18, find the average rate of change of the function from x_1 to x_2.

13. $f(x) = 3x$ from $x_1 = 0$ to $x_2 = 5$

14. $f(x) = 6x$ from $x_1 = 0$ to $x_2 = 4$

15. $f(x) = x^2 + 2x$ from $x_1 = 3$ to $x_2 = 5$

16. $f(x) = x^2 - 2x$ from $x_2 = 3$ to $x_2 = 6$

17. $f(x) = \sqrt{x}$ from $x_1 = 4$ to $x_2 = 9$

18. $f(x) = \sqrt{x}$ from $x_1 = 9$ to $x_2 = 16$